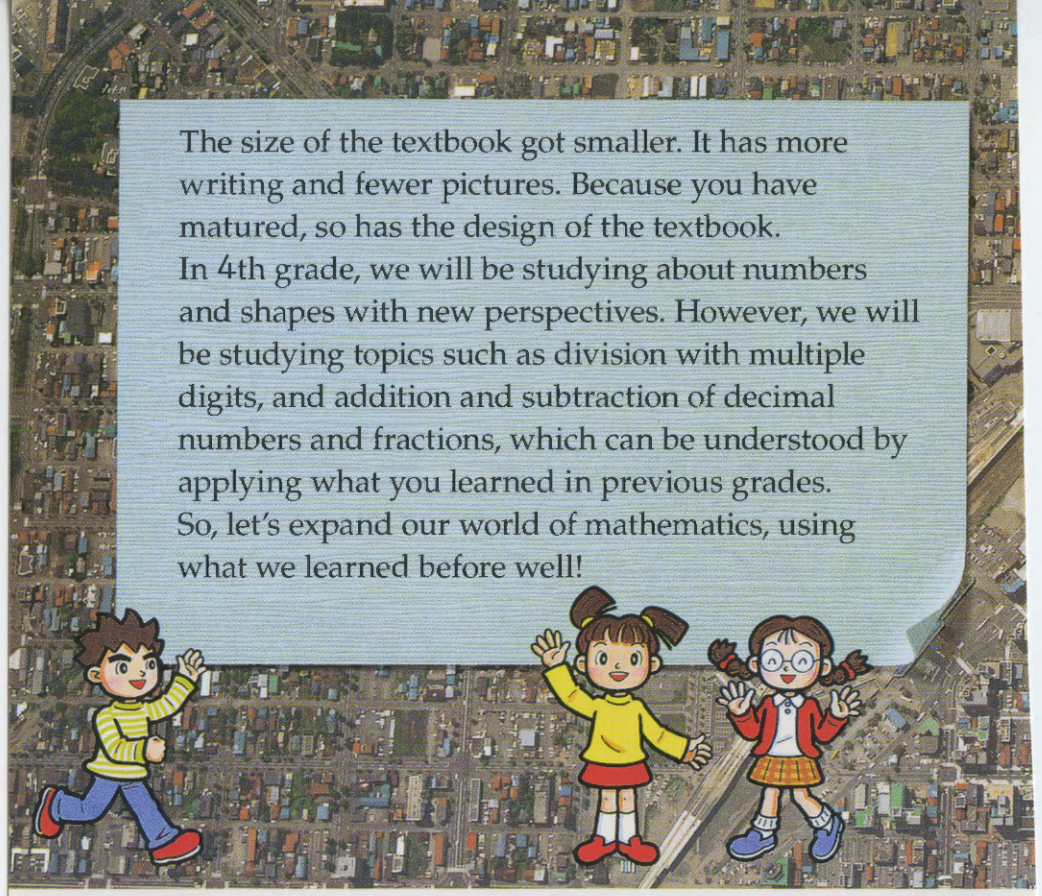
Mathematics 4A

for Elementary School



TOKYO SHOSEKI



The size of the textbook got smaller. It has more writing and fewer pictures. Because you have matured, so has the design of the textbook.
In 4th grade, we will be studying about numbers and shapes with new perspectives. However, we will be studying topics such as division with multiple digits, and addition and subtraction of decimal numbers and fractions, which can be understood by applying what you learned in previous grades.
So, let's expand our world of mathematics, using what we learned before well!

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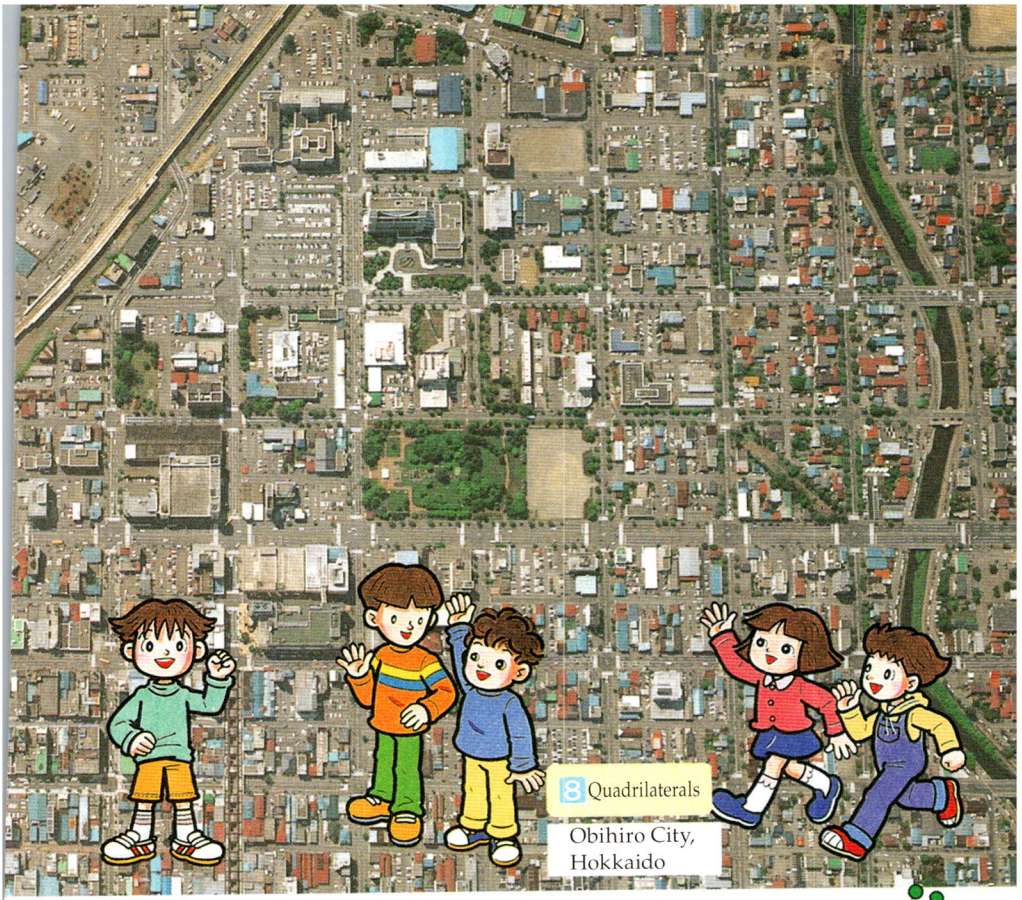

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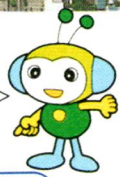








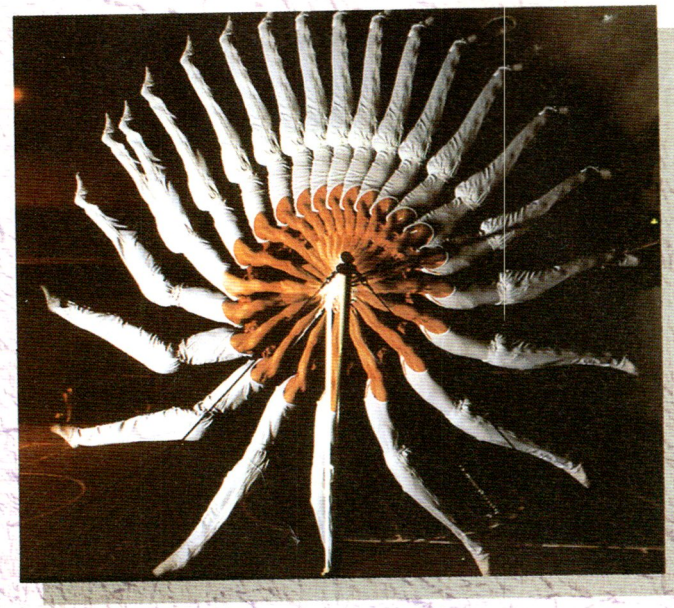




8 Quadrilaterals Osaka



Germany 8 Quadrilaterals



2 Angles



Tokyo



8 Quadrilaterals

1 Large Numbers



1 The System of Large Numbers

▶ ***Oku*** （億）

1 The population of Japan is shown on the right. Let's find out how to read the number!

125257061 people
(March 31, 1997)

Hokkaido
5692055 people

Miyagi Prefecture
2324066 people

The Metropolis of Tokyo
11574931 people

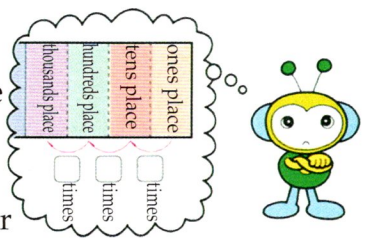
The population of Japan is more than 1000 *man*, isn't it?

Japan
125257061 people

Let's investigate large numbers larger than one thousand *man*!

1 What digit is in the thousand *man*s place?

2 Look at the digit "1" in the place to the left of the thousand *man*s place. How many times is it larger than one thousand *man*?



The number that is 10 times one thousand *man* is called **one *oku***(1億)and is written as **100000000**. Also, the number that is 10 times one *oku* is called **ten *oku***(10億), the number that is 10 times ten *oku* is called **one hundred *oku***(100億). And the number that is 10 times one hundred *oku* is called **one thousand *oku***(1000億).

			1	2	5	2	5	7	0	6	1
thousand *okus* place	hundred *okus* place	ten *okus* place	one *okus* place	thousand *mans* place	hundred *mans* place	ten *mans* place	one *mans* place	thousands place	hundreds place	tens place	ones place

3 How much more than 1 *oku* is the population of Japan?

125257061 people can be read in the following way. "One *oku* two thousand five hundred twenty five *man* seven thousand sixty-one" people.

4 Please write the following numbers in the below:

	oku (億)	*man* (万)	
ten *oku* (10億)			
one hundred *oku* (100億)			
one thousand *oku* (1000億)			

① Please read the following numbers.

(1) 813476151 (2) 320050601070

▶ *Cho* (兆)

2 The 1998 government budget for the protection of the environment in Japan is shown in the box below.
Let's think about how to read the number!



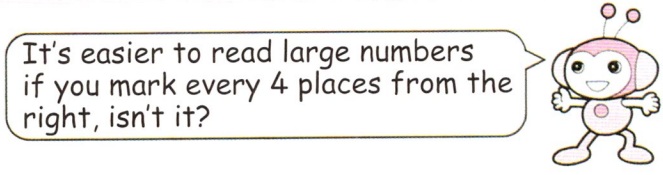
2728700000000 yen

In Japan, the number that is 10 times one thousand *oku* is called **one *cho*** (1兆) and is written as 1000000000000.
Also, the number that is 10 times one *cho* is called **ten *cho*** (10兆), the number that is 10 times ten *cho* is called **one hundred *cho*** (100兆), and the number that is 10 times one hundred *cho* is called **one thousand *cho*** (1000兆).

			2	7	2	8	7	0	0	0	0	0	0	0	0
hundreds	tens	ones		thousands	hundreds	tens	ones	thousands	hundreds	tens	ones	thousands	hundreds	tens	ones
cho (兆)				*oku* (億)				*man* (万)							

1 How many one *cho* and how many one *oku* do you need to make the number above?

2728700000000 yen can be read as follows:
"Two *cho* seven thousand two hundred eighty-seven *oku*" yen.



① Please read the following numbers.

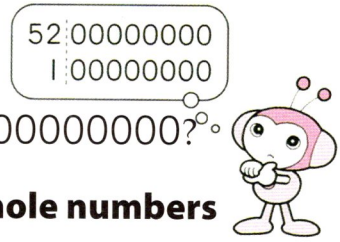
(1) 5840000560000 (2) 70300430200000

② Please write the following numbers.

(1) Seven hundred five *cho* four thousand nine hundred eighty *oku*

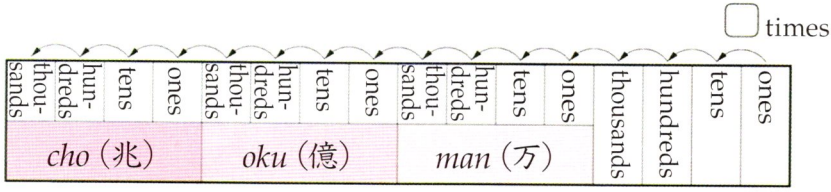
(2) A number made of 3 one *cho* and 75 one *oku*.

③ How many one *oku* do you need to make 5200000000?



The number system for whole numbers

3 Look at the table below and investigate the number system for whole numbers!

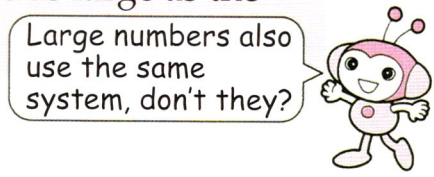


1 Each time you move to the left one place, how many times larger is the new place value?

2 Each time you move to the right one place, how many times smaller is the new place value?

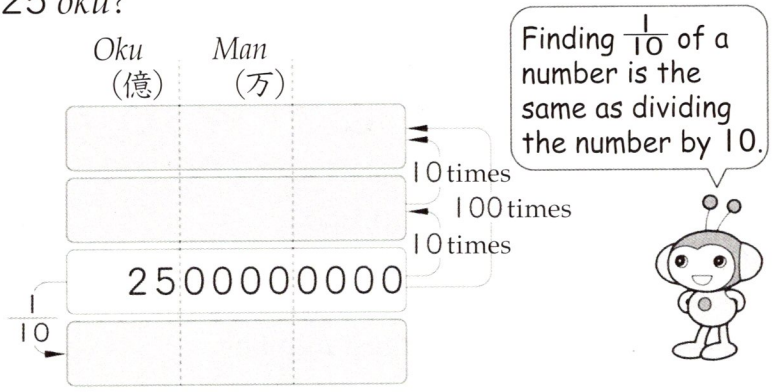
3 How many times as large as one *oku* is one *cho*?

! For any place in a whole number, the place value on the left is 10 times as large as the place value on the right.



▶ Numbers that are 10 times or $\frac{1}{10}$ as large and their place value

4 What number is 10 times and 100 times as large as 25 *oku*? Also, what number is $\frac{1}{10}$ of 25 *oku*?



Let's find out what happens to the digits and their place value when a number becomes 10 times or $\frac{1}{10}$ as large!

1 When 25 *oku* becomes 10 times as large, what happen to its digits?

2 When 25 *oku* becomes $\frac{1}{10}$ as large, what happen to its digits?

When you make a whole number 10 times as large, each digit moves up one place. Also, when you make a whole number $\frac{1}{10}$ as large, each digit moves down one place.

1 What numbers are these?

(1) 10 times 70 *oku*

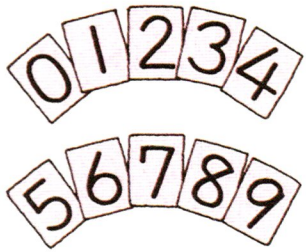
(2) 10 times 5000 *oku*

(3) $\frac{1}{10}$ of 80 *oku*

(4) $\frac{1}{10}$ of 3 *cho*

How to express whole numbers

5 The numbers 0 to 9 are written on cards, one on each card.
Let's make different 10-digit whole numbers!
Remember you can use each card only once.



1 Please make the largest number.

2 Please make the smallest number.

You can express any whole number, no matter how big it is, using these 10 digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.

Story of Math

Numbers Larger Than *Cho*

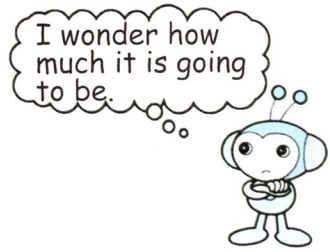
There are numbers larger than *cho*(兆). 10 times one thousand *cho* is called "one *kei*(1京) and numbers continue.

Sai 載	Sei 正	Kan 澗	Ko 溝	Jo 穰	Shi 秭	Gai 垓	京	兆	Oku 億	万	千	百	十	一
	Goku 極	Gogasha 恒河沙		Asogi 阿僧祇		Nayuta 那由他		Fukashigi 不可思議		Muryotaisu 無量大数				

2 Multiplication

1 We are going to buy 139 sketchbooks. Each sketchbook costs 275 yen.
What is the total price?

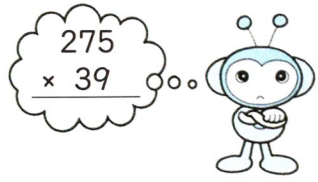




1 Please write a math sentence.

Math Sentence

Let's think about how to calculate!



How to calculate 275×139

$$
\begin{array}{r}
275 \\
\times\ 139 \\
\hline
2475 \\
825 \\
275 \\
\hline
38225
\end{array}
\qquad
\begin{array}{l}
\cdots\cdots 275 \times\ \ 9 = \ \ 2475 \\
\cdots\cdots 275 \times\ 30 = \ \ 8250 \\
\cdots\cdots 275 \times 100 = 27500
\end{array}
$$

2 Why was 275 moved two places to the left?

$275 \times 139 = 38225$ Answer 38225 yen

The answer in multiplication is called the **product**.

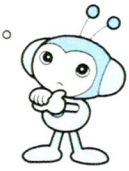
1 (1) $\begin{array}{r} 384 \\ \times\ 563 \end{array}$ (2) $\begin{array}{r} 776 \\ \times\ 892 \end{array}$ (3) $\begin{array}{r} 407 \\ \times\ 935 \end{array}$ (4) $\begin{array}{r} 950 \\ \times\ 413 \end{array}$

Ideas for multiplication calculation

2 Let's think about a better way to calculate 784×403!

$$\begin{array}{r} 784 \\ \times\ 403 \\ \hline 2352 \\ 3136 \\ \hline 315952 \end{array}$$

$$\begin{array}{r} 784 \\ \times\ 403 \\ \hline 2352 \\ 000 \\ 3136 \\ \hline 315952 \end{array}$$



1 (1) $\begin{array}{r} 274 \\ \times\ 608 \\ \hline \end{array}$ (2) $\begin{array}{r} 796 \\ \times\ 408 \\ \hline \end{array}$ (3) $\begin{array}{r} 208 \\ \times\ 305 \\ \hline \end{array}$ (4) $\begin{array}{r} 930 \\ \times\ 502 \\ \hline \end{array}$

3 Let's think about a better way to calculate 5400×320!

$$\begin{array}{r} 5400 \\ \times\ 320 \\ \hline 108 \\ 162 \\ \hline 1728000 \end{array}$$

$$\begin{aligned} 5400 \times 320 &= 54 \times 100 \times 32 \times 10 \\ &= 54 \times 32 \times 100 \times 10 \\ &= 54 \times 32 \times 1000 \end{aligned}$$



When you multiply numbers that end with one or more 0's, first multiply them as if there were no 0's. Then, add the 0's you omitted to the right of the product.

1 (1) 690×6700 (2) 14000×250

Practice

① Please read the following numbers.

(1) 54019001763 (2) 80024000052061 4

② Please order the following numbers from smallest to greatest.

(1) (9985640, 110054000, 32546310)

(2) (123685000, 124570365, 123489254)

③ Using the digits 0 to 9 only once, make a number as close to 10 *oku* as you can.

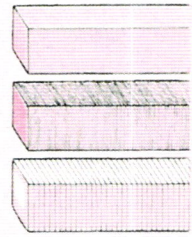
④ Instead of multiplying 658 by 309, I accidentally multiplied by 307. How is this different from the correct product?

Challenge

Length of 1 *Oku* . . .

What is the total length if you line up 1 *oku* of something?

	Length of 1 *oku* of them
0.1 *mm* thick paper	
1 *mm* thick cardboard	
1 *cm* thick book	



Distance from Tokyo to Atami ... 105*km*
Distance from Tokyo to Hakata ... 1176*km*

Check

1 Please answer the following questions about the number to the right.

4003867300620

(1) What is the place value of the left-most digit?

(2) The digit 8 in the number shows 8 of what?

(3) How many times as large is the number 3 in the fourth place from the left as the number 3 in the eighth place from the left?

(4) Please read the number.

2 Please write the following numbers using numerals.

(1) One *cho* seven thousand five hundred *oku* nine thousand two hundred fifty-one *man*

(2) 126 one *oku* and 79 one *man* together

(3) 1 smaller than ten *oku*

3 Please calculate

(1) 637×395 (2) 283×504 (3) 670×900

End of the Unit

◎ Write about something that you understood and thought was good.

◎ Write about something that you want to study in the future.

Let's write about these things in your notebook!



Review · 1

① Please calculate.

(1) $6\overline{)84}$ (2) $2\overline{)57}$ (3) $7\overline{)73}$ (4) $3\overline{)97}$

(5) $3\overline{)60}$ (6) $6\overline{)71}$ (7) $2\overline{)68}$ (8) $5\overline{)80}$

(9) $4\overline{)176}$ (10) $8\overline{)830}$ (11) $6\overline{)105}$ (12) $7\overline{)980}$

(13) $5\overline{)748}$ (14) $7\overline{)362}$ (15) $3\overline{)693}$ (16) $9\overline{)184}$

② Please calculate.

(1) $1.7 + 0.2$ (2) $6.3 + 1.5$ (3) $0.4 + 0.7$

(4) $5.4 - 4.3$ (5) $7.4 - 5.1$ (6) $1.4 - 0.6$

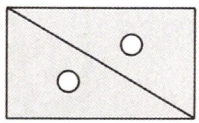
③ Please calculate.

(1) $\frac{2}{5} + \frac{2}{5}$ (2) $\frac{1}{7} + \frac{3}{7}$ (3) $\frac{3}{8} + \frac{5}{8}$

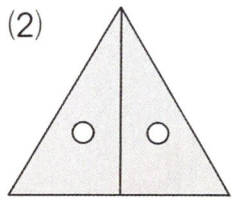
(4) $\frac{7}{9} - \frac{4}{9}$ (5) $\frac{5}{6} - \frac{1}{6}$ (6) $1 - \frac{2}{3}$

④ The pictures below show how two set-squares are put together. What kind of shapes are they?

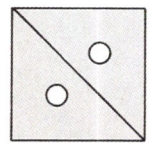
(1)



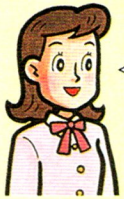
(2)



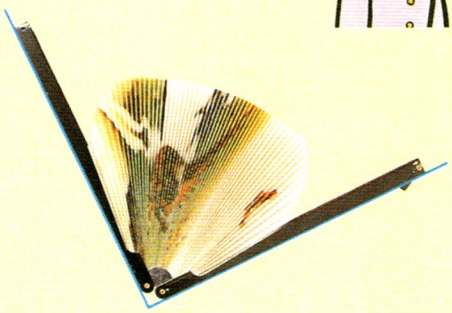
(3)

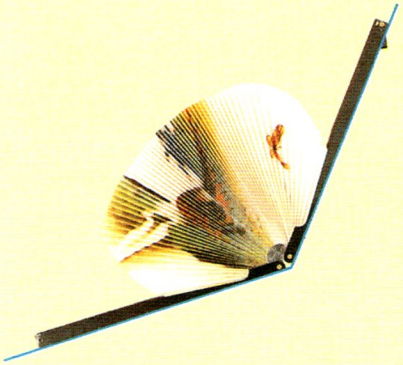


2 Angles



Which fan is open the widest?





3 angles

angles and a little more

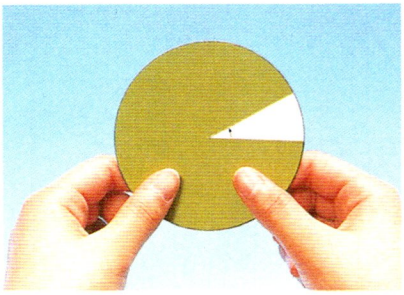


We cannot measure these angles that are formed by the fans exactly with a set-square.

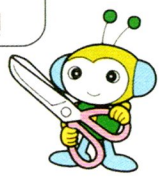
Sayuri

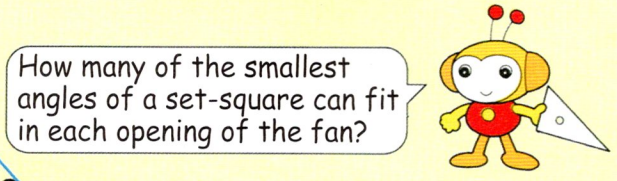
Angles formed by rotation

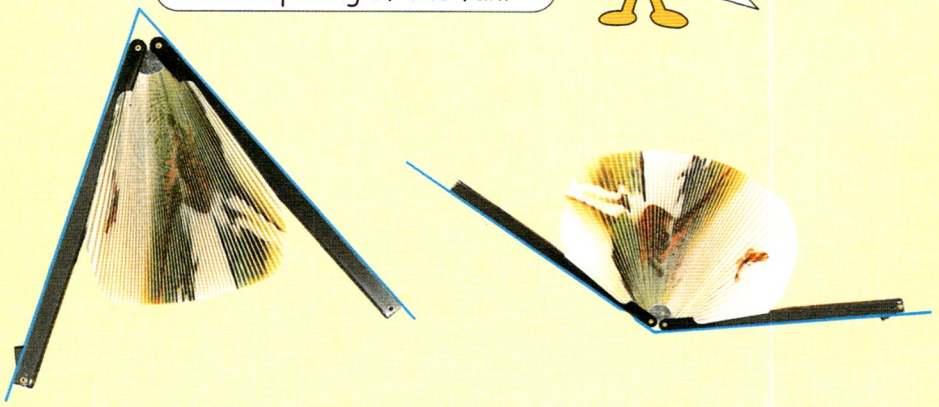
1 Put together two cardboard circles as shown below and form many angles by rotating one circle.



Let's cut out the circles on page 106 and make many different angles!



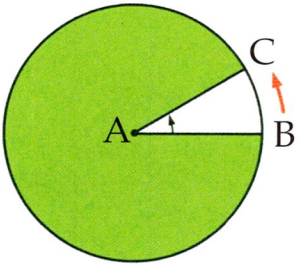


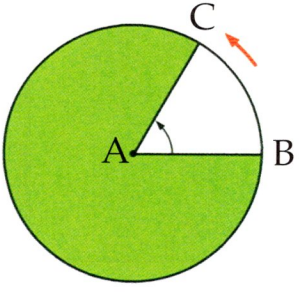


☐ angles

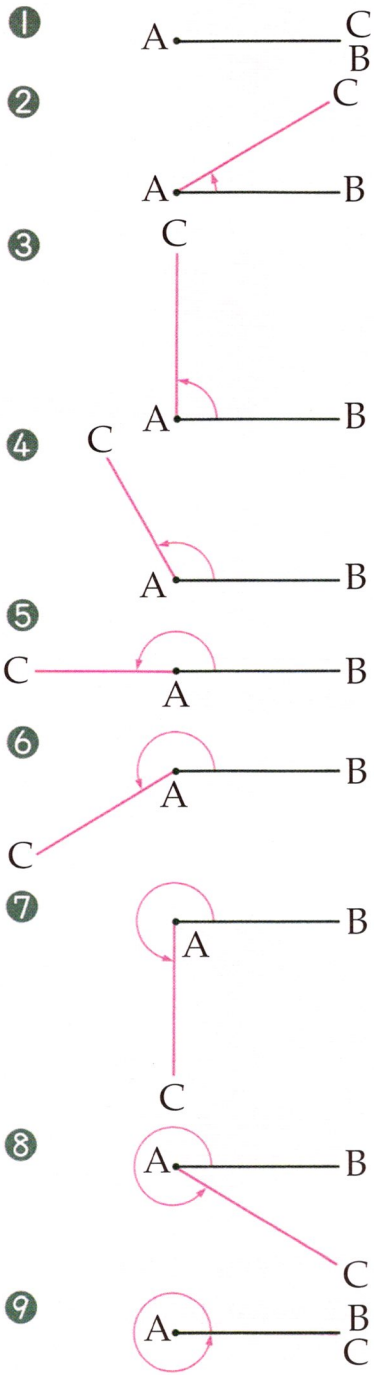
☐ angles and a little more

Let's investigate how to express the size of angles!





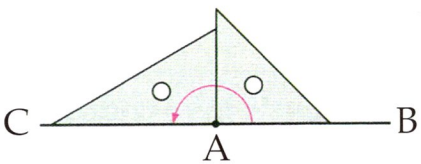
1 How does the size of the angle change as line segment AC moves in the direction of the arrow?



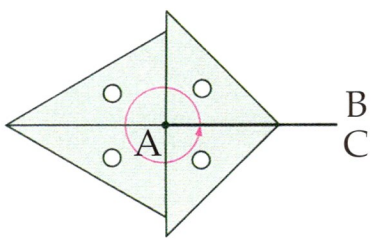
2 Which of the angles from ❶ through ❾ is a right angle?

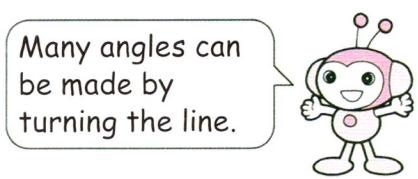
3 How many right angles are in angles ❺, ❼, and ❾?

An angle formed by a half turn is two right angles as in angle ❺.



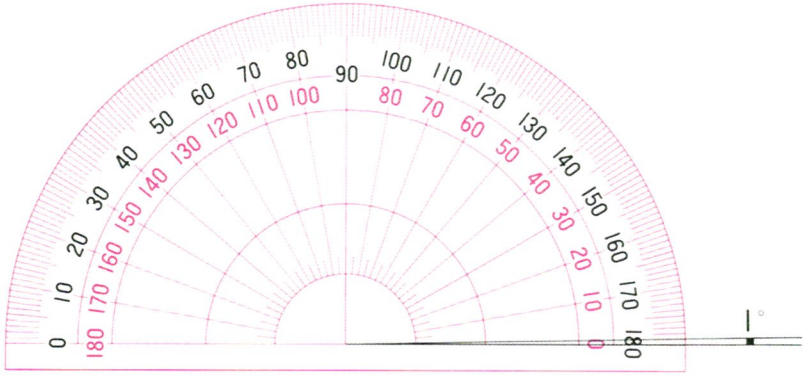
The angle formed by a full turn is four right angles as in angle ❾.





Measuring angles

2 In order to measure angles, we use a protractor. Let's investigate how measuring scales are marked on the protractor!

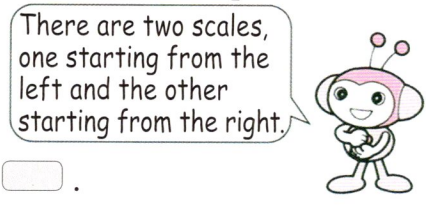


When you divide a right angle into 90 equal parts, each part is called 1 **degree** and is written as 1°. A degree is a unit used to express angles.

1 right angle = 90°

1 How big is the smallest interval on the protractor?

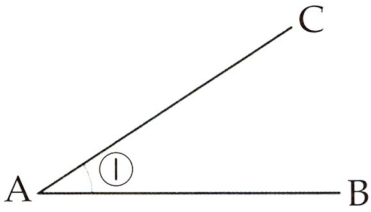
2 From 0° to how many degrees is the protractor marked?



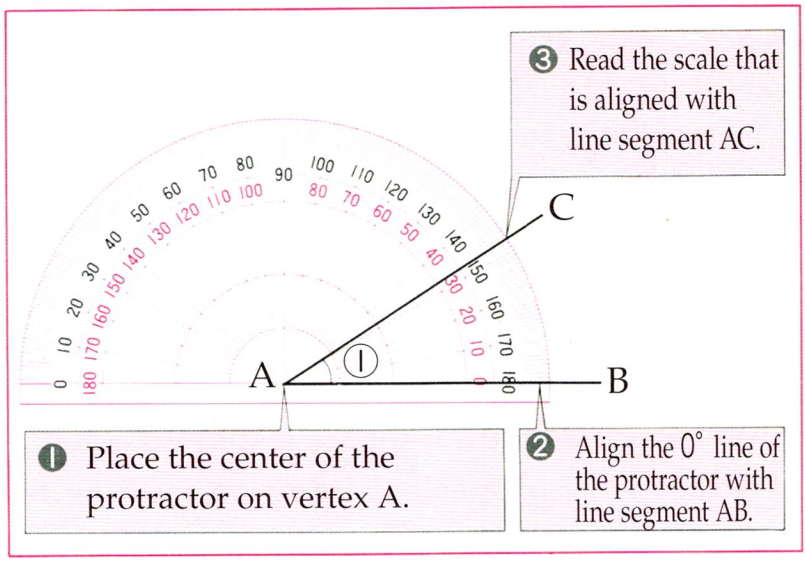
1 Write numbers in the ☐.

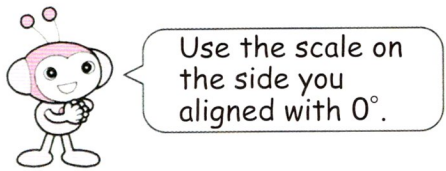
(1) The angle formed by a half turn ..... 2 right angles = ☐ degrees

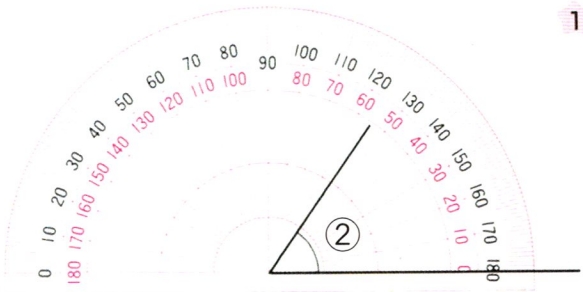
(2) The angle formed by a full turn ..... 4 right angles = ☐ degrees



3 Measure angle ① with a protractor.

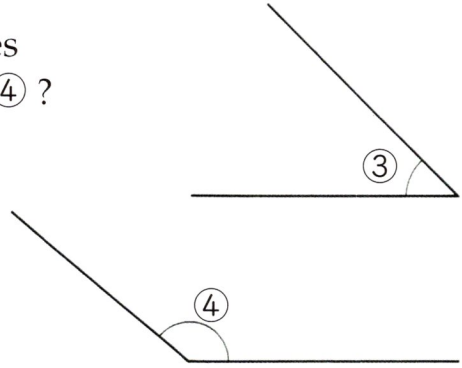




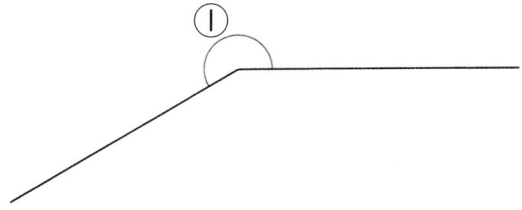


1 Think about how to measure angle ② .

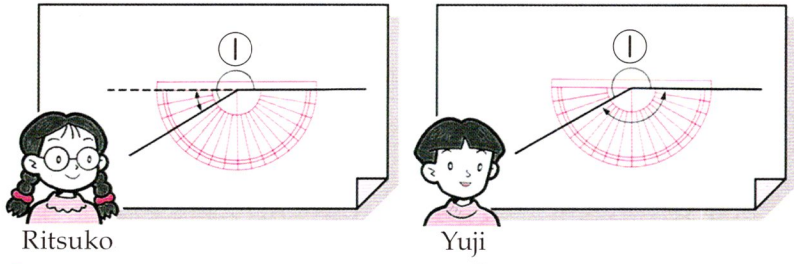
① How many degrees are angles ③ and ④ ? First estimate, and then measure.



4 Think of a way to measure angle ①.



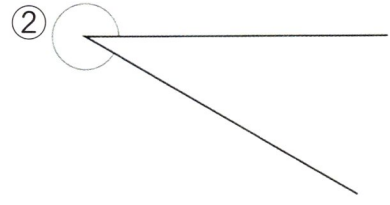
1 Explain Ritsuko's and Yuji's ways of measuring.

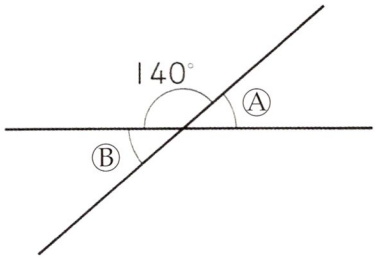


Ritsuko Yuji

2 How many degrees is angle ① ?

① How many degrees is angle ② ?



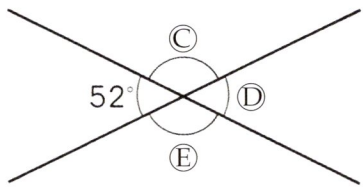


5 Compare opposite angles Ⓐ and Ⓑ.

1 Measure angles Ⓐ and Ⓑ with a protractor.

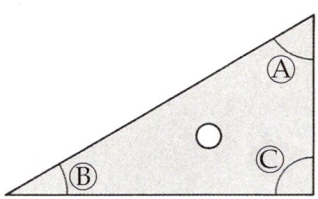
2 Find the measure of angles Ⓐ and Ⓑ by calculation.

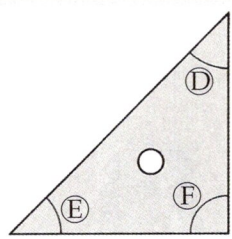
1 How many degrees are angles Ⓒ, Ⓓ, and Ⓔ, respectively?



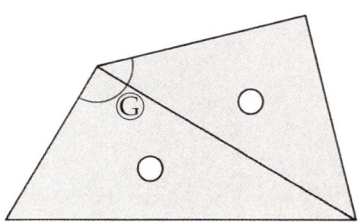
▶ Angles of set-squares

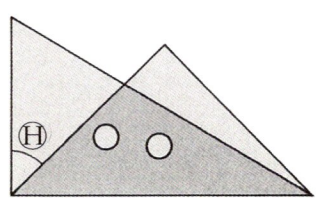
6 Let's investigate the angles of set-squares!





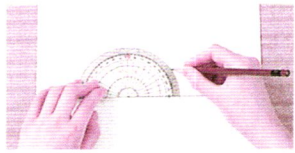
1 How many degrees are angles Ⓖ and Ⓗ that are formed by putting two set-squares together?

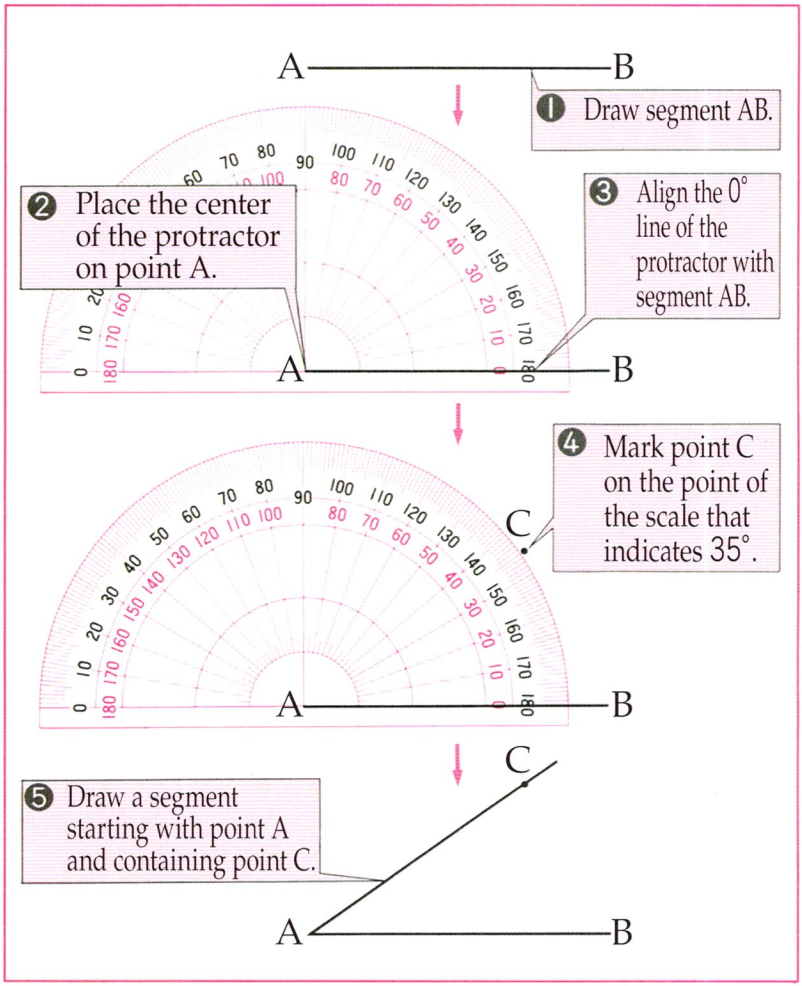




How to Draw Angles

7 Let's draw a 35° angle using a protractor.





1. Draw the following three angles: 72°, 110°, and 165°.
2. Think of a way to draw a 260° angle.

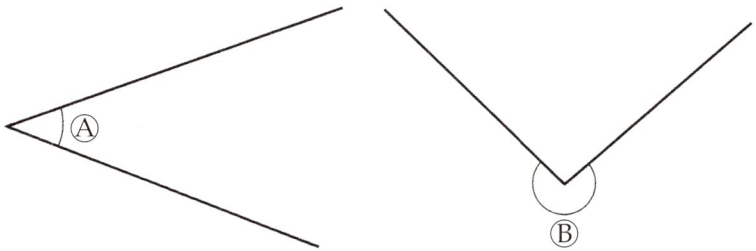
Check

1 Fill in the ☐ with the appropriate numbers.

(1) 1 right angle = ☐ degrees.

(2) An angle formed by 1 complete turn = ☐ degrees.

2 Measure angles Ⓐ and Ⓑ.

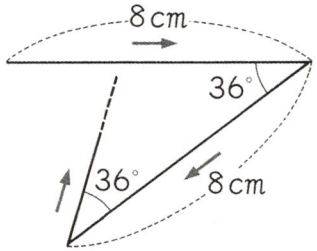


3 Draw the angles below.

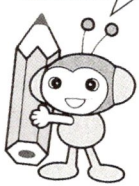
(1) 50° (2) 83° (3) 125° (4) 190°

Challenge

Let's Draw Many Shapes!

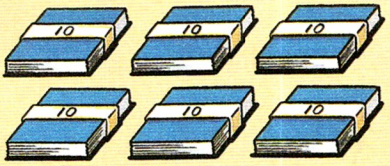


As shown on the left, first draw a segment that is 8 cm long. Next, draw a 36° angle from its end point. Then repeat the process over and over. What kind of shape do you get?



What happens if you change the angle to 45° or 30°, instead of 36°?

3 Division





There are 60 pieces of origami paper.



Minoru

If each person gets 5 papers, how many friends can share them?

Math Sentence



Ritsuko

If each person gets 20 papers, how many friends can share them?

The divisor is a two-digit number, isn't it?



Let's find the way to divide by two-digit numbers!

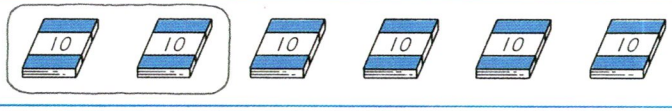
1 Division by a Multiple of 10

1 There are 60 pieces of origami paper. When you give 20 papers to each person, how many people can share the origami papers?

Math Sentence

? Let's think about how to calculate!

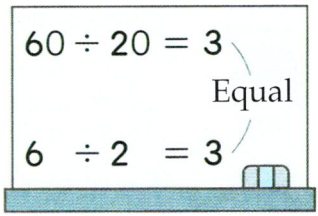
1 How many 20's are in 60?



$60 \div 20 =$ ☐ Answer: ☐

2 How many groups of 10 are in 20? How many are in 60?

The answer for $60 \div 20$ can be determined by $6 \div 2$ if you think about the groups of 10 in 60 and 20.

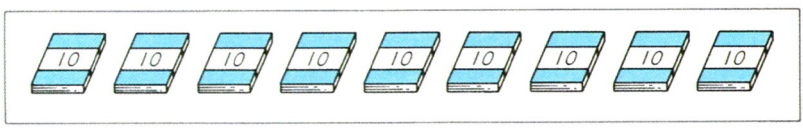


1 Please calculate mentally.

(1) $120 \div 30$ (2) $280 \div 70$ (3) $300 \div 60$

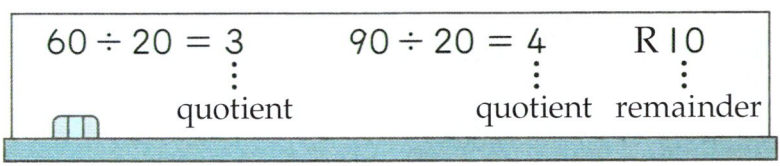
2 Let's think about how to calculate $90 \div 20$!

1 How many 20's are in 90?



$90 \div 20 = 4$ R 10

The answer in division is called the **quotient**.



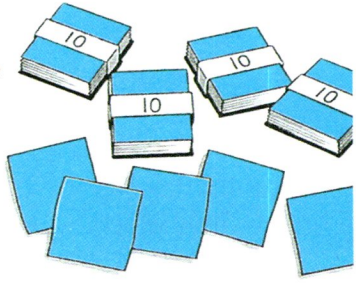
1 Please find the quotients and remainders mentally.

(1) $80 \div 30$ (2) $250 \div 70$ (3) $420 \div 80$

2 Division with Two-digit Numbers (1)

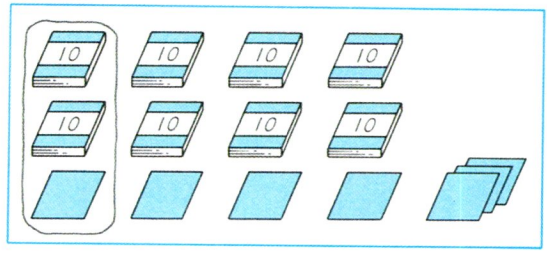
How to find the quotient (1)

1 There are 87 pieces of origami paper. If you give 21 papers to each person, how many people can share these papers?



Math Sentence ☐

1 How many 21's are in 87?



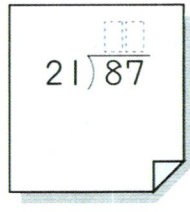
87 ÷ 21 = ☐ R ☐

Answer: ☐ people can share these papers and ☐ pieces of origami paper are left

? Let's find out how to calculate 87÷21!

2 In which place should the quotient be written?

3 Estimate the quotient by using 20 as the divisor instead of 21.



How to calculate 87÷21

❶ $21\overline{)87}$ → ❷ $\begin{array}{r}4\\21\overline{)87}\\84\end{array}$ → ❸ $\begin{array}{r}4\\21\overline{)87}\\84\\\hline3\end{array}$

Write the quotient 4 in the ones place.

Multiply 21 by 4.

Subtract 84 from 87.

4 Let's check the answer: 87÷21=4 R 3

$$21 \times 4 + 3 = 87$$

Divisor×Quotient+Remainder=Dividend

① Which one shows the quotient in the correct place?

(1) $\begin{array}{r}3\\23\overline{)76}\\69\\\hline7\end{array}$ (2) $\begin{array}{r}3\\23\overline{)76}\\69\\\hline7\end{array}$

② Please calculate and check your answers.

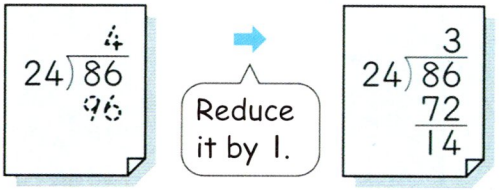
(1) $31\overline{)98}$ (2) $43\overline{)89}$ (3) $22\overline{)68}$ (4) $12\overline{)48}$

(5) $24\overline{)76}$ (6) $64\overline{)93}$ (7) $52\overline{)72}$ (8) $45\overline{)90}$

2 Let's think about how to calculate 86÷24!

1 Estimate the quotient using 20 instead of 24.

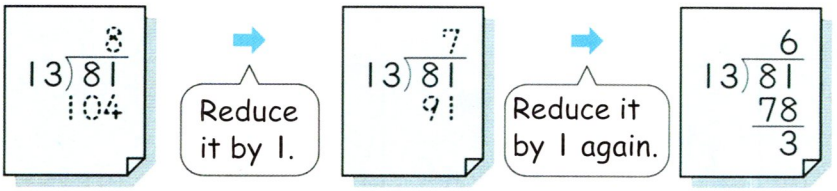
2 Can you subtract the product of 24 and 4 from 86?



If your estimated quotient is too large, reduce it by 1.

1 (1) 14)58 (2) 32)63 (3) 22)61 (4) 13)48

3 Let's think about how to calculate 81÷13!



When your estimated quotient is too large, reduce it one by one.

1 (1) 12)92 (2) 15)41 (3) 14)73 (4) 13)86

4 Let's think about how to calculate 275÷42!

1 In which place should you put the quotient?

2 Estimate the quotient by using 40 as the divisor instead of 42.

$$42\overline{)275}$$

$$\begin{array}{r} 6 \\ 42\overline{)275} \\ \underline{252} \\ 23 \end{array}$$



1 (1) $34\overline{)205}$ (2) $73\overline{)528}$ (3) $62\overline{)320}$

(4) $23\overline{)149}$ (5) $63\overline{)551}$ (6) $86\overline{)653}$

(7) $24\overline{)165}$ (8) $35\overline{)276}$ (9) $58\overline{)405}$

5 Let's think about how to calculate 317÷32!

$$\begin{array}{r} 10 \\ 32\overline{)317} \\ 320 \end{array}$$

Reduce it by 1.

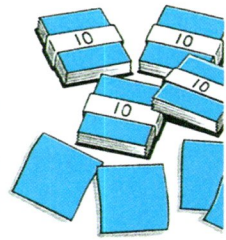
$$\begin{array}{r} 9 \\ 32\overline{)317} \\ \underline{288} \\ 29 \end{array}$$

In this case, you can use 9 as the quotient.

1 (1) $53\overline{)501}$ (2) $35\overline{)312}$ (3) $26\overline{)203}$

How to find the quotient (2)

6 There are 74 pieces of origami paper. When you give 18 papers to each person, how many people can share the papers?



Math Sentence

If you think about 10 instead of 18, you can estimate the quotient as 7...



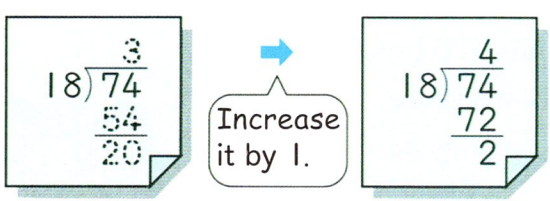
Naoko

1 Please calculate.

Makoto thinks that 18 and 20 are close. So he tried 3 as the quotient.

2 Is the quotient 3?

The remainder 20 is greater than the divisor 18, so use 4 as the quotient.



When your estimated quotient is too small, increase it one by one.

1 (1) 27)85 (2) 18)93 (3) 38)267 (4) 47)389

Practice 1

① Please calculate the following.

(1) 89 ÷ 40 (2) 51 ÷ 17 (3) 92 ÷ 36 (4) 90 ÷ 23

(5) 83 ÷ 28 (6) 93 ÷ 31 (7) 76 ÷ 16 (8) 48 ÷ 14

(9) 224 ÷ 30 (10) 140 ÷ 28 (11) 365 ÷ 43

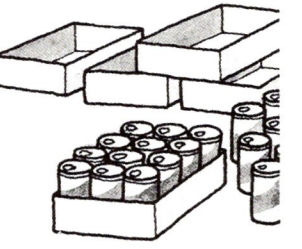
(12) 404 ÷ 58 (13) 815 ÷ 82 (14) 216 ÷ 36

② Let's calculate the following and check the answers!

(1) 70 ÷ 34 (2) 94 ÷ 12 (3) 81 ÷ 27 (4) 92 ÷ 27

(5) 128 ÷ 29 (6) 506 ÷ 53 (7) 426 ÷ 68

③ There are 86 cans of juice. If you put 12 cans in one box, how many boxes can you make? Do you have any remainder?



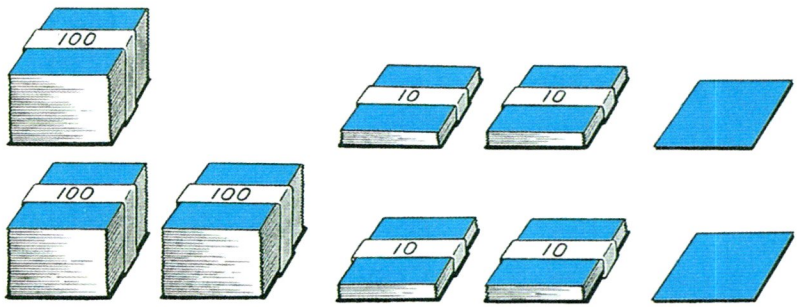
④ The school has 120 brooms. If 23 classrooms share the brooms equally, how many brooms will each class have? Is there any remainder?



⑤ There are 342 pieces of paper. If 38 friends share the papers equally, how many papers will each one get?

3 Division with Two-digit Numbers (2)

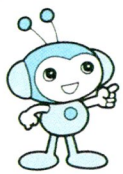
1 There are 342 pieces of origami paper. If 21 people share these papers equally, how many will each person receive?



Math Sentence

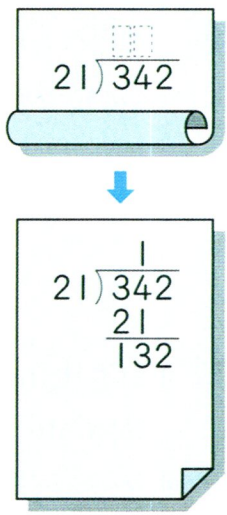
Let's think about how to calculate!

1. In which place should the quotient begin?
2. Calculate 34÷21 and write the quotient in the tens place.



When 21 people share 34 bundles of 10 papers . . .

3. Please calculate 132÷21 and find the ones place of the quotient.



How to calculate 342÷21

❶

$$\begin{array}{r} 1 \\ 21\overline{)342} \\ 21 \\ \hline 13 \end{array}$$

34 is bigger than 21, so the quotient starts in the tens place.

❷

$$\begin{array}{r} 16 \\ 21\overline{)342} \\ 21 \\ \hline 132 \\ 126 \\ \hline 6 \end{array}$$

1 (1) $43\overline{)926}$ (2) $52\overline{)875}$ (3) $36\overline{)684}$

(4) $28\overline{)883}$ (5) $32\overline{)603}$ (6) $35\overline{)630}$

2 Let's think about how to calculate 7876÷32!

1 In which place does the quotient start?

1 (1) $36\overline{)7725}$ (2) $21\overline{)3538}$

(3) $45\overline{)9835}$ (4) $23\overline{)8304}$

$$\begin{array}{r} 246 \\ 32\overline{)7876} \\ 64 \\ \hline 147 \\ 128 \\ \hline 196 \\ 192 \\ \hline 4 \end{array}$$

3 Let's think about more efficient ways to calculate 941÷23 and 9412÷23!



$$\begin{array}{r} 40 \\ 23\overline{)941} \\ 92 \\ \hline 21 \\ 00 \\ \hline 21 \end{array}$$

(1) $\begin{array}{r} 40 \\ 23\overline{)941} \\ 92 \\ \hline 21 \end{array}$ (2) $\begin{array}{r} 409 \\ 23\overline{)9412} \\ 92 \\ \hline 212 \\ 207 \\ \hline 5 \end{array}$

$$\begin{array}{r} 409 \\ 23\overline{)9412} \\ 92 \\ \hline 21 \\ 00 \\ \hline 212 \\ 207 \\ \hline 5 \end{array}$$



When the quotient has 0, you can simplify the calculation like the examples above.

1 (1) $35\overline{)734}$ (2) $14\overline{)572}$ (3) $42\overline{)8758}$

4 Let's think about how to calculate 1336÷36!

1 In which place does the quotient start?

How to calculate 1336÷36

❶
$$\begin{array}{r} 3 \\ 36\overline{)1336} \\ 108 \\ \hline 25 \end{array}$$

13 is smaller than 36, so use 133 as the initial dividend.

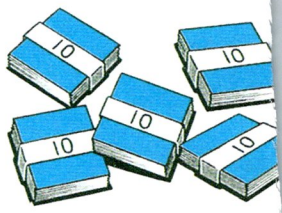
❷
$$\begin{array}{r} 37 \\ 36\overline{)1336} \\ 108 \\ \hline 256 \\ 252 \\ \hline 4 \end{array}$$

1 (1) $72\overline{)6239}$ (2) $94\overline{)6942}$ (3) $39\overline{)2886}$

4 Different Ways to Divide

▶ The properties of division

1 There are 150 pieces of origami paper. If each person gets 50 papers, how many people can share the papers?



Math Sentence ▶ ____

1 Let's think about the problem by imagining that the papers are in bundles of 10!

$150 \div 50 = 3$
$\downarrow \div 10 \quad \downarrow \div 10$ equal
$15 \div 5 = 3$

$150 \div 50 = 3$
$\uparrow \times 10 \quad \uparrow \times 10$ equal
$15 \div 5 = 3$

2 Let's think about the problem by imagining that the papers are in bundles of 5!

$150 \div 50 = 3$
$\downarrow \div 5 \quad \downarrow \div 5$ equal
$30 \div 10 = 3$

$150 \div 50 = 3$
$\uparrow \times 5 \quad \uparrow \times 5$ equal
$30 \div 10 = 3$

! In division, when you multiply (or divide) both the dividend and the divisor by the same number, the quotients will remain the same.

1 (1) $90 \div 30$ (2) $4800 \div 80$ (3) $800 \div 25$

▶ Simplifying the division method

2 Let's think about more efficient ways to calculate 24000÷500!

$$\begin{array}{r} 48 \\ 500\overline{)24000} \\ 20 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

When both the dividend and the divisor end with 0's, you can delete the same number of 0's from both before dividing.

① (1) $40\overline{)2800}$ (2) $300\overline{)7500}$ (3) $850\overline{)6800}$

3 Let's think about more efficient ways to calculate 2700÷400!

1 Which remainder is correct? Use the math sentence below to check.

(1) $\begin{array}{r} 6 \\ 400\overline{)2700} \\ 24 \\ \hline 3 \end{array}$ (2) $\begin{array}{r} 6 \\ 400\overline{)2700} \\ 24 \\ \hline 300 \end{array}$

Divisor×Quotient+Remainder=Dividend

When you simplify division by deleting some 0's at the end of the dividend and the divisor, you have to add on the same number of 0's to determine the correct remainder.

① (1) $600\overline{)8500}$ (2) $3000\overline{)14800}$

Practice 2

① Please calculate the following.

(1) $826 \div 42$ (2) $515 \div 14$ (3) $627 \div 27$

(4) $741 \div 13$ (5) $602 \div 35$ (6) $814 \div 16$

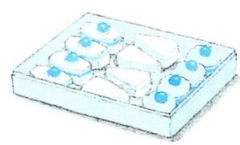
(7) $8939 \div 48$ (8) $3456 \div 75$ (9) $7061 \div 23$

② Please calculate the following in an efficient way.

(1) $28800 \div 4800$ (2) $180 \div 5$

③ When a number is divided by 54, the quotient is 18 and the remainder is 28. When you divide this number by 73, what is the answer?

④ You bought 12 cakes and had them put in a box. The total cost was 2200 yen. If the box costs 40 yen, how much did each cake cost?



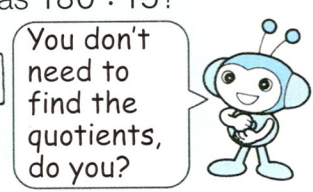
Challenge

Let's Find the Same Quotient!

$180 \div 15$ Can you find the division problem that has the same quotient as $180 \div 15$?

$2700 \div 180$ $360 \div 30$ $1000 \div 90$

$900 \div 750$ $1400 \div 120$ $60 \div 5$



Check

1 Find the errors and correct them.

(1)
$$\begin{array}{r} 120 \\ 47\overline{)564} \\ 47 \\ \hline 94 \\ 94 \\ \hline 0 \end{array}$$

(2)
$$\begin{array}{r} 32 \\ 32\overline{)9664} \\ 96 \\ \hline 64 \\ 64 \\ \hline 0 \end{array}$$

(3)
$$\begin{array}{r} 109 \\ 68\overline{)7510} \\ 68 \\ \hline 710 \\ 612 \\ \hline 98 \end{array}$$

2 We want to make the quotient less than 10. What number can you put in the tens place?

$74\overline{)7\square5}$

3 Please calculate the following.

(1) $32\overline{)98}$ (2) $35\overline{)70}$ (3) $75\overline{)504}$ (4) $16\overline{)112}$

(5) $47\overline{)843}$ (6) $34\overline{)689}$ (7) $26\overline{)6897}$ (8) $82\overline{)5176}$

Challenge

Let's Fill in the Boxes to Find the Message!

① $98 \div 32 =$ ☐ R ☐ → ○

② $281 \div 46 =$ ☐ R ☐ → ○

③ $433 \div 12 =$ ☐ R ☐ → ○

④ $4050 \div 19 =$ ☐ R ☐ → ○

⑤ $1222 \div 14 =$ ☐ R ☐ → ○

Fill in the circles using the key below.



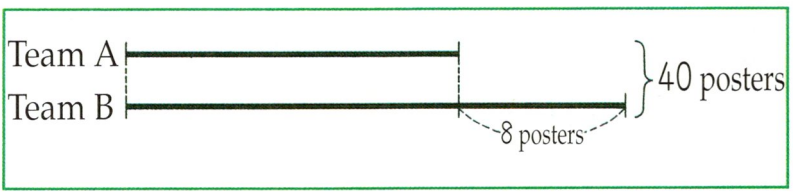
R 1 → O
R 2 → G
R 3 → D
R 4 → !
R 5 → O

Problem Solving

Sports Festival

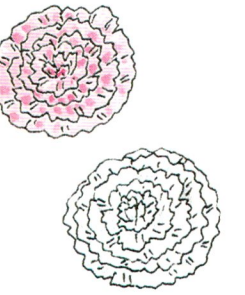


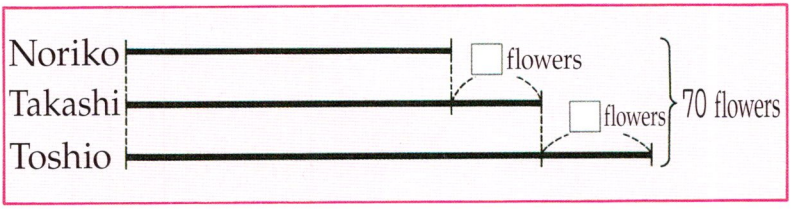
1 Teams A and B are making 40 posters for the sports festival. Team B will make 8 more than team A. How many posters will they each make?



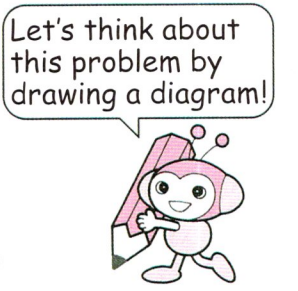
① Noriko, Takashi and Toshio are making 70 paper flowers together. Takashi will make 5 more flowers than Noriko. Toshio will make 6 more flowers than Takashi.

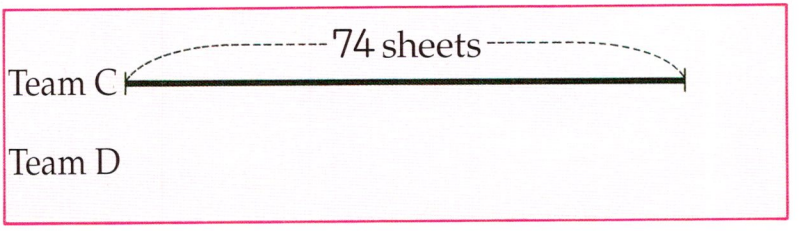
How many paper flowers will they each make?





2 Teams C and D are making paper flags together. Team C has 74 sheets of paper and Team D has 46 sheets. How many sheets of paper does Team C need to give to Team D if the two teams make the same number of flags?





4 How to Organize Data





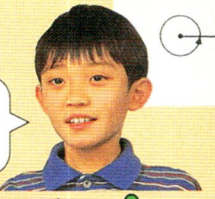


I wonder what types of injuries are happening.

Natsuko



Look at the table on page 46.



Where are they getting injured?

Keisuke



Let's use tally marks!

Kind of injury	number of people (people)
Scratch	
Bruise	
Cut	
Sprain	
Total	

Place	number of people (people)
Playground	
Gymnasium	
Classroom	
Hallway	
Total	



What types of injuries are likely to happen at each place?

Let's investigate how to organize data effectively!

1 Let's think about effective ways to show what types of injuries happen at each place!

1 Pay attention to the types of injuries and the places where they happened, and complete the table below.

Types and Places of Injuries (May) (people)

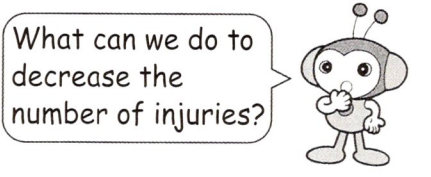
Kind of injury \ Place	Playground	Classroom	Gymnasium	Hallway	Total
Scratch					
Bruise					
Cut					
Sprain					
Total					Ⓐ

2 Calculate the total number of each type of injury and the places they happened.

3 What number goes in the cell marked Ⓐ?

4 Where and what type of injury happened most often?

5 Looking at the table, describe what you notice.



1 Organize the data on page 46 by paying attention to the grade levels and types of injuries.

Use for pages 43 and 44

Injury Survey (May)

Grade Level	Name	Type of Injury	Place
2	Tanaka	Scratch	Playground
4	Hayashi	Bruise	Gymnasium
5	Yokota	Cut	Classroom
5	Takahashi	Scratch	Classroom
2	Yamashita	Bruise	Playground
3	Nakamura	Scratch	Playground
5	Tani	Cut	Playground
1	Yamada	Scratch	Classroom
4	Takada	Sprain	Playground
2	Ishikawa	Bruise	Playground
1	Nagashima	Scratch	Playground
6	Nishimura	Cut	Classroom
2	Ishino	Bruise	Classroom
4	Uchimura	Scratch	Playground
3	Kobayashi	Scratch	Classroom
6	Ishida	Sprain	Hallway
1	Nakashima	Scratch	Playground
2	Hayashi	Bruise	Gymnasium
4	Ogawa	Scratch	Classroom
4	Murakami	Bruise	Classroom
2	Shimada	Scratch	Classroom
6	Takenaka	Cut	Classroom
3	Kawakami	Scratch	Playground
4	Aoki	Bruise	Playground
1	Oshima	Scratch	Gymnasium
3	Uchida	Sprain	Classroom
5	Yamaguchi	Scratch	Playground
4	Tachibana	Bruise	Hallway

Use for page 45

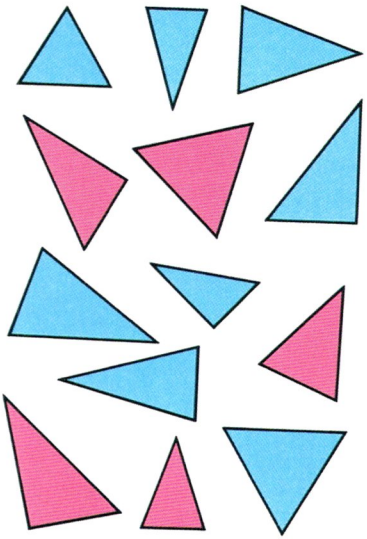
Library Usage

Number	Last week	This week
1	○	○
2	○	×
3	×	○
4	×	×
5	○	○
6	○	×
7	○	○
8	×	○
9	○	×
10	×	○
11	○	○
12	×	×
13	×	○
14	○	×
15	○	○
16	○	×
17	×	○
18	○	○
19	×	○
20	×	×
21	○	×
22	×	○
23	○	×
24	×	○
25	○	×
26	×	○
27	○	○
28	○	×
29	×	○
30	×	○
31	×	×
32	○	×

○ ··· Borrowed
× ··· Not Borrowed

Check

1 Looking at the shapes and colors of the triangles on the right, organize them in the table below.

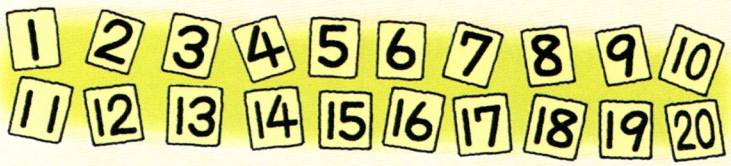


Shapes and Colors

Shape \ Color			Total
Total			

Challenge

Let's Organize Numbers in a Table!

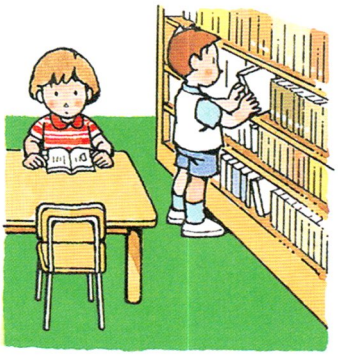


Check the numbers from 1 to 20 to see if they can be divided by 2 and 3. Then, organize the results in the table on the right.

Divisible or Not divisible

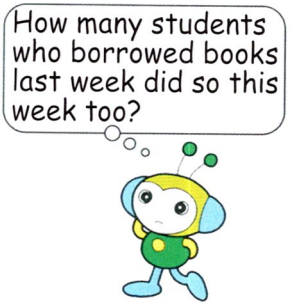
Divided by 2 \ Divided by 3	Divisible	Not divisible
Divisible		2, 4
Not divisible	3	1

2 Akira's class investigated how students in his class used the library last week and this week.



1 Looking at the table on page 46, complete the table below.

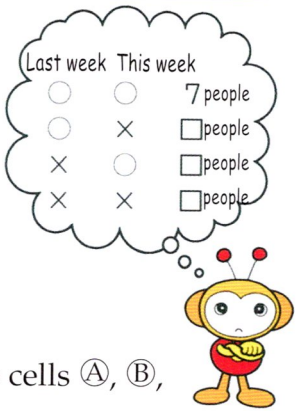
Last week	Borrowed	17 people
	Not Borrowed	
This week	Borrowed	
	Not Borrowed	



Let's think about an effective way to show how the library was used last week and this week!

Library Usage

This week \ Last week	Borrowed	Not Borrowed
Borrowed	Ⓐ 7	Ⓑ
Not Borrowed	Ⓒ	Ⓓ

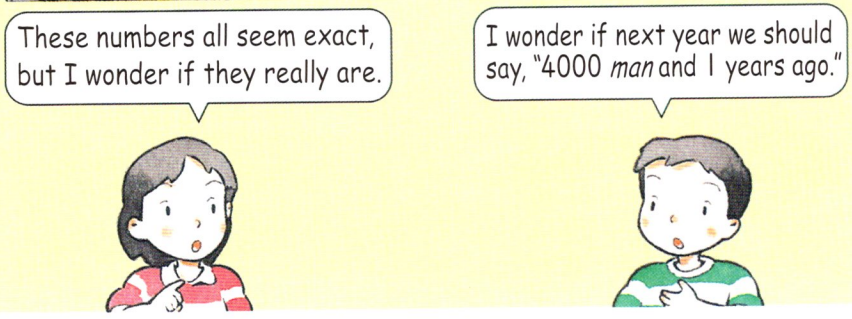


2 What kind of students are in the cells Ⓐ, Ⓑ, Ⓒ, and Ⓓ?

3 Fill in the numbers in cells Ⓑ, Ⓒ, and Ⓓ.

5 Approximate Numbers



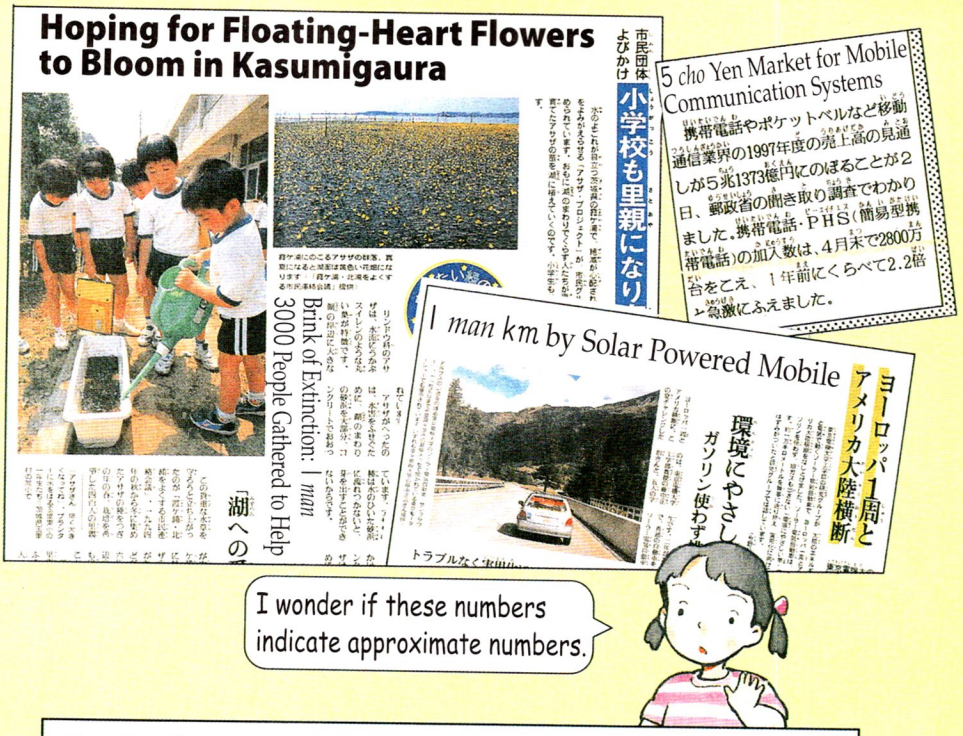


1 Approximate Numbers

▶ Approximate numbers

1 Hiroshi went to three soccer games.
Let's show the attendance at the games on the 8th and 15th, respectively, on the pictograph on the next page!





Attendance at Soccer Games (⚽ … 1 *man* (10 thousand) people)

1st	20214people	⚽⚽⚽⚽⚽⚽
8th	28418people	⚽⚽⚽⚽⚽⚽
15th	41756people	⚽⚽⚽⚽⚽⚽

1 About how many *man* (万)(10 thousand) visitors were there on the 8th and 15th?

2 Is 28418 closer to 2 *man* (20 thousand) or 3 *man* (30 thousand)?



Because 28418 is close to 3 *man*, we can consider this number about 3 *man*. "About 3 *man*" can also be said as "approximately 3 *man*." The number of visitors on the 8th was approximately 3 *man* (30 thousand) people.

3 About how many *man* (10 thousand) is 41756? Please look at the number line below.



"About 3 *man*" and "approximately 3 *man*" are called **approximate numbers**.

4 Complete the pictograph on page 49 by using approximate numbers of attendance. Please color the appropriate number of ⚽.

▶ Rounding

2 Let's think about how to show the populations of Higashiyama and Kitakawa Cities with approximate numbers!

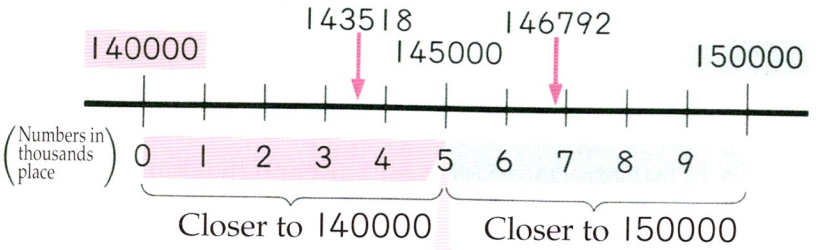
Population of Higashiyama and Kitakawa

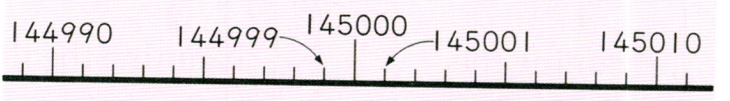
Higashiyama	146792
Kitakawa	143518



1 In order to find out if these numbers are closer to 14 *man* (140 thousand) or 15 *man* (150 thousand), which place should we examine?

2 What numbers for the thousands place will make the number approximately 14 *man* (140 thousand)? Also, what numbers for the thousands place will make the number approximately 15 *man* (150 thousand)?





When we indicate the numbers between 14 *man* (140 thousand) and 15 *man* (150 thousand) as approximate numbers to the nearest *man* (10 thousand), if the digit in thousands place is:0, 1, 2, 3, or 4, then round down to approximately 140000.
5, 6, 7, 8, or 9, then round up to approximately 150000.
The process for finding approximate numbers is called **rounding**.

① Which numbers would be 120000 when the digits in the thousands place are rounded?

Ⓐ 120438　　Ⓑ 126343　　Ⓒ 115000

② Round the digits in the thousands place.

(1) 36982　　(2) 120938　　(3) 527154

When we round 146792 to 150000 by rounding the digit in the thousands place, there are other ways to describe the process.

Round the number to the nearest *mans* (10 thousands) place.

1 4 6 7 9 2

Examine the thousands place.

Round the number to the second highest place.

1 4 6 7 9 2

Examine the third highest place.

③ Round the following numbers to the nearest *mans* (10 thousands) place.

(1) 368253　(2) 241493　(3) 504524　(4) 9557

④ Round the following numbers to the second highest place.

(1) 245260　　(2) 6835291　　(3) 45491836

2 Calculation with Approximate Numbers

1 Approximately how many thousand is the total number of school children in the three towns?

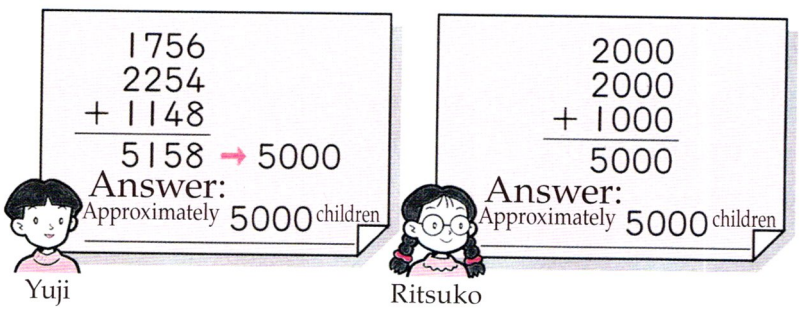


Numbers of School Children

Higashita town	1756 children
Nishikawa town	2254 children
Kitayama town	1148 children

? Let's think about a way to calculate when approximate numbers are used as answers!

1 Explain these two students' calculation methods.



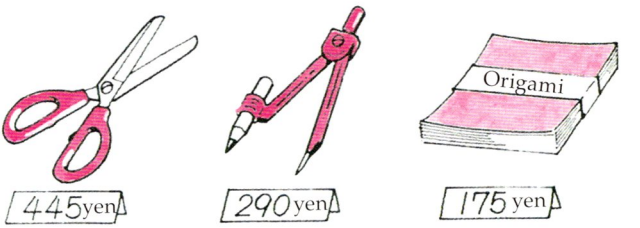
We call the answer in addition the **sum**, and the answer in subtraction the **difference**.

! To find an approximate sum or difference, first round each number to its indicated place and then calculate.

1 Find the answers rounded to the nearest *mans* (10 thousands) place.

(1) 122653 + 278546 (2) 98478 − 31486

2 Minoru is going to buy a pair of scissors, a compass, and a pack of origami paper. Can he buy all of them with 1000 yen?



1 Explain Minoru's way of estimating the price.

$$\begin{array}{r} 500 \\ 300 \\ +\ 200 \\ \hline 1000 \end{array}$$



Minoru

3 Sayuri is saving money every month to buy a 1000 yen book.

How much money should she save in July to buy the book?

Monthly Savings

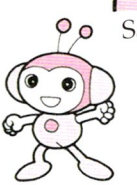
April···	213 yen
May ···	352 yen
June ···	326 yen

1 Explain Sayuri's way of estimating the amount.

$$\begin{array}{r} 200 \\ 300 \\ +\ 300 \\ \hline 800 \end{array}$$



Sayuri



Practice

① Round the following numbers by paying attention to the *mans* (10 thousands) place.

(1) 1254324 (2) 739831 (3) 1972100

② Round 67480 to the nearest hundreds place.

③ Round 362591 to the second highest place.

④ Appproximately how many thousand is the total population of Takeshi's town? What is the approximate difference in thousands between the male and female populations? Calculate after rounding the numbers first.

the total population of Takeshi's town

Male ……18176people

Female ……20713people

⑤ We are going to make a bar graph showing the lengths of the tunnels by using 1*cm* to indicate 1000*m*. Find the length of each bar by rounding the lengths of each tunnel to the nearest hundred.

Lengths of Tunnels

Name	Length (*m*)
Kanetsu Tunnel	11055
Tokyo Bay Aqua Tunnel	9583
Enasan Tunnel	8649
Shin-Kobe Tunnel	7175



Check

1 Which of the following can we show with approximate numbers?

Ⓐ Your body temperature when you are sick

Ⓑ The walking distance on a school trip

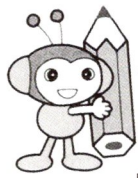
Ⓒ The time it takes to run 50m

2 Round the following numbers.

	78518	81704	695280
By paying attention to the thousands place			
By paying attention to the *man*s place			
Round to the second highest place			

Challenge

Let's Draw a Bar Graph with Approximate Numbers!



Find the student populations for your school and your neighboring schools and draw a bar graph showing the information.

❶ Use a 1cm length to represent every 100 students.

❷ Round the number to the nearest tens place.

❸ Calculate the length of each bar.

Number of Students

School Name	Actual Number (people)	Approximate Number (people)	Length of Bar

Review · 2

① Write the following numbers using numerals.

(1) Eight hundred six *cho* three thousand seven hundred fifty *oku*

(2) 563 one *oku*s (100 millions) and 7428 one *man*s (10 thousands) combined.

② Please calculate the following.

(1) $\begin{array}{r} 684 \\ \times\ 731 \\ \hline \end{array}$ (2) $\begin{array}{r} 276 \\ \times\ 320 \\ \hline \end{array}$ (3) $\begin{array}{r} 659 \\ \times\ 454 \\ \hline \end{array}$ (4) $\begin{array}{r} 548 \\ \times\ 503 \\ \hline \end{array}$

(5) $\begin{array}{r} 749 \\ \times\ 600 \\ \hline \end{array}$ (6) $\begin{array}{r} 225 \\ \times\ 889 \\ \hline \end{array}$ (7) $\begin{array}{r} 806 \\ \times\ 370 \\ \hline \end{array}$ (8) $\begin{array}{r} 312 \\ \times\ 759 \\ \hline \end{array}$

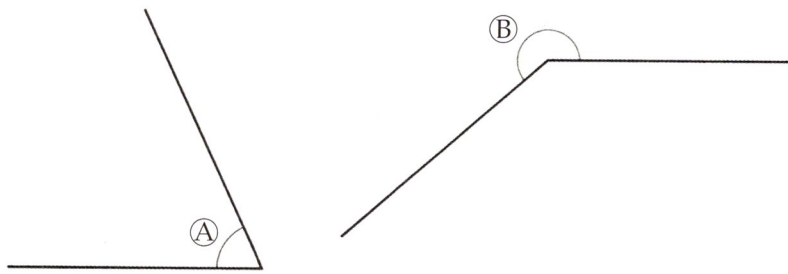
③ Please calculate the following.

(1) $45\overline{)970}$ (2) $72\overline{)564}$ (3) $58\overline{)652}$ (4) $93\overline{)147}$

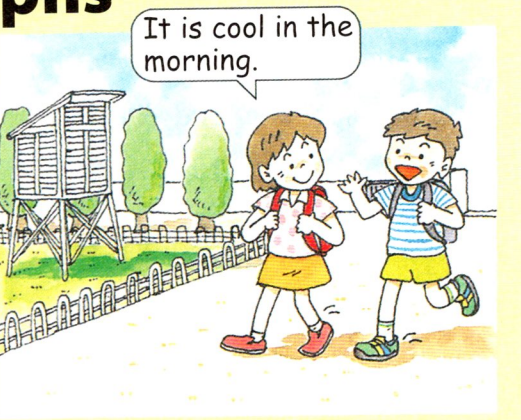
(5) $86\overline{)729}$ (6) $24\overline{)806}$ (7) $69\overline{)508}$ (8) $31\overline{)546}$

(9) $19\overline{)247}$ (10) $55\overline{)632}$ (11) $73\overline{)295}$ (12) $22\overline{)153}$

④ Measure the angles Ⓐ and Ⓑ after estimating them first.

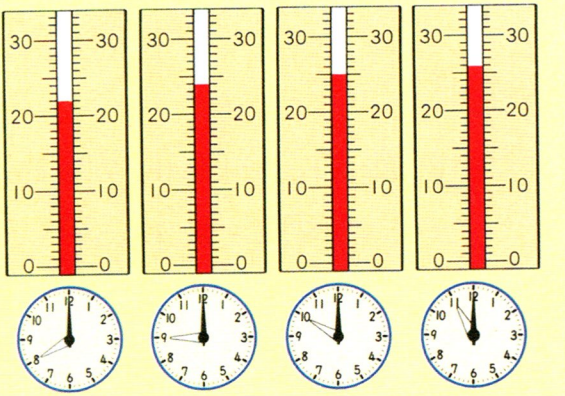


6 Line Graphs





Temperature of July 6th



Change in Temperature (Recorded on July 6th)

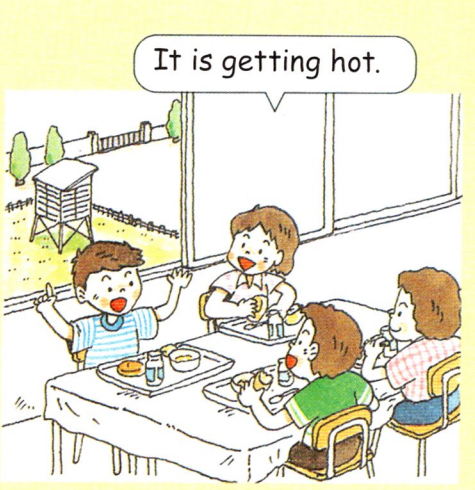
Time	a.m. 8	9	10	11	12	p.m. 1	2	3	4	5
Temperature (degrees)(°C)	22									

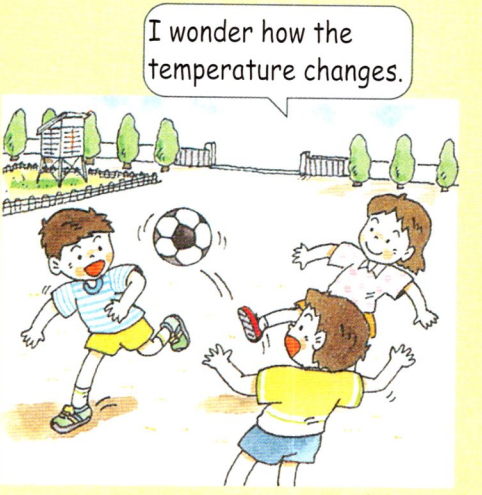


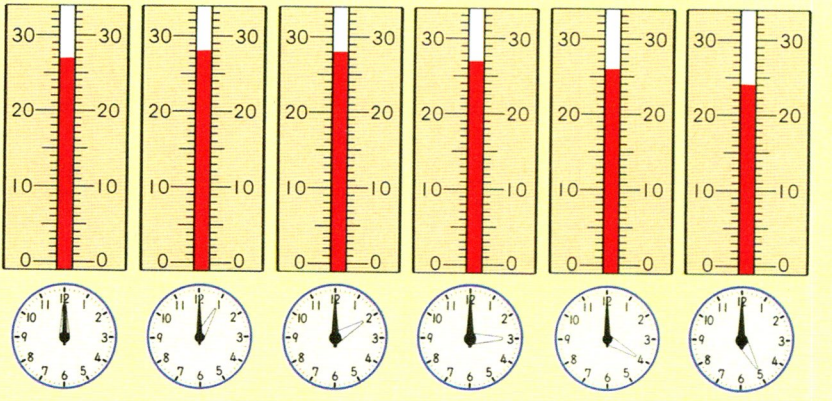
How can we show change in temperature effectively?

When we were in the 3rd grade, we made bar graphs.







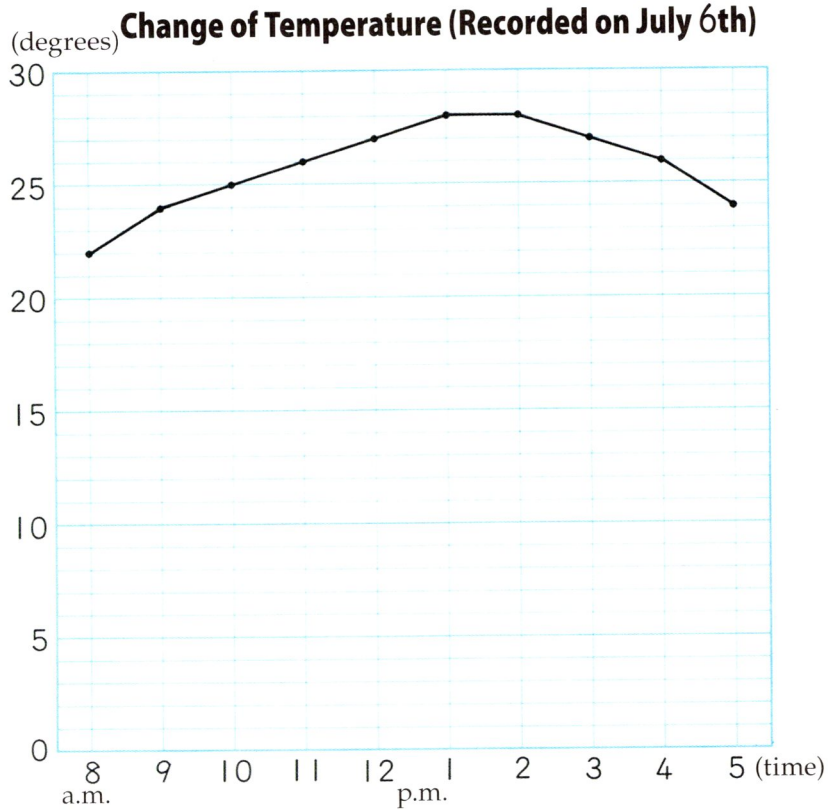


Let's think about graphs that can show change effectively!

Line graphs

1 Let's think about what kind of graph is the best way to show change of temperature!

We use **line graphs** when showing quantities, like temperature, that change over time.



1. What does the horizontal axis indicate? What does the vertical axis indicate?

2. What does one interval on the vertical axis indicate?

3. What was the temperature at 8:00 a.m.? At what time in the morning was the temperature 26 degrees?

4. What was the highest temperature of the day?

▶ Slope and change of lines

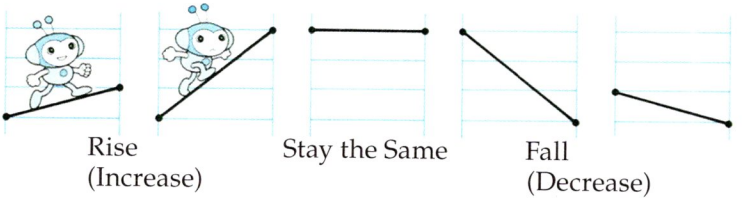


2 Let's investigate the changes in temperature on a line graph!

1. From 8 o'clock to what time was the temperature rising? From what time to what time did the temperature fall?
2. From what time to what time did the temperature stay the same?
3. From what time to what time did the temperature rise the fastest?

   And from what time to what time did the temperature fall the fastest?

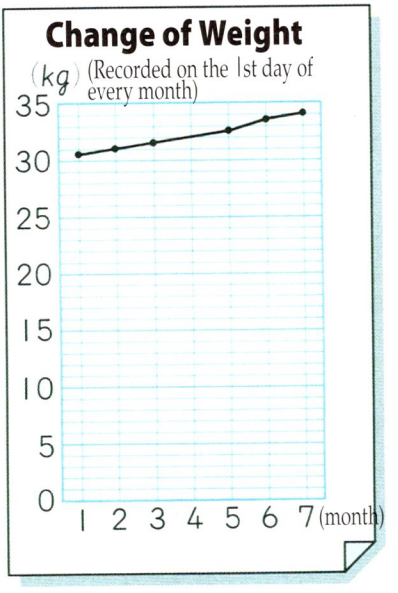
! In line graphs, the slope of the line indicates the direction of the change. Moreover, the steeper the line, the greater the change.

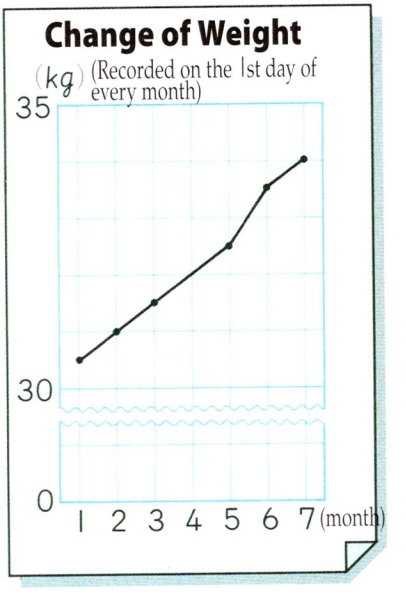


Line graphs are a convenient way to show changes over time.

3 Both line graphs below show changes in Tadashi's weight.
Which graph shows the changes more clearly?

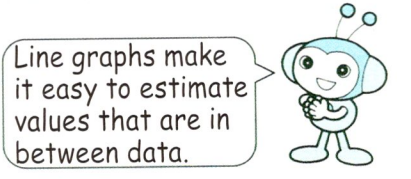






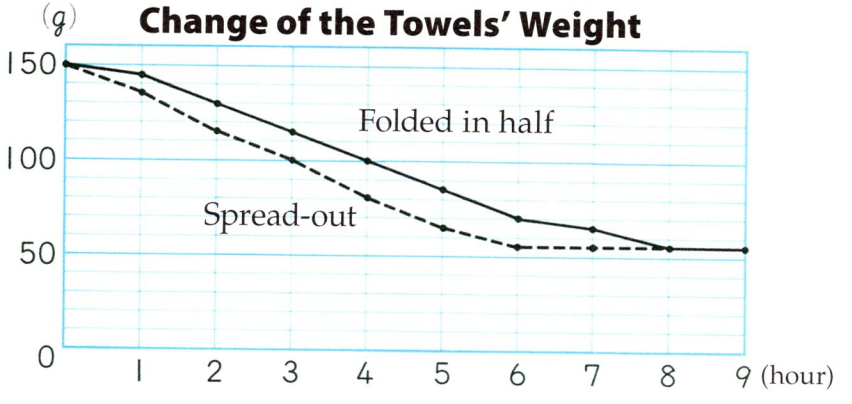
In line graphs, we can show change in slope more clearly by using the symbol ≈ to cut the scale on the vertical axis to emphasize the slope of the line.

1 About how many kg was his weight in April?



4 Michiko soaked 2 towels that weighed 55g each. Then, she hung them to dry, one spread-out and the other folded in half. Looking at the graph below, let's compare how the two towels dried!





1 How many g were the towels when they were wet?

2 Which towel dropped in weight to 100g first?

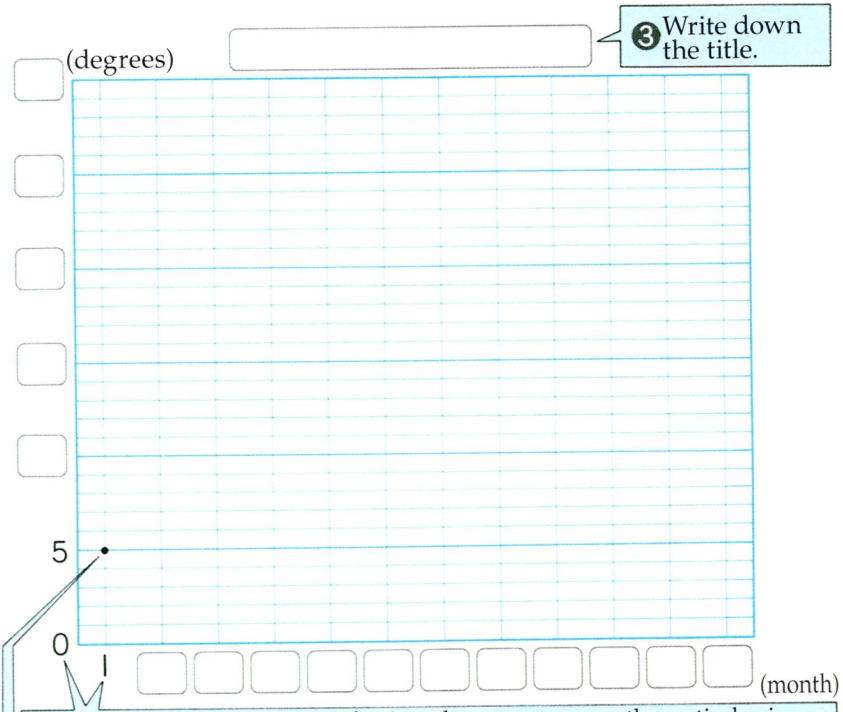
3 After how many hours of hanging did the spread-out towel become completely dry? And how was the folded towel?

How to draw line graphs

5 The table below shows temperatures recorded throughout the year.
Let's draw a line graph based on this set of data and investigate the change in temperature throughout the year!

Temperatures throughout the Year
(Recorded at 9:00 a.m. on the first day of the month)

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (degrees) (°C)	5	4	7	16	19	22	24	29	26	18	14	8



❸ Write down the title.

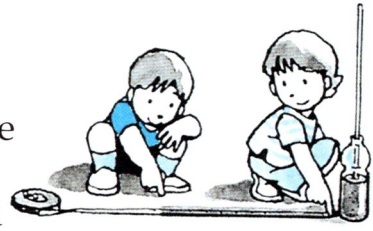
❶ Put months on the horizontal axis and temperatures on the vertical axis, and then scale each axis appropriately.

❷ Plot points for each month's temperature, and then, connect the dots with a line.

1 Draw a line graph for temperatures throughout the year.

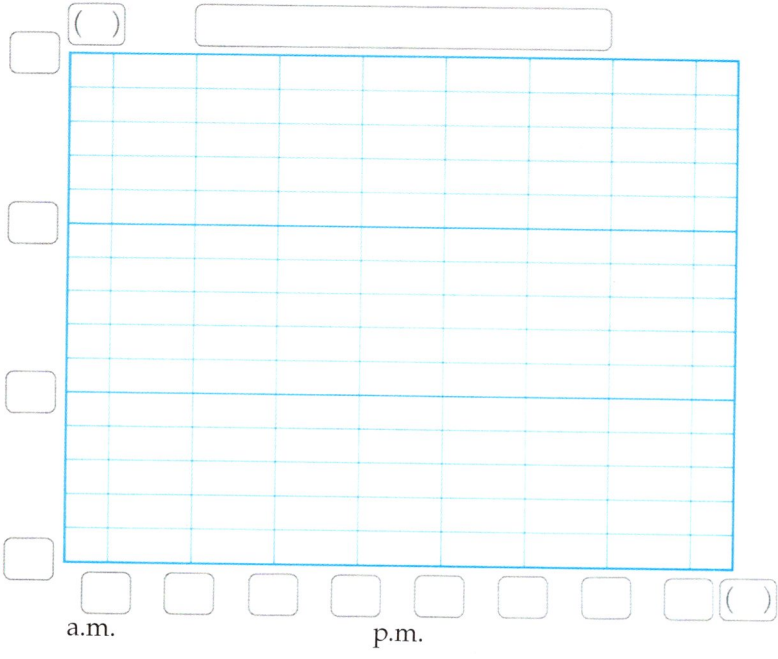
2 Look at the graph and talk about what you observe.

1 The table below shows an investigation of the length of a shadow made by a meter stick. Draw a line graph based on this set of data.



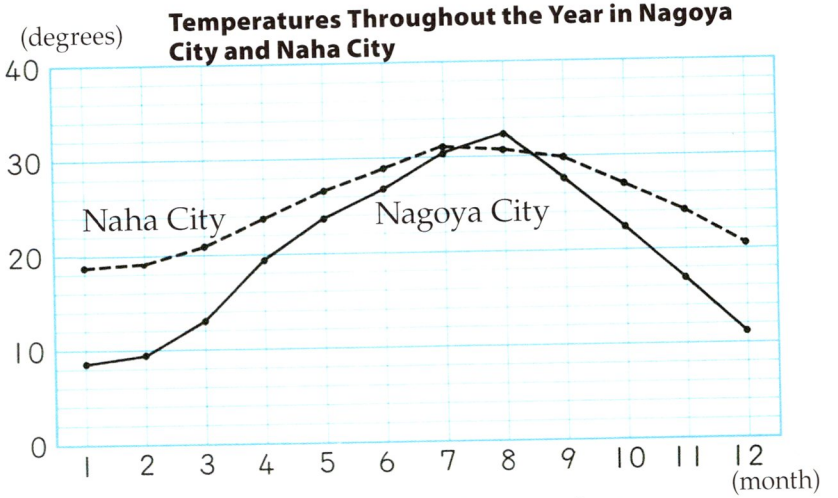
Length of the Shadow of a Meter Stick

Time	a.m. 9	10	11	12	p.m. 1	2	3	4
Length of Shadow (*m*)	0.7	0.5	0.3	0.2	0.3	0.5	0.8	1.3



Practice

① The line graph below shows change of temperature throughout the year in Nagoya City and Naha City.



(1) In October, approximately what are the temperatures in Nagoya City and Naha City, respectively?

(2) Which months show that Nagoya City recorded higher temperatures than Naha City?

② The table below shows the length of a gourd's stalk every week while it grew.
Draw a line graph based on this set of data.

Length of a Gourd's Stalk

Date	June 9th	16th	23th	30th	July 7th	14th
Length (cm)	3	7	18	39	70	99

Check

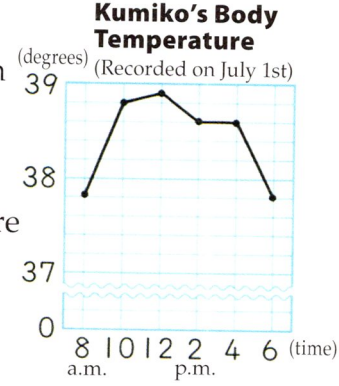
1 Which of the following can be appropriately shown on a line graph?

Ⓐ The student population in different schools in April

Ⓑ Your height recorded every April

Ⓒ Temperatures at 10:00 a.m. in various places

2 The graph on the right shows Kumiko's body temperature taken on the day she was absent from school due to a cold.



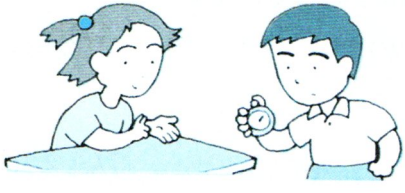
(1) What was her body temperature at 10:00 a.m.?

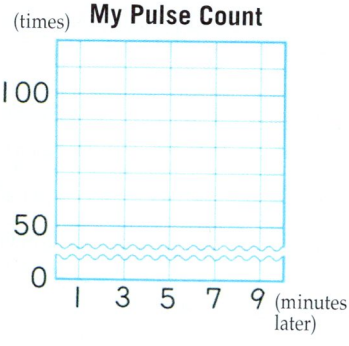
(2) From what time to what time did her temperature stay the same?

Challenge

Let's Show Information with a Line Graph!

After some physical exercise, count your pulse for one minute several times, then draw a line graph to show how it changes.





Review ◦ 3

① Calculate the following.

(1) $\begin{array}{r} 843 \\ \times\ 726 \\ \hline \end{array}$ (2) $\begin{array}{r} 536 \\ \times\ 408 \\ \hline \end{array}$ (3) $\begin{array}{r} 395 \\ \times\ 700 \\ \hline \end{array}$ (4) $\begin{array}{r} 602 \\ \times\ 537 \\ \hline \end{array}$

(5) $\begin{array}{r} 590 \\ \times\ 500 \\ \hline \end{array}$ (6) $\begin{array}{r} 241 \\ \times\ 473 \\ \hline \end{array}$ (7) $\begin{array}{r} 860 \\ \times\ 920 \\ \hline \end{array}$ (8) $\begin{array}{r} 409 \\ \times\ 781 \\ \hline \end{array}$

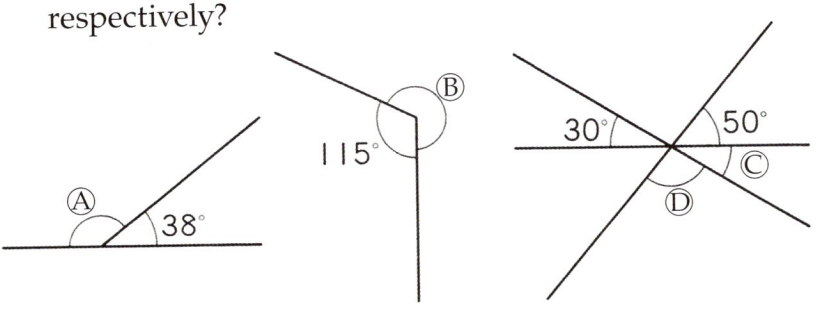
② Calculate the following.

(1) $29\overline{)87}$ (2) $16\overline{)97}$ (3) $82\overline{)738}$ (4) $37\overline{)265}$

(5) $58\overline{)242}$ (6) $46\overline{)192}$ (7) $25\overline{)967}$ (8) $39\overline{)878}$

(9) $34\overline{)948}$ (10) $28\overline{)672}$ (11) $29\overline{)8874}$ (12) $64\overline{)5773}$

③ How many degrees are angles Ⓐ, Ⓑ, Ⓒ, and Ⓓ, respectively?



④ Round the two numbers in the ( ) to the nearest *mans* (10 thousands) place.
Then, find the sum and difference of the two rounded numbers mentally.

(1) (97083, 65434) (2) (38056, 741276)

⑤ What is the place value of the left-most digit in each of the following numbers?
Also, please read the numbers.

(1) 6125490000 (2) 3800705004200

⑥ Express the following numbers using numerals.

(1) Three *cho* six hundred *oku* four thousand five hundred one *man*

(2) 4 one *oku* and 7 one hundred *man* combined

(3) A number that is one bigger than one hundred *oku*

(4) 10 times 8000 *man*

⑦ Fill in the empty cells in the table on the right.

Population of the 4th Grade (children)

Gender / Class	Boys	Girls	Total
Class #1	14		29
Class #2			31
Total	29		

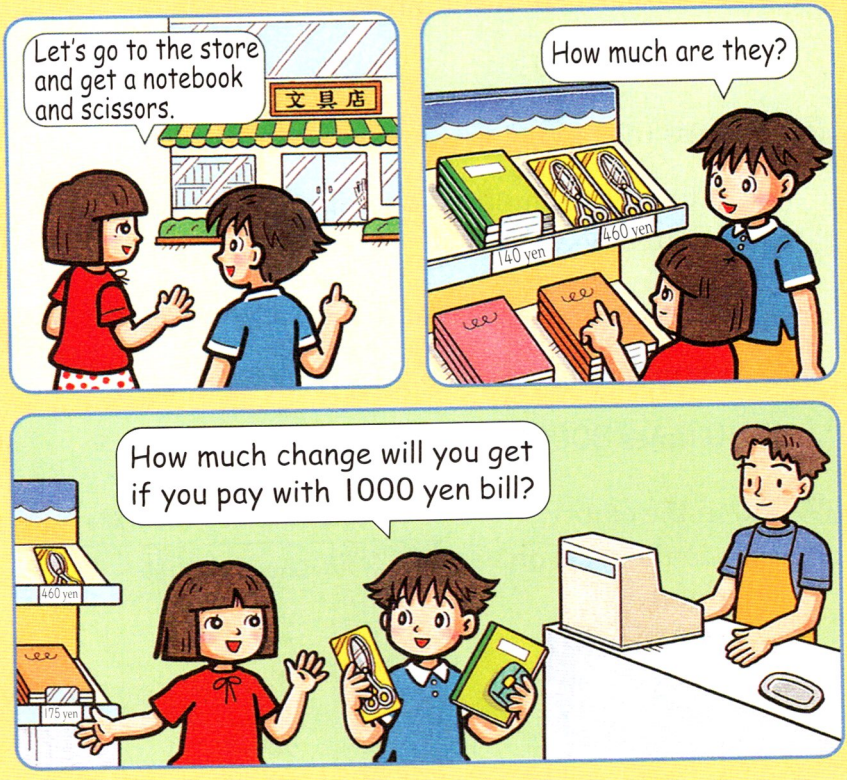
⑧ I bought 42 notebooks that cost 60 yen each. The sales clerk gave me a discount price of 2310 yen. How much of a dicount per notebook did I get?

⑨ Fill in the □ with the appropriate numbers.

(1) □ + 6 = 84 (2) 34 + □ = 43

(3) □ − 4 = 17 (4) □ − 27 = 35

7 Math Sentences and Their Calculation



1 Calculations with Parentheses

1 Makoto had a 1000 yen bill. He bought a 140 yen notebook and a 460 yen pair of scissors. How much change did he get back? Let's combine Makoto's two math sentences above into one!

1000−140=860
860−460=400

140+460=600
1000−600=400



Naoko

I did two calculations separately.

I added the cost of the notebook and the scissors first.



Makoto



Both of you needed two math sentences. I wonder if we could make one math sentence.



If I combine my two math sentences into one it could be like ...

Math Sentence



I wonder how I could combine my two math sentences into one.

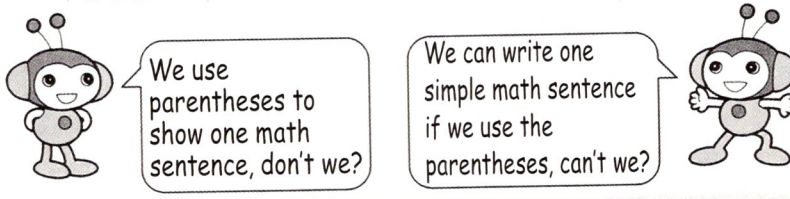
Let's think about how to combine two math sentences into one and the order of calculation!

1 Use the word sentence below to write a combined math sentence.

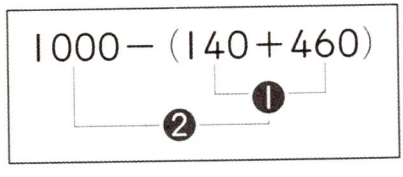
Money paid − Total price = Change

You can combine Makoto's two math sentences into one as in the example below.

$1000-(140+460)=400$



? Let's think about the order of calculation for $1000-(140+460)$.



$$1000-(140+460)=1000-600$$
$$=400$$

! When you have parentheses, you calculate the part of the math sentence inside the parentheses first.

① (1) $1000-(700+50)$ (2) $1000-(700-50)$

(3) $630-(130-50)$ (4) $510+(480-270)$

2 Minoru buys a 50 yen pencil and a 15 yen pencil cap. How many pairs of pencils with caps can he buy with 520 yen?





Minoru

$50 + 15 = 65$

$520 \div 65 = 8$ Answer : 8 pairs

1 Let's put Minoru's math sentences into one sentence using the following word sentence.
Calculate the answer also.

Amount of money you have	÷	Price you pay for one pair	=	Number of pairs you can buy

1 There are 54 sheets of graph paper. You want to buy 60 more and share all the papers among 38 people equally.
Make one math sentence to determine how many sheets of paper each person will receive.

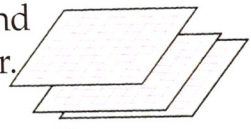
2 (1) $(26 + 18) \times 8$ (2) $23 \times (53 - 45)$

(3) $(135 - 30) \div 35$ (4) $637 \div (46 + 45)$

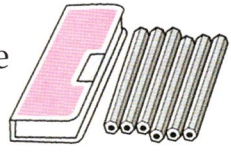
2 Math Sentences with Multiplication and Division

1 Let's make one math sentence for each of the following problems, then find the answers.

(1) You have a 100-yen coin and buy 3 sheets of graph paper. One sheet costs 25 yen. How much change will you get?



(2) You buy a 500 yen pen case and a half dozen pencils. One dozen pencils costs 480 yen. How much will you pay?



(1) Math Sentence

(2) Math Sentence

In a math sentence, multiplication and division can be considered as one quantity and you do not have to use parentheses.

(1) $100 - 25 \times 3 = 25$

(2) $500 + 480 \div 2 = 740$

In a math sentence, you calculate multiplication and division before addition and subtraction.

1 (1) $24 + 12 \times 5$ (2) $350 - 175 \div 25$

2 Let's think about the order of calculation!

(1) $180 \times 4 + 150 \times 6$

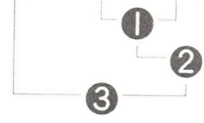
(2) $15 - 36 \div 6 \times 2$

(3) $90 - (50 - 5 \times 3)$

(1) $180 \times 4 + 150 \times 6 =$ ☐



(2) $15 - 36 \div 6 \times 2 =$ ☐



(3) $90 - (50 - 5 \times 3) =$ ☐



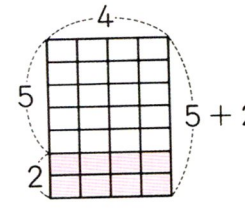
Order of calculation:
(1) Usually we calculate from left to right.
(2) When there are parentheses, we calculate the part inside the parentheses first.
(3) We calculate multiplication and division before addition and subtraction.

① (1) $100 - 75 + 48 \div 3$ (2) $40 + 3 \times (2 + 6)$

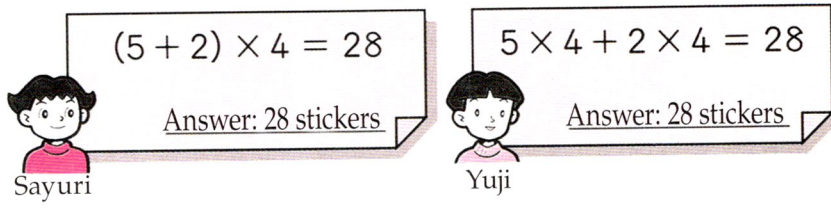
(3) $(37 - 15) \times (8 + 14)$ (4) $40 \div (12 - 4) \times 2$

3 Properties of Operations

1 How many red and white stickers are there altogether? Write one math sentence to answer the question.



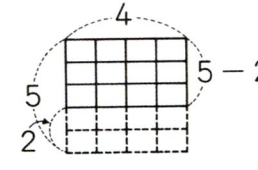
1 Let's explain Sayuri's and Yuji's methods!



Sayuri's and Yuji's math sentences express the same amount, so they can be connected by an equal sign.

$$(5+2)\times 4 = 5\times 4 + 2\times 4$$

2 Can you use an equal sign for $(5-2)\times 4$ and $5\times 4-2\times 4$?



These are two properties of operations that are related to math sentences with parentheses.

$$(■+●)\times ▲ = ■\times ▲ + ●\times ▲$$

$$(■-●)\times ▲ = ■\times ▲ - ●\times ▲$$

3 Use different numbers in ■, ●, and ▲ above to verify the sentences above.

Practice

① Please calculate the following.

(1) $175 - (98 + 29)$

(2) $900 - (600 - 143)$

(3) $40 \times (23 - 15)$

(4) $(36 + 24) \div 5$

(5) $38 \times 2 - 7 \times 4$

(6) $6 \times (43 - 38) \div 2$

② Please fill in the ☐. Also find the answers with mental arithmetic.

(1) $57 \times 4 - 17 \times 4 = (\square - 17) \times 4$

$= \square \times 4$

(2) $102 \times 9 = (\square + 2) \times 9$

$= \square \times 9 + 2 \times 9$

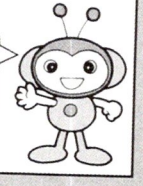
③ Tomoko has 24 counters, Haruko has 28, and Kazuko has 23. If all the counters are shared equally among the three girls, how many will each girl get? Please write one math sentence.

Story of Math

The Multiplication Algorithm and the Property of Operations

$$\begin{array}{r} 12 \\ \times\ 58 \\ \hline 96 \\ 60 \\ \hline 696 \end{array}$$

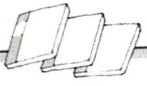
$12 \times (8 + 50) = 12 \times 8 + 12 \times 50$



4 Relationships Between Operations

1 Please solve the following problems and investigate the relationships between the operations.

What is the total price of three 85 yen notebooks?

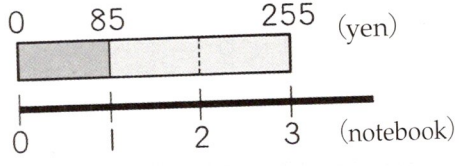


What is the price of one notebook if you pay 255 yen for 3 notebooks?

1 Let's compare the following two math sentences.

$85 \times 3 = 255$

$255 \div 3 = 85$



There are two relationships between multiplication and division.

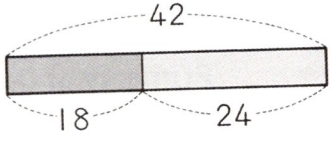
$\square \times 3 = 255 \Rightarrow 255 \div 3 = \square$

$85 \times \square = 255 \Rightarrow 255 \div 85 = \square$

2 Let's compare the following two math sentences.

$18 + 24 = 42$

$42 - 24 = 18$



These are two relationships between addition and subtraction.

$\square + 24 = 42 \Rightarrow 42 - 24 = \square$

$18 + \square = 42 \Rightarrow 42 - 18 = \square$

Check

1 Please find the errors and correct them.

(1) $16+4\times8=20\times8$
$=160$

(2) $12\div4+9-2=3+9$
$=12-2$
$=10$

2 Please calculate the following.

(1) $67-(100-52)$

(2) $(84-32)\div4$

(3) $72\div6+18\div3$

(4) $47-28\div7\times8$

3 What numbers can you put in the $\square$?

(1) $(30+8)\times25=30\times25+8\times\square$

(2) $18\times4=(\square-2)\times4$

Challenge

Which Problem Goes with Each Math Sentence?



$30\times(40+50)$	Price of one 30 yen pencil and fifty 40 yen caps
$30+40\times50$	Price of forty 30 yen pencils and one 50 yen eraser
$30\times40+50$	Price of forty 30 yen red pencils and fifty 30 yen blue pencils

8 Quadrilaterals



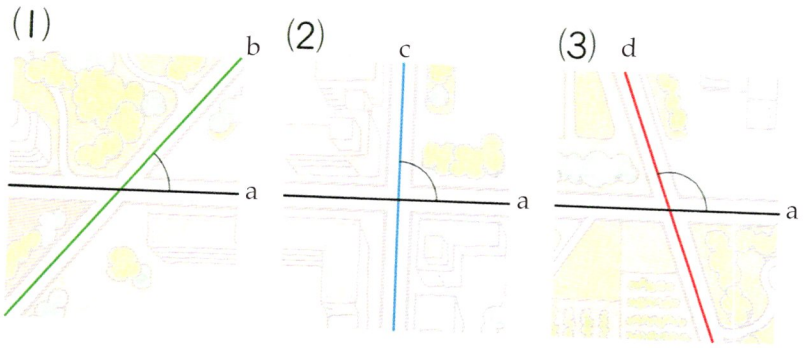
1 Perpendicular and Parallel Lines

▶ Perpendicular lines

1 Let's investigate how two straight lines intersect in (1), (2), and (3) on the following page!

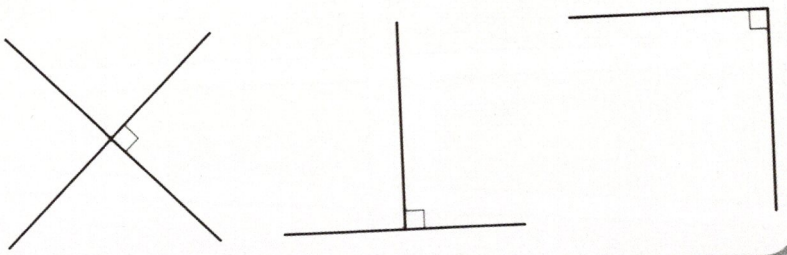
1. Please measure the angles formed by the two lines.
2. In which case do the two lines intersect at a right angle?

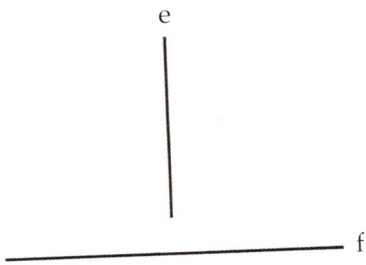
Let's investigate how two straight lines intersect and line up!



In case (2), the 2 straight lines, a and c, intersect to form a right angle.

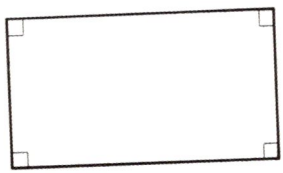
Two straight lines that intersect to form a right angle are said to be **perpendicular** to each other.

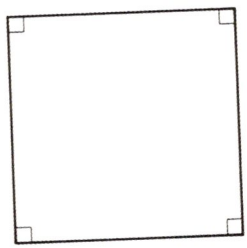




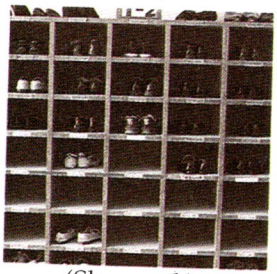
3 In the figure on the left, are lines e and f perpendicular?
Let's find out by extending line e.

Any two adjacent sides of rectangles and squares are perpendicular.





1 Find two straight lines that are perpendicular to each other around you.

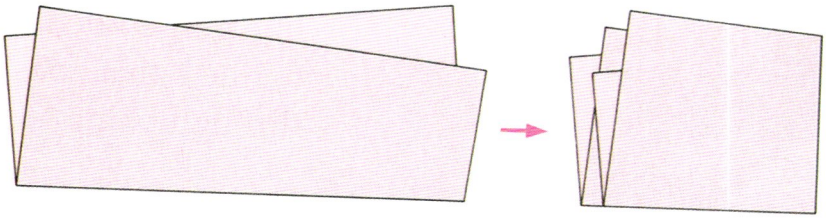


(Shoe rack)

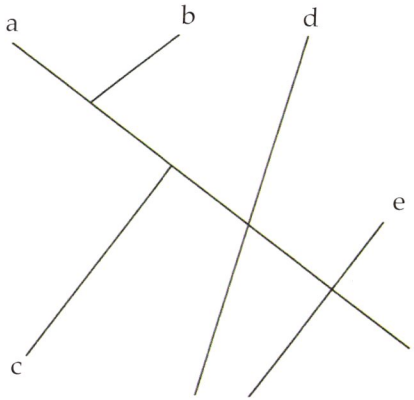


(School building)

2 Fold one sheet of paper in quarters like the picture below, and then open it completely. How do the two crease lines appear?

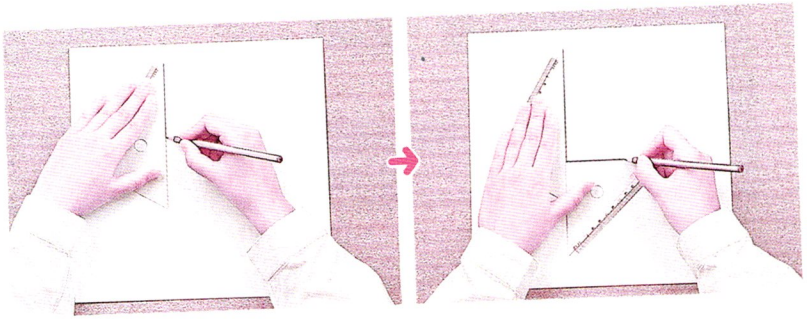


3 In the figure on the right, which lines are perpendicular to line a? Please check them with a set-square.

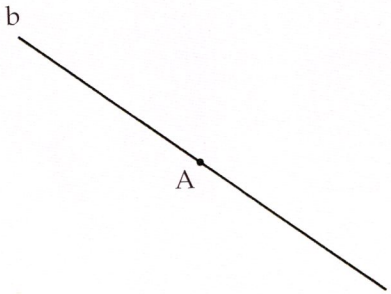


How to draw perpendicular lines

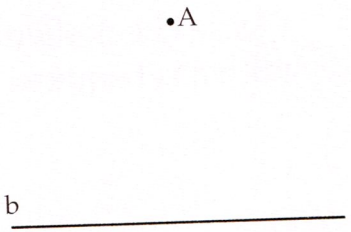
2 Let's draw a pair of perpendicular lines by using two set-squares!



① Draw a straight line perpendicular to line b that goes through point A.







Parallel lines

3 Let's investigate how the two lines in (1) and (2) below are lined up.

(1)



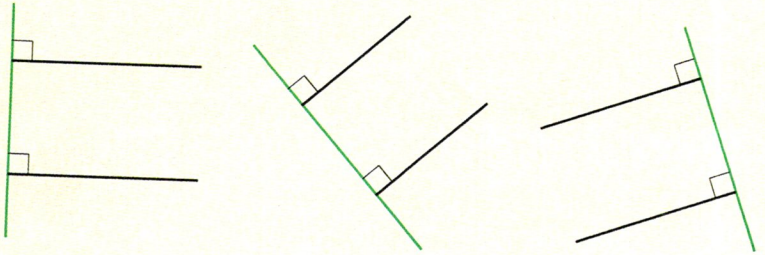
(2)

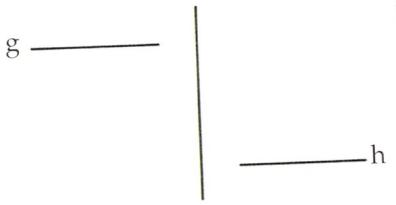


1 How do lines a and b intersect line c?
Also, how do lines d and e intersect line f?

Lines a and b are both perpendicular to the line c.

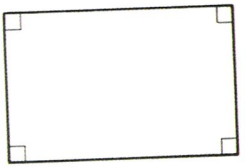
Two lines that are perpendicular to a third line are said to be **parallel** to each other.





1 In the figure to the left, are lines g and h parallel?
Please investigate by extending lines g and h.

Opposite sides of rectangles and squares are parallel to each other.

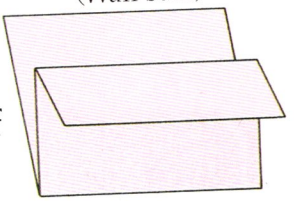


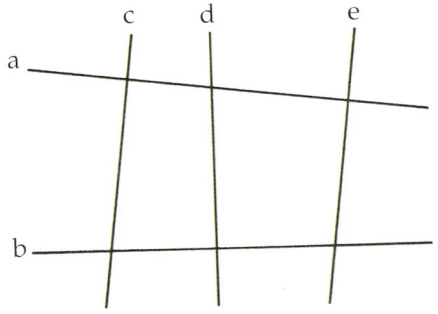
1 Let's find parallel lines around you!


(Crosswalk)


(Wall bars)

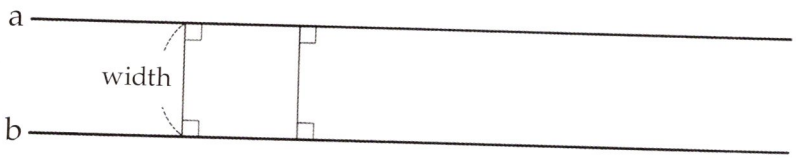
2 As shown in the picture to the right, fold a sheet of paper twice with the vertical sides of the edges aligned.
How do the two crease lines appear?





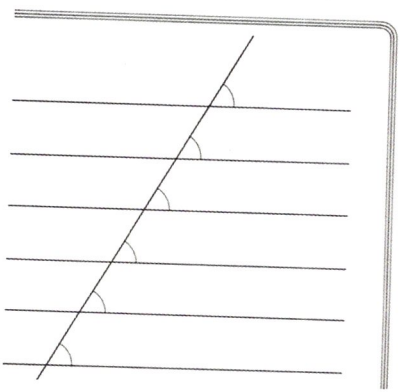
3 Which pair of lines are parallel?
Please check using set-squares.

4 Lines a and b below are parallel.
Draw several perpendicular lines between them and compare the widths between lines a and b.



The widths between parallel lines are always equal.
Two parallel lines never intersect no matter how far they are extended.

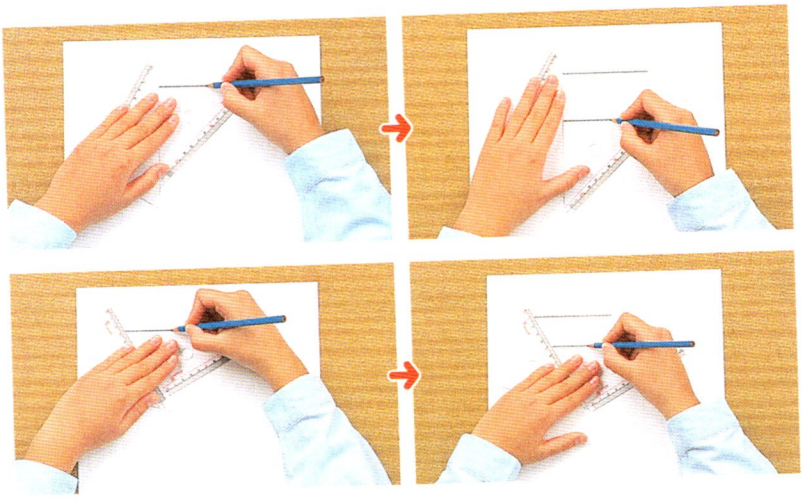
5 The horizontal lines of a notebook are parallel.
Draw a slanted line intersecting the horizontal lines and compare the angles formed at the intersections.



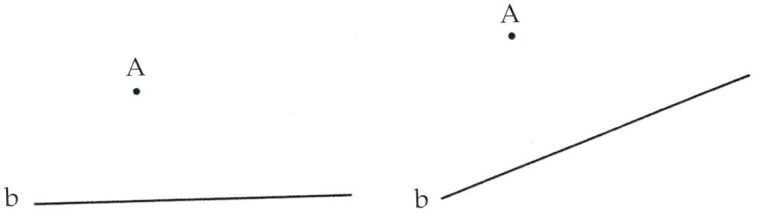
Parallel lines intersect straight lines at equal angles.

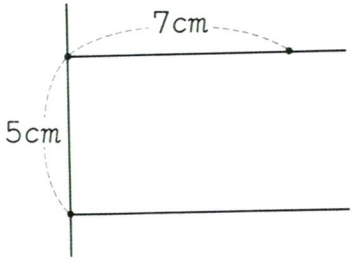
How to draw parallel lines

6 Let's draw parallel lines using two set-squares!



① Draw a line parallel to line b that goes through point A.

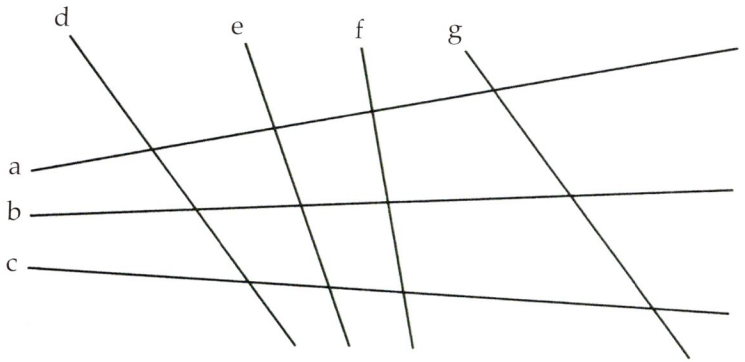




② Please draw a rectangle 5cm long and 7cm wide using two set-squares.

Practice 1

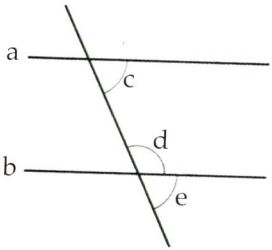
① Which straight lines are parallel to each other? And which straight line is perpendicular to line a?



② In the figure to the right, lines a and b are parallel, and angle c is 65°.

(1) How many degrees is angle e?

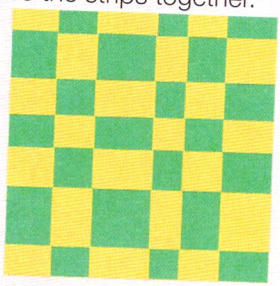
(2) How many degrees will it be if you add angle c and angle d?

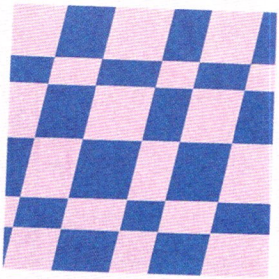


Challenge

Let's Make Many Designs by Weaving Colored Paper!

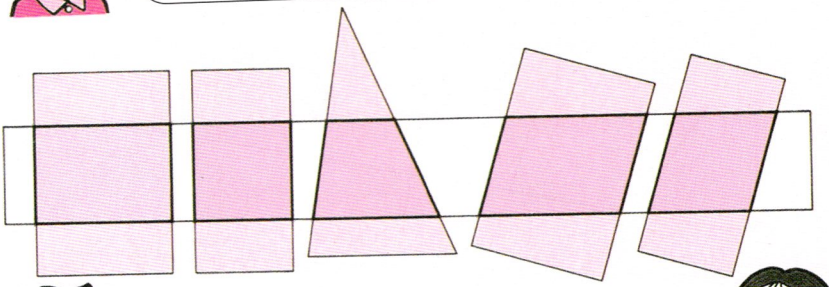
Cut two sheets of colored paper along parallel lines and weave the strips together.







What kinds of shapes can we make if we overlap a rectangle with rectangles and triangles?





Kazuya

We can make a rectangle and a square.

We can make some other quadrilaterals.



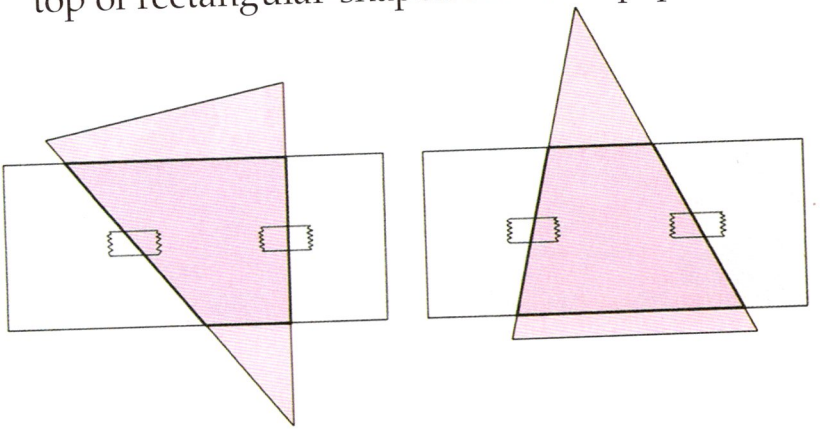
Ritsuko

Let's investigate the characteristics of many quadrilaterals!

2 Trapezoids, Parallelograms, and Rhombuses

▶ Trapezoids

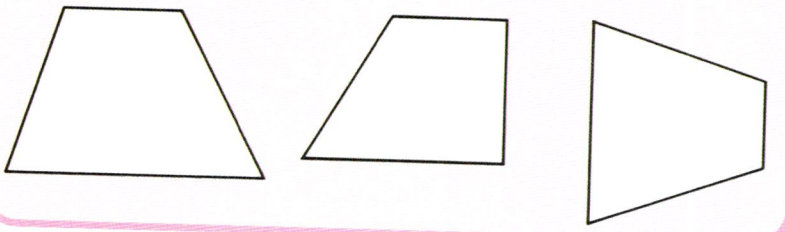
1 As shown below, let's make quadrilaterals by placing triangular-shaped sheets of paper on top of rectangular-shaped sheets of paper!



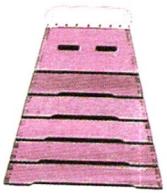
Let's investigate the characteristics of the quadrilaterals we made!

1 Please investigate how the opposite sides are lined up.

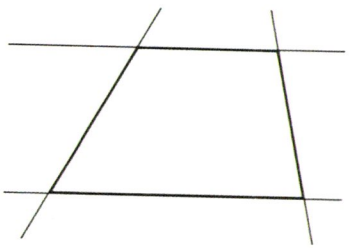
Quadrilaterals with a pair of opposite sides that are parallel are called **trapezoids**.



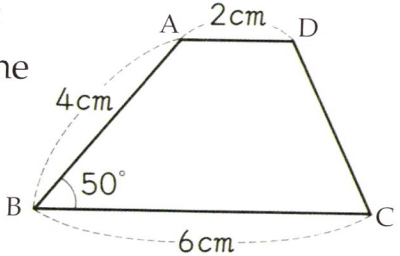
1 Let's find trapezoid-shaped objects around you!



2 Draw a trapezoid by drawing two parallel lines and two other lines that intersect those lines as shown on the right.

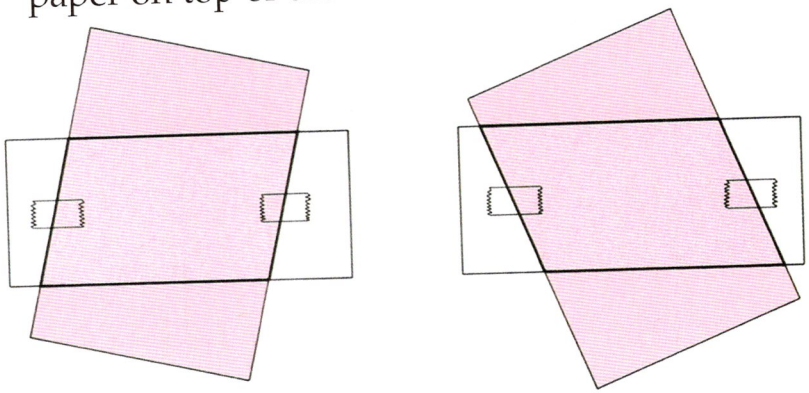


3 Draw a trapezoid just like the drawing on the right.



▶ Parallelograms

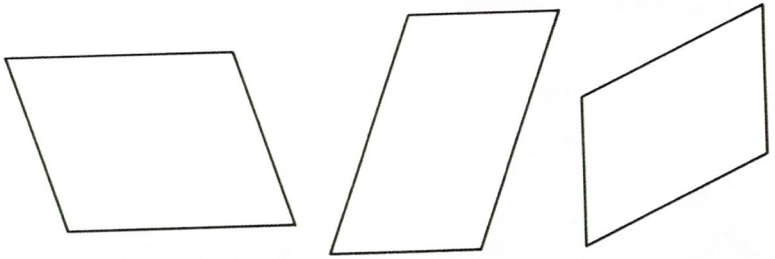
2 As shown below, let's make quadrilaterals by placing two rectangular-shaped sheets of paper on top of each other!



? Let's investigate the characteristics of the quadrilaterals we made!

1 Please investigate how opposite sides are lined up.

Quadrilaterals with two pairs of opposite sides that are parallel are called **parallelograms**.



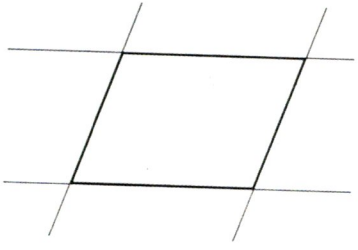
2 Let's investigate the lengths of opposite sides and the sizes of opposite angles!

! The lengths of opposite sides in a parallelogram are the same. Also, the measures of opposite angles are the same.

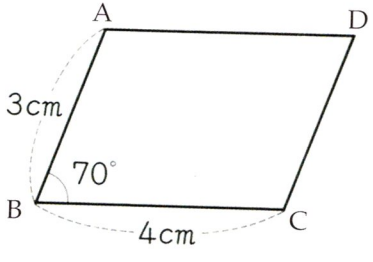
1 Find parallelogram-shaped objects around you.



2 Draw a parallelogram by drawing two pairs of parallel lines, as shown on the right.

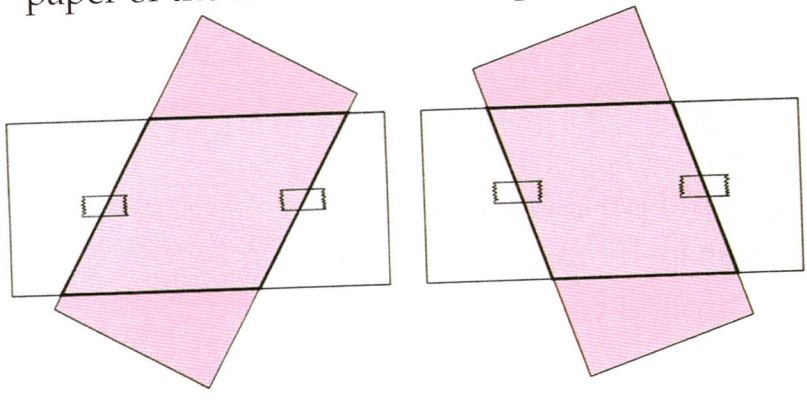


3 Please draw a parallelogram just like the drawing on the right.



Rhombuses

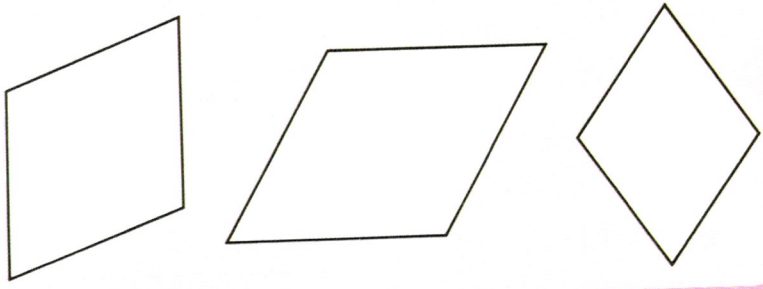
3 As shown below, let's make quadrilaterals by placing two rectangular-shaped sheets of paper of the same width on top of each other!



? Let's investigate the characteristics of the quadrilaterals we made!

1 Please investigate the lengths of the four sides.

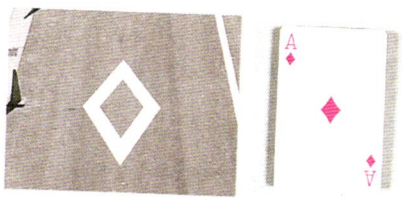
Quadrilaterals whose four sides all have the same lengths are called **rhombuses (or rhombi)**.



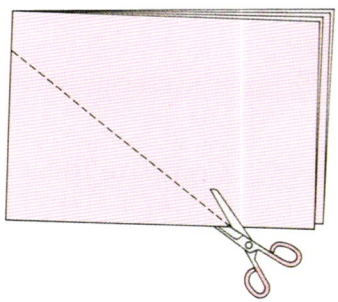
2 Let's investigate how opposite sides are lined up as well as the measures of opposite angles!

The opposite sides of rhombuses are parallel. Also, their opposite angles are the same size.

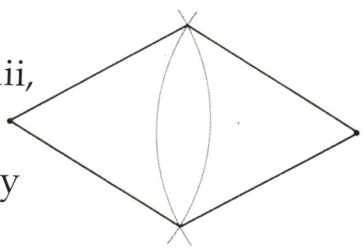
1 Let's find rhombus-shaped objects around you!



2 Fold a sheet of paper in quarters and cut it out along the dotted line as shown on the right. When you open it, you will get a rhombus. Please explain why.

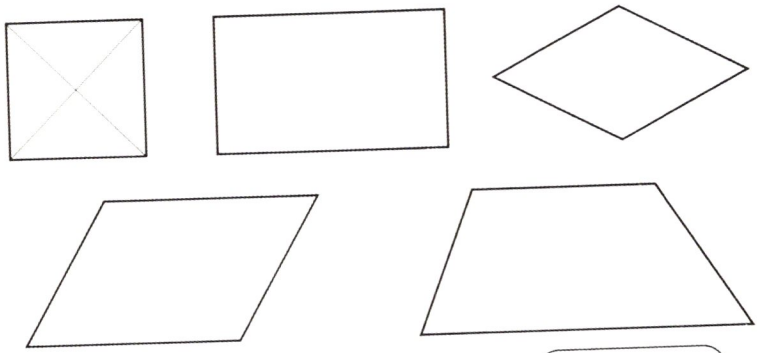


3 Draw two intersecting circles with identical radii, as shown on the right. Then, draw a rhombus by connecting the points created by the intersection and the centers of the circles.



▶ Diagonals

4 Draw straight lines in the quadrilaterals below by connecting opposite vertices.



Line segments that connect two opposite vertices are called **diagonals**.



? Let's investigate the characteristics of diagonals in quadrilaterals!

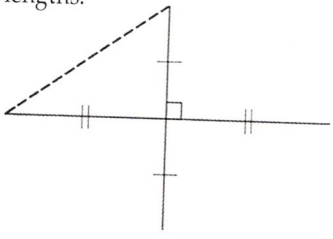
1 Compare the lengths of the two diagonals.

2 Compare the lengths of the four line segments between the intersection of the diagonals and the four vertices.

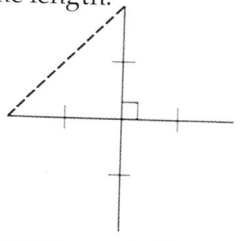
3 Compare the angles formed by the two diagonals at the point of intersection.

(1) In the drawings on the next page, two line segments intersect at a right angle at their midpoints.
If you draw quadrilaterals using the line segments as diagonals, what kind of quadrilateral will you get?

(1) The diagonal lines are different lengths.



(2) The diagonal lines are the same length.

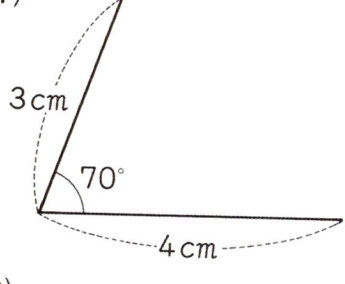




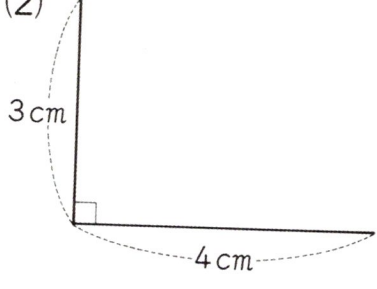
▶ How to Draw Quadrilaterals

5 In the drawings (1) ~ (4) below, draw parallel lines to each side.
What kind of quadrilaterals do we get?

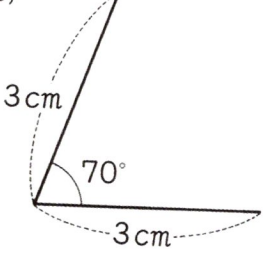
(1)



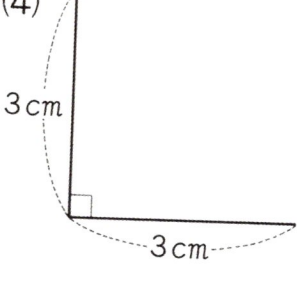
(2)



(3)

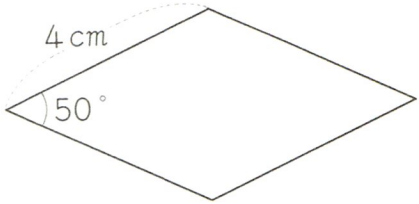


(4)

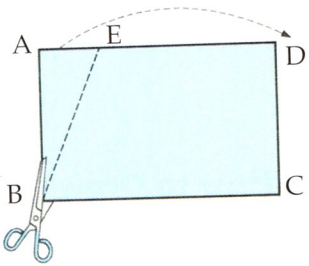


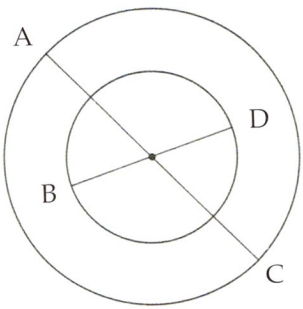
Practice 2

① Please draw a rhombus like the one on the right.



② Cut out a right triangle from a rectangular sheet of paper as shown on the right.
Then, align side AB of the triangle to side DC.
What kind of quadrilateral do we get?

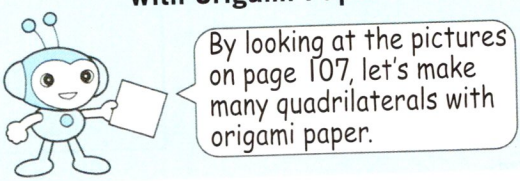




③ Draw the diameters for the circles that share the same center as shown on the left.
If we connect the end points of diameters, A, B, C, and D, what kind of quadrilateral will we get?

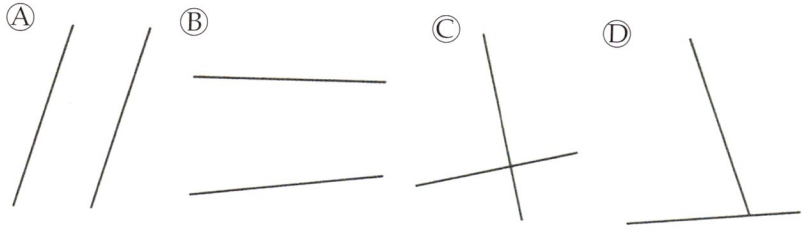
Challenge

Let's Make Many Different Quadrilaterals with Origami Paper!



Check

1 Among the drawings below, which two lines are parallel? Which two lines are perpendicular?



2 Which of the following characteristics are always true about parallelograms and rhombuses? Please indicate by placing a ○ in the box.

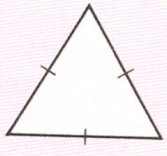
	Parallelograms	Rhombuses
• 2 pairs of opposite sides are parallel		
• 2 diagonals are the same length		
• Opposite sides are the same length		
• Opposite angles are the same size		
• All four sides are the same length		

Challenge

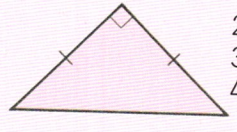
Let's Make Shapes!



If we combine 2, 3, or 4 of the triangles below, what kind of shapes can we make? Use the triangles on page 105 to check your answer.



2 triangles
3 triangles
4 triangles



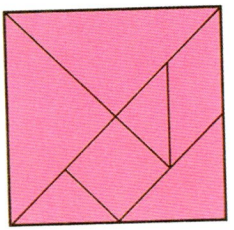
2 triangles
3 triangles
4 triangles

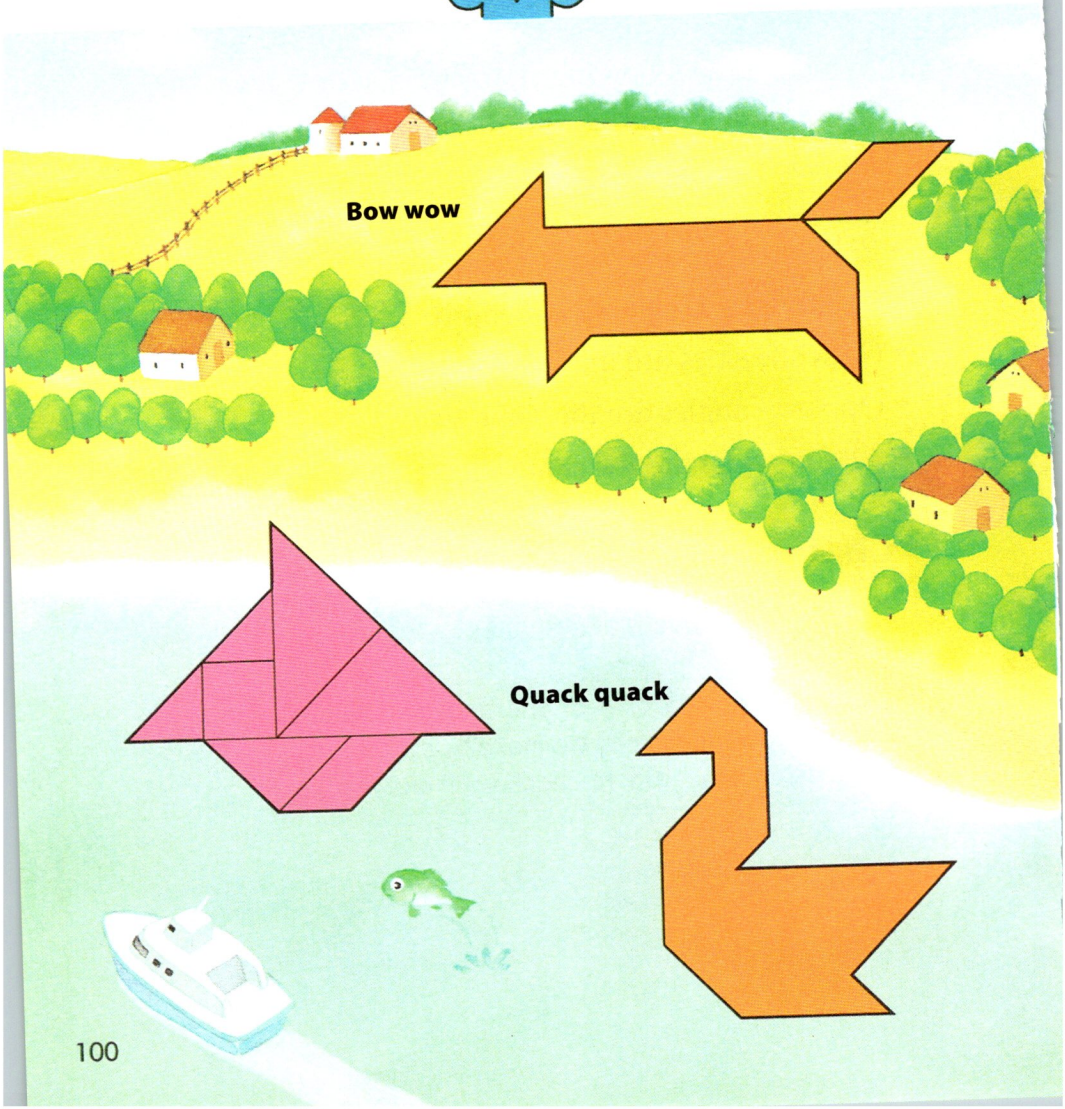
Math Plaza

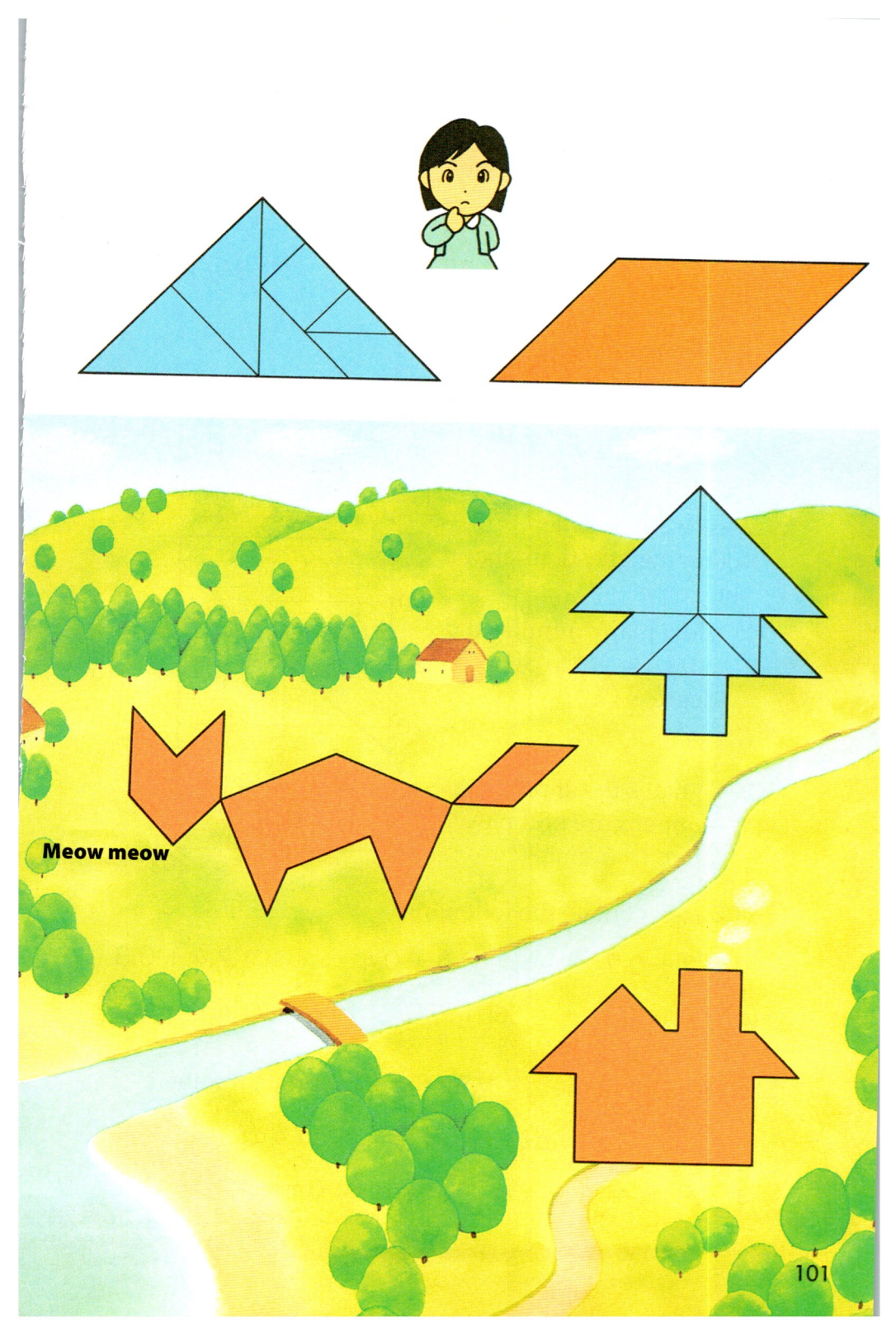
Tangrams

Cut out the square on page 105 along the lines. Then, make many shapes by placing the pieces in different ways.








Meow meow

Review ◦ 4

① Please calculate the following.

(1) $72\overline{)648}$ (2) $29\overline{)724}$ (3) $35\overline{)279}$ (4) $16\overline{)864}$

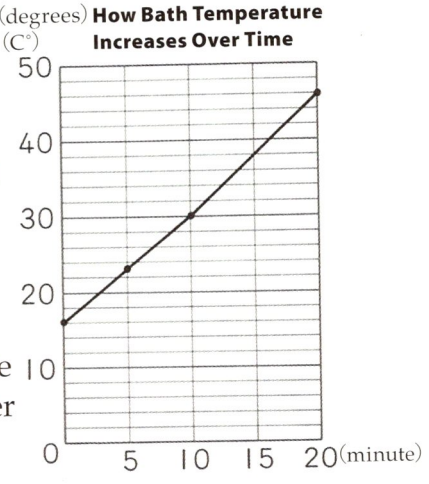
(5) $48\overline{)8192}$ (6) $63\overline{)4284}$ (7) $34\overline{)7004}$

② Please calculate the following.

(1) $89 - (75 - 26)$ (2) $(54 + 83) \times 9$

(3) $36 \times 6 + 47$ (4) $75 \div (25 - 10)$

③ Please answer the following questions about the graph.



(1) How many minutes did it take to get to 30 degrees (°C)?

(2) What is the approximate temperature of the water after 15 minutes?

④ Please calculate the following.

(1) $0.2 + 0.7$ (2) $0.6 + 0.4$ (3) $2.4 + 0.3$

(4) $0.8 - 0.3$ (5) $1 - 0.7$ (6) $4.8 - 0.5$

⑤ Please fill in the ☐ with the appropriate numbers.

(1) $0.8\ell =$ ☐ $d\ell$ (2) $5\ell 4d\ell =$ ☐ ℓ

(3) $4.9cm =$ ☐ cm ☐ mm (4) $6mm =$ ☐ cm

Calculation Practice

① (1) 404×446 (2) 807×341 (3) 580×476 (4) 678×892

(5) 962×264 (6) 760×350 (7) 182×750 (8) 126×700

② (1) $25\overline{)98}$ (2) $40\overline{)83}$ (3) $17\overline{)59}$ (4) $14\overline{)94}$

③ (1) $57\overline{)613}$ (2) $24\overline{)216}$ (3) $25\overline{)318}$ (4) $57\overline{)336}$

(5) $73\overline{)820}$ (6) $14\overline{)476}$ (7) $49\overline{)578}$ (8) $78\overline{)948}$

④ (1) $42\overline{)1008}$ (2) $54\overline{)1261}$ (3) $73\overline{)8550}$

(4) $59\overline{)8378}$ (5) $37\overline{)3034}$ (6) $91\overline{)1819}$

⑤ (1) $700 - (250 - 50)$ (2) $48 \times 16 \div (30 + 2)$

(3) $50 \times 3 + 25 \times 6$ (4) $12 \times 15 \div (120 - 30)$

①	276080	253968	180184	604776
	266000	275187	88200	136500
②	2, R3	3, R8	3, R23	6, R10
③	12, R18	5, R51	9	12, R12
	34	11, R39	10, R43	11, R17
④	23, R19	82	19, R90	
	142	117, R9	24	
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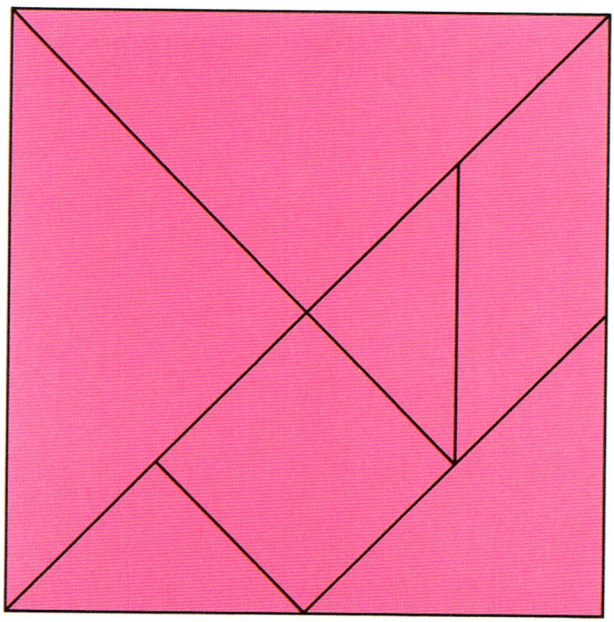

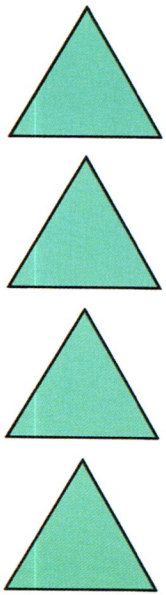

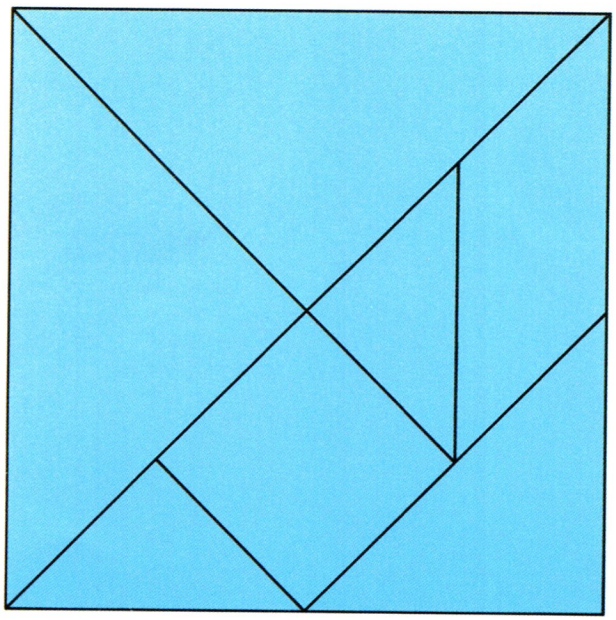

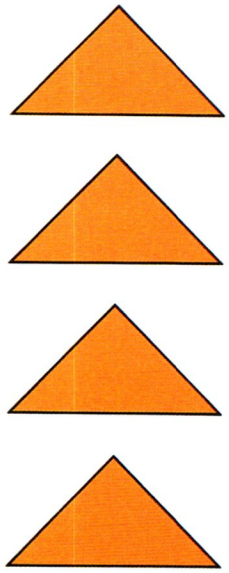

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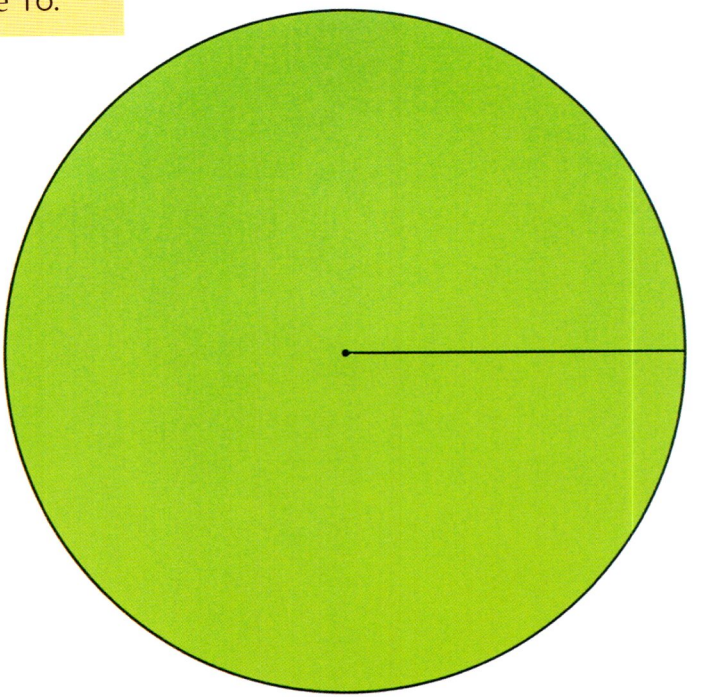
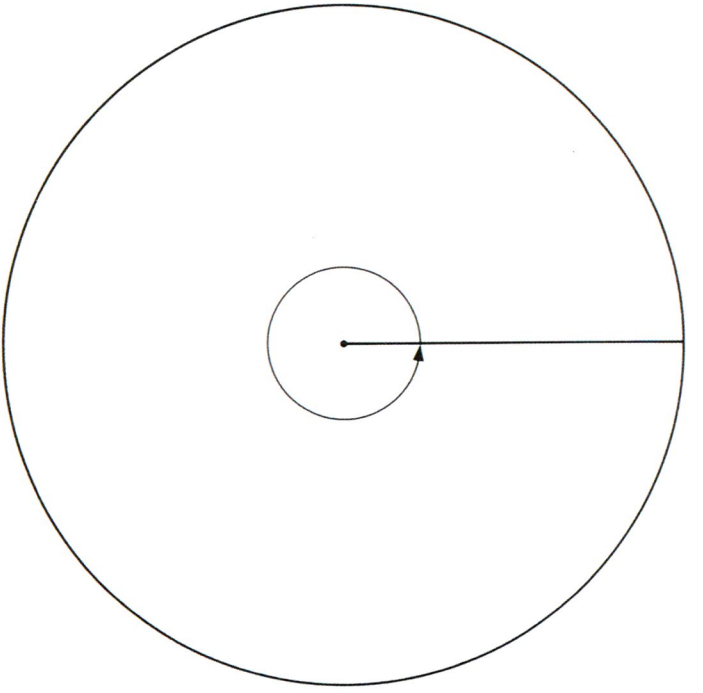
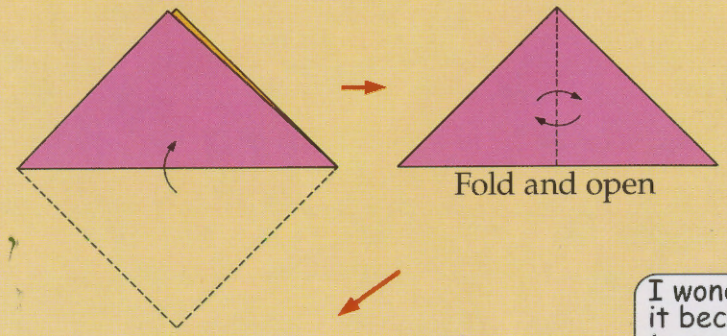
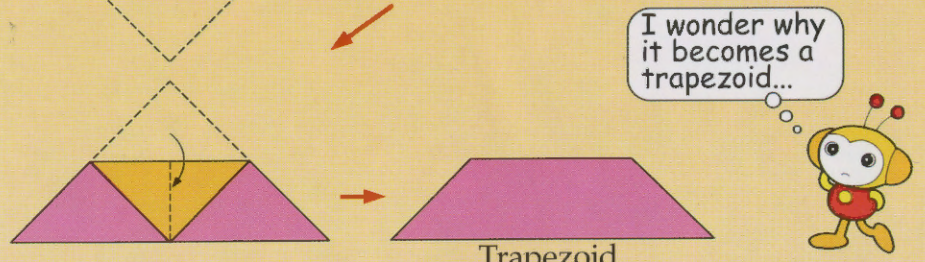
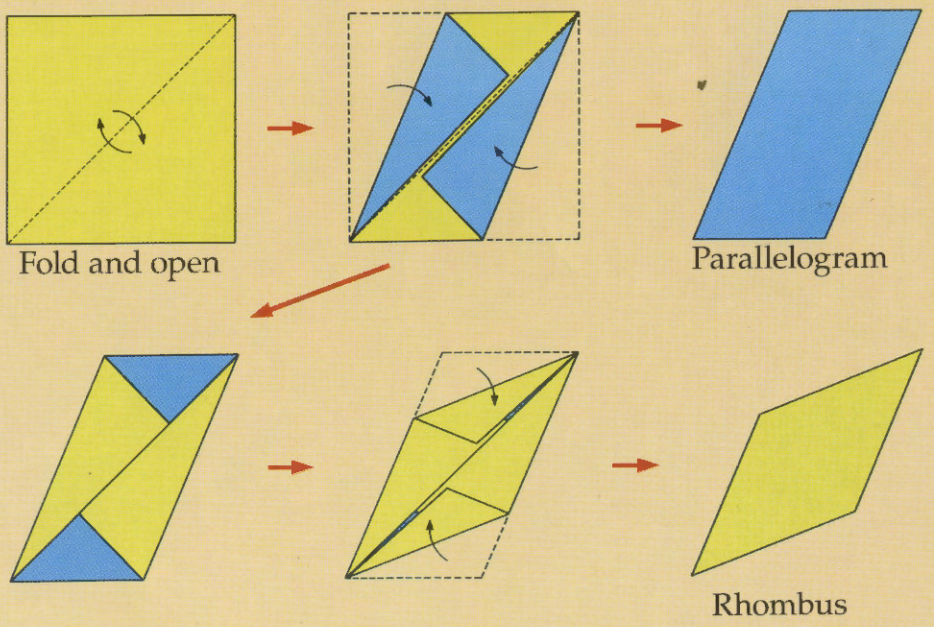




Making Various Quadrilaterals with Origami Paper







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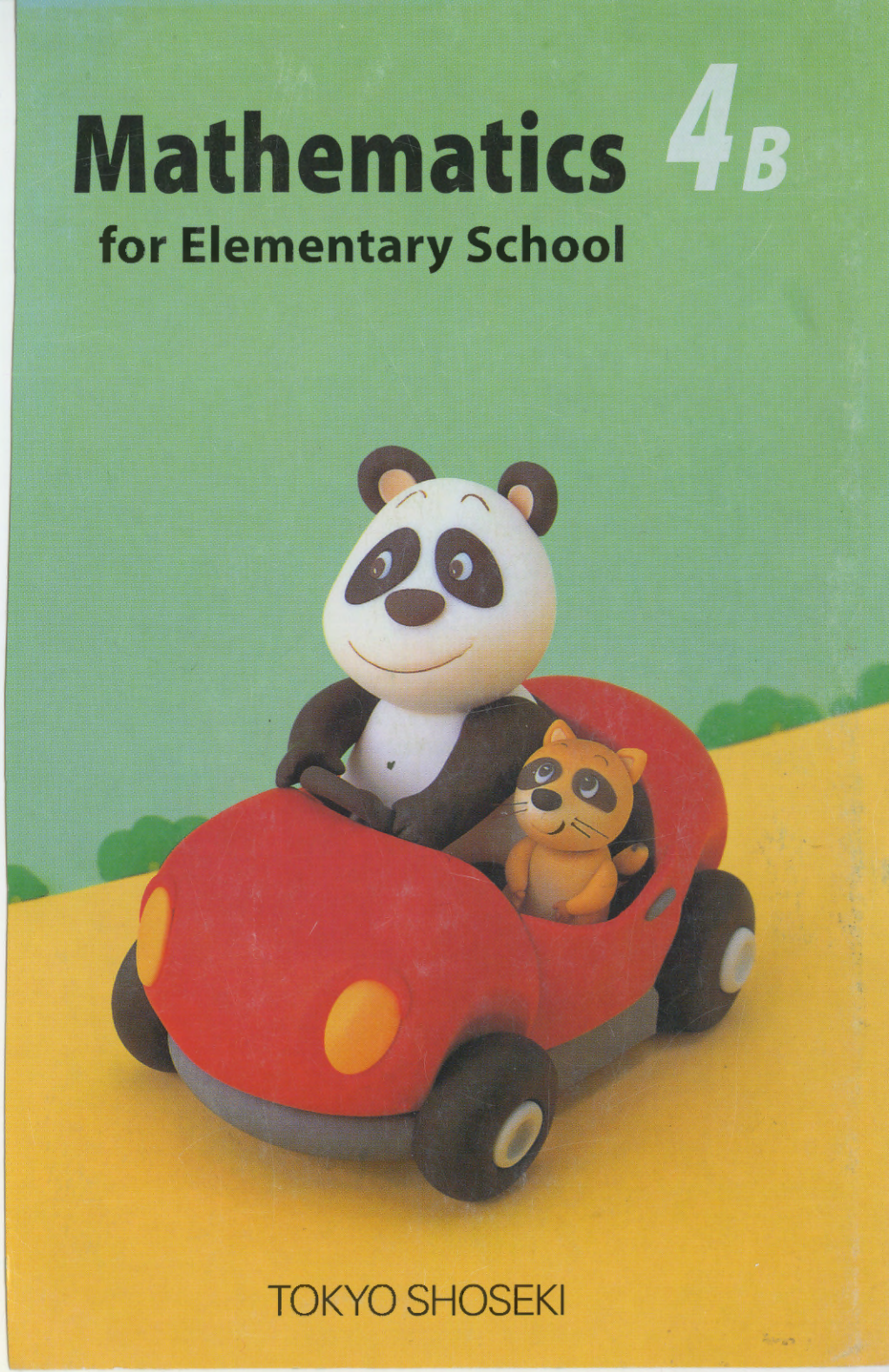
for Elementary School



TOKYO SHOSEKI



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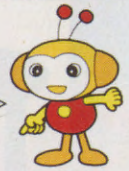

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14 Rectangular Prisms and Cubes

Tokyo





Tsukigata-cho, Hokkaido

10 Area

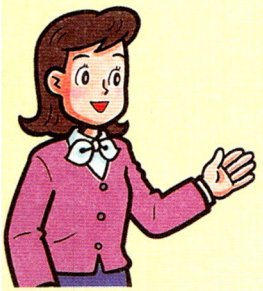
9 Decimal Numbers



1 How to Express Quantities with Decimal Numbers

1 Let's express the quantity of juice in liters!

? Let's think about how to express fractional parts that are less than 0.1!

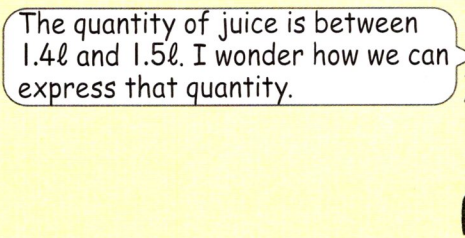


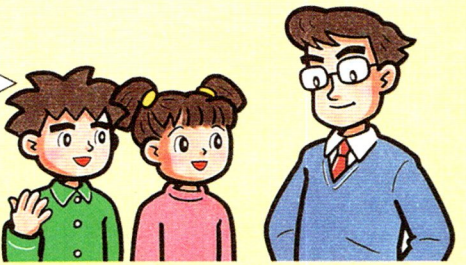
The quantity of milk

1ℓ and a little bit

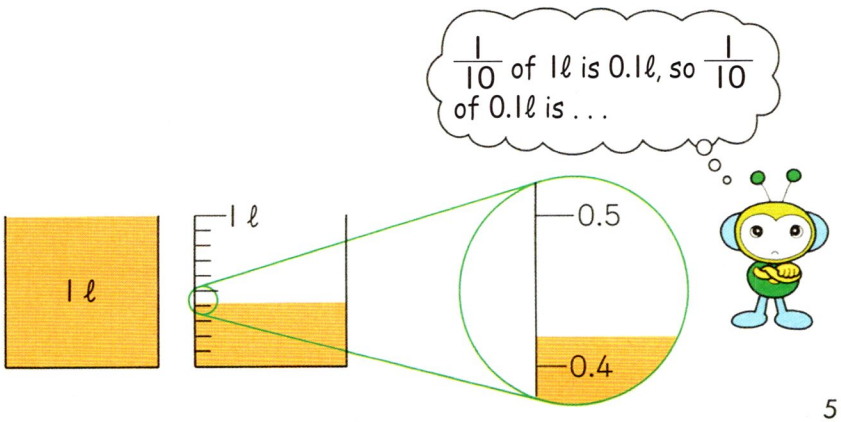
□ 0.1ℓ makes □ ℓ

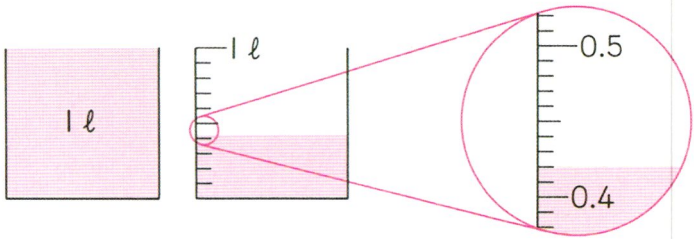
Total is □ ℓ





Let's think about how to express the fractional parts that are less than 0.1!





? $\frac{1}{10}$ of 0.1ℓ is written as 0.01ℓ and is read as "zero point zero one liters."

$\frac{1}{10}$ of 1ℓ 0.1ℓ

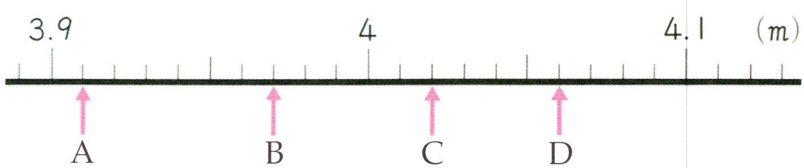
$\frac{1}{10}$ of 0.1ℓ 0.01ℓ

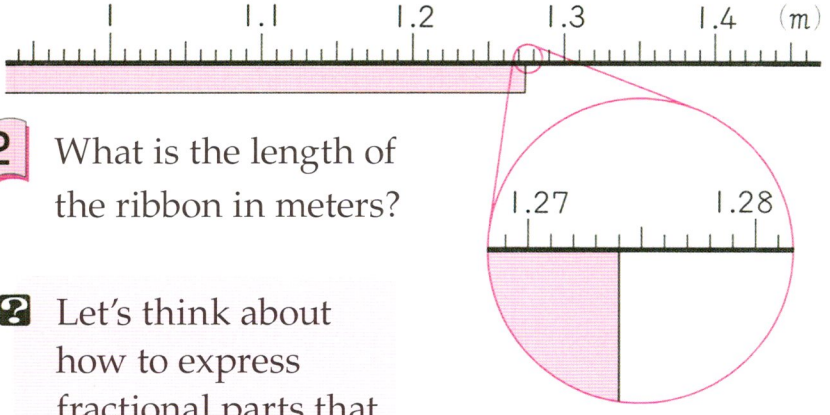
1 How many liters are two 0.01ℓ?

The combined quantity of 1.4ℓ and 0.02ℓ is written as 1.42ℓ, and read as "one point four two liters."

1 How many 0.01ℓ are in 0.05ℓ, 0.09ℓ, and 0.1ℓ, respectively?

2 Express the lengths at each mark A, B, C, and D in meters.





2 What is the length of the ribbon in meters?

Let's think about how to express fractional parts that are less than 0.01.

$\frac{1}{10}$ of 0.01m is written as 0.001m and is read as "zero point zero zero one meters."

$\frac{1}{10}$ of 0.1m 0.01m

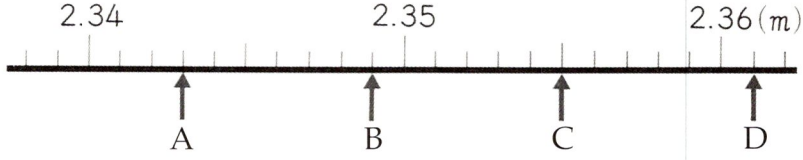
$\frac{1}{10}$ of 0.01m 0.001m

1 How many meters are four 0.001m?

The combined length of 1.27m and 0.004m is written as 1.274m and is read as "one point two seven four meters."

1 How many 0.001m are in 0.003m, 0.006m, and 0.01m, respectively?

② Please express the lengths at each mark A, B, C, and D in meters.



3 Let's express 1kg 325g using kilograms!

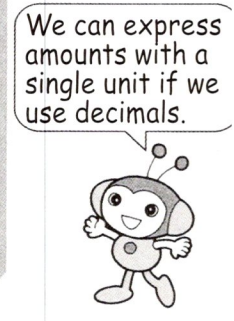
100g ···($\frac{1}{10}$ of 1kg)····· 0.1 kg

10g ···($\frac{1}{10}$ of 0.1kg)····· 0.01 kg

1g ···($\frac{1}{10}$ of 0.01kg)··· 0.001 kg

1

1 kg ·····	1	kg
300g ·····		kg
20g ·····		kg
5g ·····		kg
1 kg 325g ·····		kg



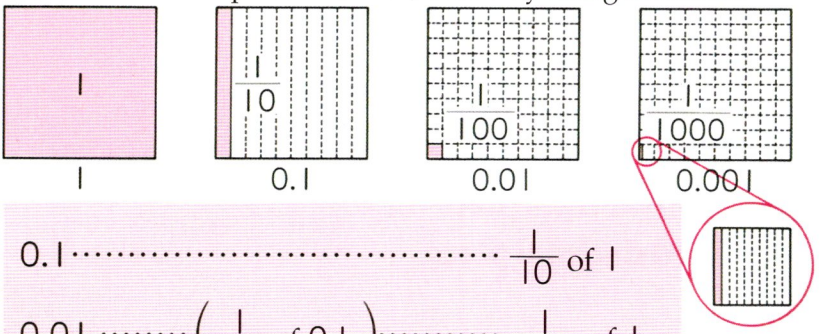
① Let's express the following weights using kilograms.

(1) 3kg 284g (2) 5kg 80g (3) 927g

2 System of Decimal Numbers

1 Let's investigate the relationships between 1, 0.1, 0.01, and 0.001!

1 How can me express 0.1, 0.01, 0.001 by using fractions?



0.1 ····· $\frac{1}{10}$ of 1

0.01 ····· ($\frac{1}{10}$ of 0.1) ····· $\frac{1}{100}$ of 1

0.001 ····· ($\frac{1}{10}$ of 0.01) ····· $\frac{1}{1000}$ of 1

2 Let's think about how to make the number 2.345!

1 How many each of 1, 0.1, 0.01, and 0.001 are combined to make the number 2.345?

2.345

2 ····· ☐ 1

0.3 ····· ☐ 0.1

0.04 ····· ☐ 0.01

0.005 ····· ☐ 0.001

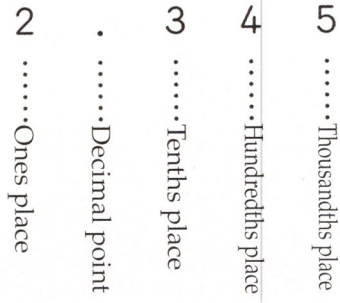
① How many each of 1, 0.1, and 0.001 are combined to make the number 9.706?

3 Let's investigate the place values of 2.345!

1 In what place is the digit 3?

2 What should we call the place to the right of the tenths place?

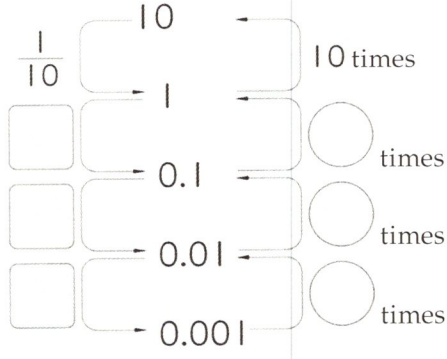
The places to the right of the tenths are called the **hundredths place**, or **second decimal place**, and the **thousandths place**, or **third decimal place**.



① What is the place value of the 7 in 6.078?

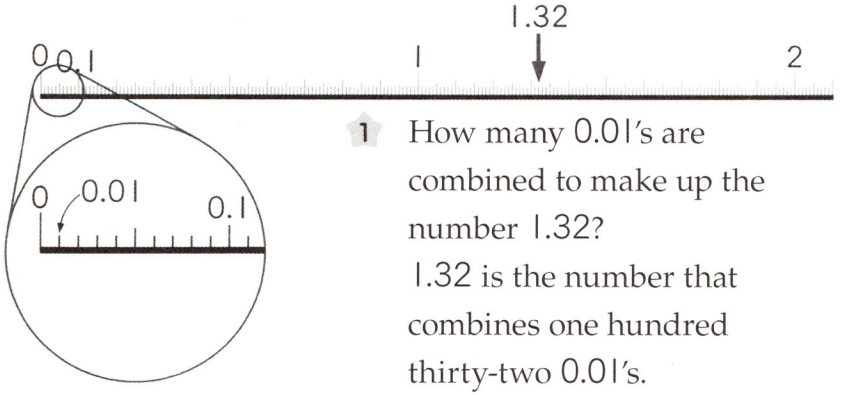
4 Let's investigate the system of decimal numbers!

1 Fill in the □ with the appropriate numbers.
Also, fill in the ○ with the appropriate numbers.



Just like whole numbers, decimal numbers have the characteristic that every time you move one place to the left the place value increases by a factor of 10.

5 How many 0.01 are in the number 1.32?



1 How many 0.01's are combined to make up the number 1.32?
1.32 is the number that combines one hundred thirty-two 0.01's.

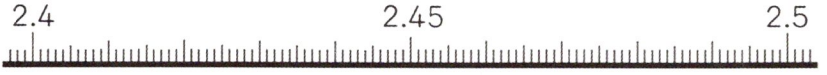
① How many 0.01 are combined to make up the following numbers?

(1) 0.04 (2) 0.19 (3) 2.51 (4) 0.1 (5) 4.2

▶ Comparing Decimal Numbers

6 Let's compare the following decimal numbers!

Ⓐ 2.43 Ⓑ 2.477 Ⓒ 2.408

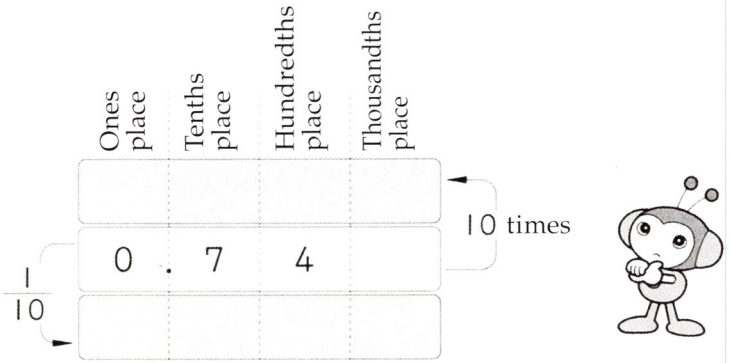


1 Mark the decimal numbers Ⓐ , Ⓑ , and Ⓒ with ↑ on the number line to compare them.

① Write the number that is 0.005 greater than 2.45. Also, write the number that is 0.03 less than 2.5.

▶ Numbers and place values that are different by a factor of 10 or $\frac{1}{10}$

7 What number is 10 times 0.74?
Also, what number is $\frac{1}{10}$ of 0.74?



Let's find what happens to the digits and their places when a decimal number is multiplied or divided by 10!

1. When 0.74 becomes 10 times bigger, by how many places does each digit increase?
2. When 0.74 becomes $\frac{1}{10}$ smaller, by how many places does each digit decrease?

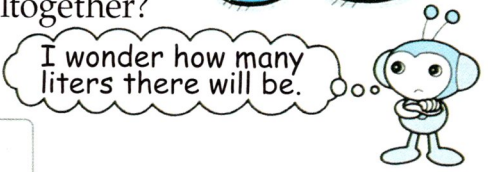
Just like whole numbers, decimal numbers have the characteristic that when you multiply a decimal number by 10 each digit increases by one place. Also when you divide a decimal number by 10, each digit decreases by one place.

① What numbers are 10 times and 100 times 0.12?
Also, what number is $\frac{1}{10}$ of 0.12?

3 Addition and Subtraction of Decimal Numbers

1 The thermos has 1.75ℓ of water, and the tea kettle has 2.64ℓ. How many liters of water are there altogether?





Math Sentence

Let's think about how to calculate!

1	0.1	0.01
□	□□□□□□□□□	□□□□□□
□□	□□□□□□	□□□□□



Kazuya

I will think about how many 0.01's, 0.1's, and 1's there are separately.

I will think about 0.01 as a unit and how to calculate 175+264.



Naoko

How to calculate 1.75+2.64

❶ 1.75 + 2.64 ➡

Write the numbers by lining up the place values.

❷ 1.75 + 2.64 = 439 ➡

Just like whole number addition, start calculating from the rightmost place.

❸ 1.75 + 2.64 = 4.39

Write the decimal point right underneath the other decimal points.

1 (1) $\begin{array}{r} 2.35 \\ +\ 4.52 \\ \hline \end{array}$ (2) $\begin{array}{r} 36.42 \\ +\ \ 1.89 \\ \hline \end{array}$ (3) $\begin{array}{r} 0.39 \\ +\ 0.63 \\ \hline \end{array}$ (4) $\begin{array}{r} 0.483 \\ +\ 7.562 \\ \hline \end{array}$

2 Let's think about how to calculate 0.574+0.226!

The answer, 0.800, is the same as 0.8.

$$\begin{array}{r} 0.574 \\ +\ 0.226 \\ \hline 0.800 \end{array}$$

1 (1) $\begin{array}{r} 8.47 \\ +\ 6.73 \\ \hline \end{array}$ (2) $\begin{array}{r} 0.074 \\ +\ 0.586 \\ \hline \end{array}$ (3) $\begin{array}{r} 0.053 \\ +\ 0.047 \\ \hline \end{array}$ (4) $\begin{array}{r} 13.98 \\ +\ \ 6.02 \\ \hline \end{array}$

3 Let's think about how to calculate 3.6+0.835!

Think of 3.6 as 3.600 to calculate.

$$\begin{array}{r} 3.600 \\ +\ 0.835 \\ \hline 4.435 \end{array}$$

1 (1) $\begin{array}{r} 18.3 \\ +\ \ 4.05 \\ \hline \end{array}$ (2) $\begin{array}{r} 0.768 \\ +\ 7.24 \\ \hline \end{array}$ (3) $\begin{array}{r} 32 \\ +\ \ 9.34 \\ \hline \end{array}$

2 (1) 7.58+62.5 (2) 12.7+0.45 (3) 4+9.47

3 Are the following math sentences on the left equal to the ones on the right?

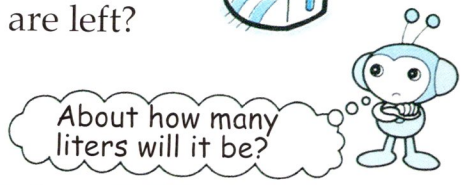
(1) 7.9+2.64 2.64+7.9

(2) (1.34+2.1)+0.86 1.34+(2.1+0.86)

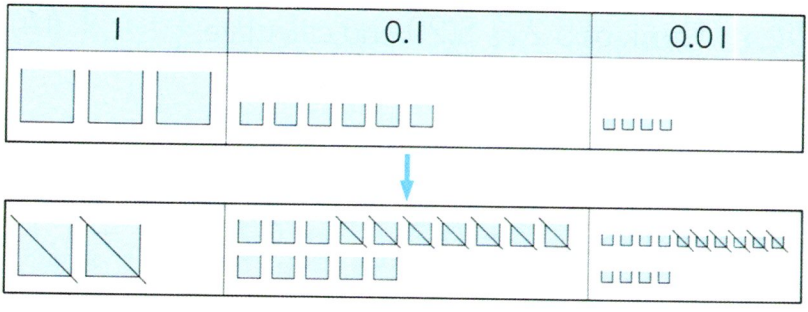
4 Out of 3.64ℓ of water, 2.76ℓ were used.
How many liters of water are left?



Math Sentence



Let's think about how to calculate!





Mami

I am thiking about how many 0.01's, 0.1's and 1's there are.

I am thinking about subtracting 364−276 by using 0.01 as a unit.



Makoto

How to calculate 3.64−2.76

❶
$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline \end{array}$$

Write the numbers, lining up the place values.

❷
$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline 88 \end{array}$$

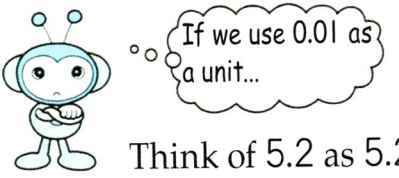
Just like whole number subtraction, start calculating from the right-most place.

❸
$$\begin{array}{r} 3.64 \\ -\ 2.76 \\ \hline 0.88 \end{array}$$

Write the decimal point right underneath the other decimal points and then write down a 0.

1 (1) $\begin{array}{r} 6.85 \\ -\ 4.96 \\ \hline \end{array}$ (2) $\begin{array}{r} 4.03 \\ -\ 0.86 \\ \hline \end{array}$ (3) $\begin{array}{r} 37.24 \\ -\ 5.9 \\ \hline \end{array}$ (4) $\begin{array}{r} 9.35 \\ -\ 8.62 \\ \hline \end{array}$

5 Let's think about how to calculate 5.2−3.76!



Think of 5.2 as 5.20 and calculate.

$$\begin{array}{r} 5.20 \\ -\ 3.76 \\ \hline 1.44 \end{array}$$

1 (1) $\begin{array}{r} 0.6 \\ -\ 0.37 \\ \hline \end{array}$ (2) $\begin{array}{r} 2.3 \\ -\ 0.21 \\ \hline \end{array}$ (3) $\begin{array}{r} 9.56 \\ -\ 0.882 \\ \hline \end{array}$ (4) $\begin{array}{r} 1.14 \\ -\ 1.137 \\ \hline \end{array}$

6 Let's think about how to calculate 3−0.42!

Think of 3 as 3.00 and calculate.

$$\begin{array}{r} 3.00 \\ -\ 0.42 \\ \hline 2.58 \end{array}$$

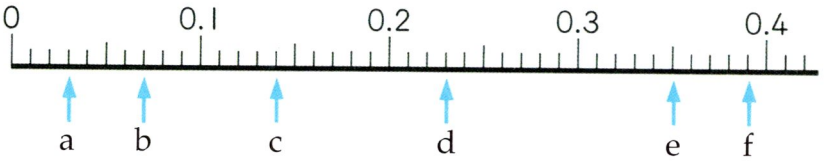
1 (1) $\begin{array}{r} 8 \\ -\ 3.54 \\ \hline \end{array}$ (2) $\begin{array}{r} 7 \\ -\ 6.38 \\ \hline \end{array}$ (3) $\begin{array}{r} 42 \\ -\ 0.96 \\ \hline \end{array}$ (4) $\begin{array}{r} 1 \\ -\ 0.997 \\ \hline \end{array}$

2 (1) $10.56-9.8$ (2) $52.8-0.84$ (3) $5-0.93$

3 (1) $2.73+14.5-16.15$ (2) $9-0.65-2.741$

Practice

① What decimal numbers are indicated by a, b, c, d, e, and f?



② Write down the following numbers.

(1) The number that is made up of three 0.01's and nine 0.001's

(2) The number that is 100 times 0.007

(3) The number that is greater than 2.8 by 0.07

③ Calculate the following.

(1) $5.7 + 19.6$ (2) $73.26 + 6.74$ (3) $4.95 + 0.136$

(4) $8.37 - 7.97$ (5) $13 - 0.82$ (6) $15.8 - 6.75$

(7) $10.34 - 5.62 + 4.08$ (8) $58 - 9.49 - 25.5$

Challenge

Let's Make Different Decimal Numbers!

Using the cards on the right, let's make different decimal numbers!

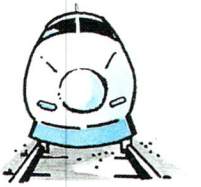


The smallest number

The largest number

The number closest to 45

Check



1 The width of the Shinkansen Bullet Train track is 1.435m. Answer the following questions about this number.

(1) What digit is in the hundredths place?

(2) The number is made up of 1.4 and what else?

(3) How many 0.001's are in the number?

2 Order the following numbers from greatest to least.

0.23 0.02 0.3 2.03 0.32

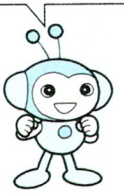
3 Find the errors in the following calculations and recalculate them correctly.

(1)	(2)	(3)	(4)
0.7 + 0.6 = 0.13	2.65 + 11.9 = 3.84	7.2 − 1.89 = 5.49	8.05 − 6.98 = 1.7

Challenge

Magic Square with Decimal Numbers

Let's fill in the blank boxes so that every row, column, and diagonal adds up to 14.4!

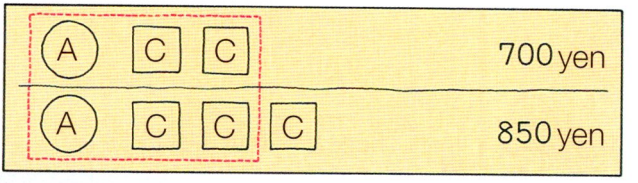


	4.8	
3.69		1.47

Problem Solving

The Botanical Garden

1 The total admission fee for one adult and two children is 700 yen. It would be 850 yen for one adult and three children.
What is the admission for one adult? One child?



① The price for two bags of morning glory seeds and three bags of cosmos seeds is 670 yen. The price for one bag of morning glory seeds and three bags of cosmos seeds is 530 yen.
What are the prices for each bag of morning glory seeds and cosmos seeds?

yen

yen



Math Plaza

Using Four 4's





There are four 4's. Let's make different math sentences by inserting +, −, ×, ÷, and ( )!

4 4 4 4

$4 \times 4 + 4 \div 4 = \square$

$(4 + 4) \times 4 + 4 = \square$

$(4 \times 4 - 4) \times 4 = \square$



What are the answers?



Let's make math sentences with the answers 0 or 1!

$4 \bigcirc 4 \bigcirc 4 \times 4 = 0$

$4 \times 4 \bigcirc (4 \bigcirc 4) = 0$

$(4 + 4) \bigcirc (4 \bigcirc 4) = 1$

$(4 \bigcirc 4 \bigcirc 4) \div 4 = 1$

There must be many more . . .





Now, let's make math sentences with the answers 2, 3, ..., 9.

$4 \bigcirc 4 + 4 \bigcirc 4 = 2$

$(4 \bigcirc 4 \bigcirc 4) \div 4 = 3$

$4 \bigcirc (4 \bigcirc 4) \times 4 = 4$

$(4 \bigcirc 4 + 4) \bigcirc 4 = 5$

$(4 \bigcirc 4) \bigcirc 4 + 4 = 6$

$4 \bigcirc 4 - 4 \bigcirc 4 = 7$

$(4 \bigcirc 4) \bigcirc 4 \div 4 = 8$

$4 \bigcirc 4 \bigcirc 4 + 4 = 9$

Let's make more of these math sentences!



10 Area



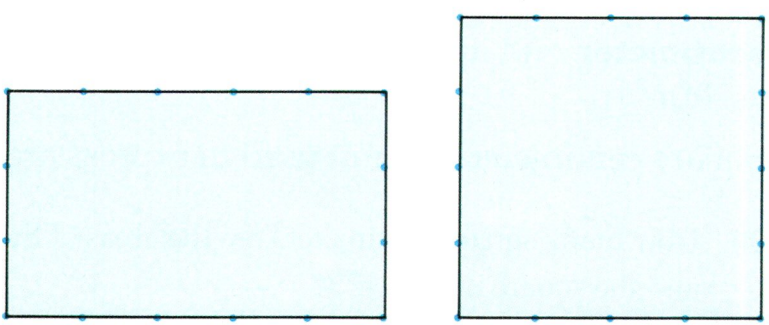
1 Area

Area

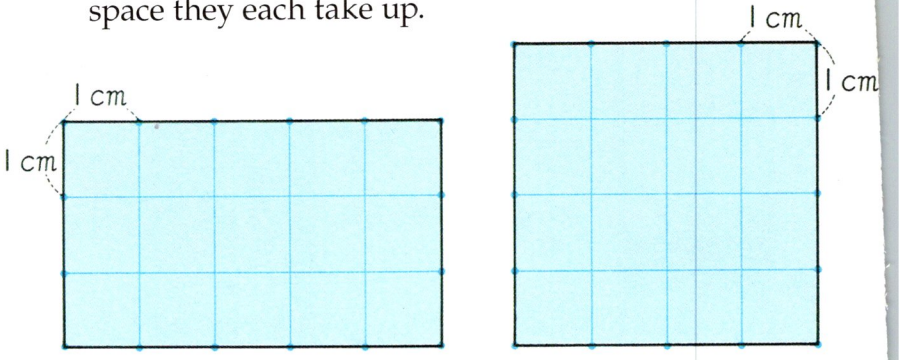
1 Which of the shapes on the right takes up more space, the rectangle or the square, and by how much? Let's think about a way to express how big they are!



Let's think about a way to compare and express the size of these shapes!



1 As shown below, divide the sides of the rectangle and the square into 1 cm segments and compare how much space they each take up.

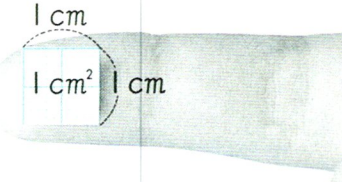


2 How many 1 cm squares are there inside the rectangle an the square?

The amount of space inside the rectangle and the square can be expressed by the number of 1 cm squares that fill up the space.

The amount of space inside a shape is called the **area**.

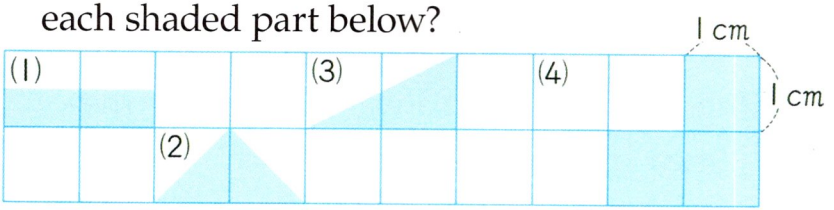
The area of a square with 1 cm sides is called **1 square centimeter**, and it is written as $1\,cm^2$.



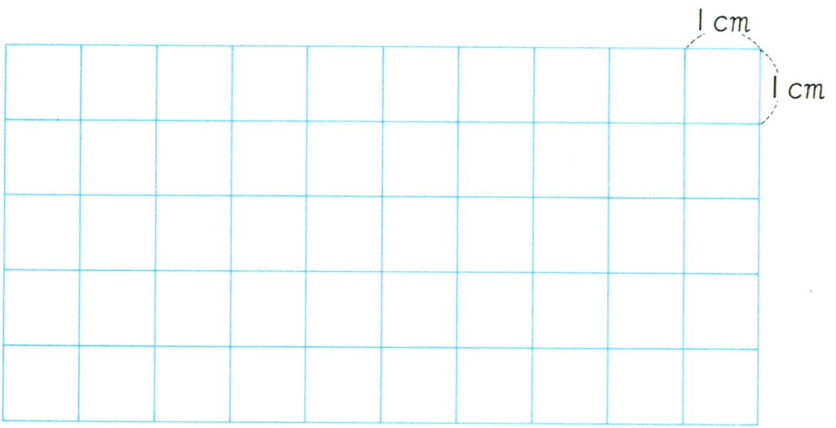
Square centimeter is a unit used to express area.

3 How many square centimeters are the areas of the rectan and the square on page 23?

1 How many square centimeters is the area of each shaded part below?



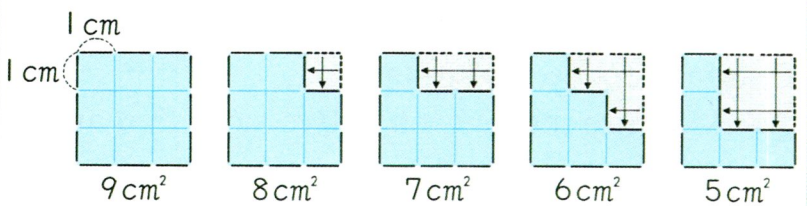
2 Draw many shapes with an area of $4cm^2$.



Math Story

Areas of Shapes with Equal Perimeters

All the shapes below have equal perimeters, but their areas are different.

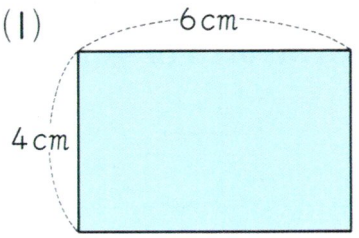


2 Area of Rectangles and Squares

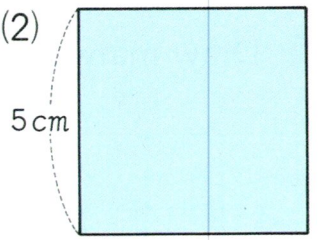
▶ Formula for the area of rectangles and squares

1 Let's think about a way to calculate the area of a rectangle or a square!

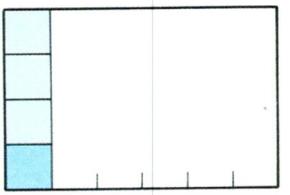
(1)



(2)



1 How many $1 cm^2$ squares can be lined up along the length (vertical side) of the rectangle? How many can be lined up along its width (horizontal side)?



2 How many $1 cm^2$ squares are there inside the whole rectangle? Let's multiply to find out! What is the area of the rectangle?

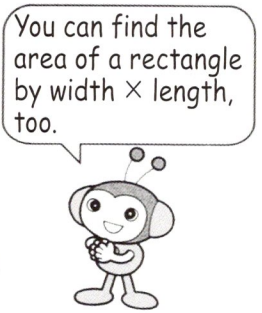
3 How many cm^2 is the area of the square? Let's multiply to find out!

! In order to calculate the area of a rectangle or a square, follow these steps:

1. Measure the lengths of two sides that are next to each other.
2. Multiply the two numbers that represent the lengths of the two sides.

Area of a rectangle = length × width

Area of a square = side × side



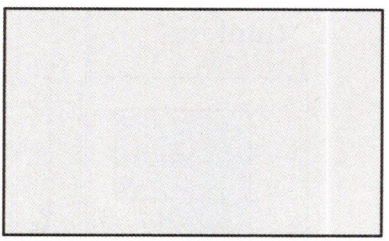
The above math sentences are called **formulas** for the area of a rectangle and a square.

① How many cm^2 are the areas of the rectangle and the square below?

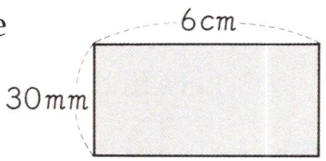
(1) A rectangle that is 15cm long and 23cm wide.

(2) A square that is 20cm on each side.

② Measure the sides of the rectangle on the right and find the area.

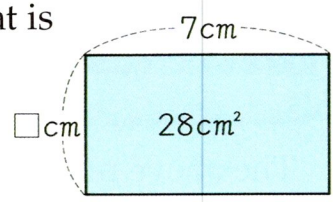


③ Find the area of a rectangle that is 30mm long and 6cm wide.



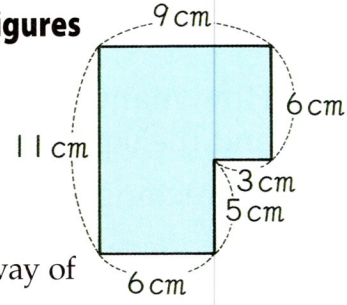
When calculating area, you need to use the same units of length for all the sides.

④ In order to draw a rectangle with an area of $28cm^2$, that is $7cm$ wide, how many cm long should it be?

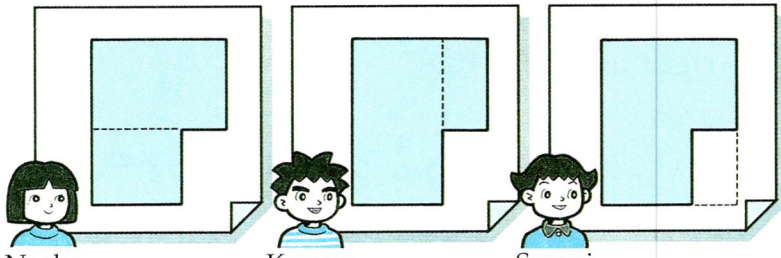


▶ Finding the area of composite figures

2 Let's think of a way to find the area of the shape on the right!

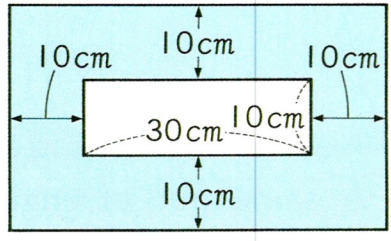


1 Let's explain each friend's way of thinking!



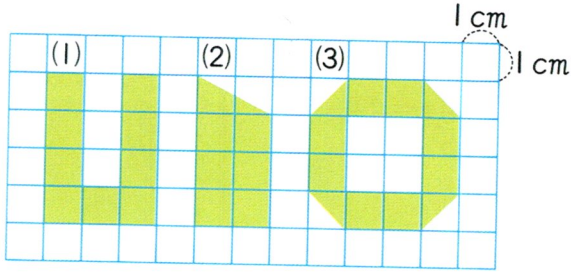
2 Let's find the area using each method!

① How many cm^2 is the area of the shape on the right?

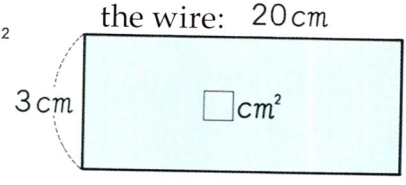


Exercise 1

① How many cm^2 is the area of shaded part?



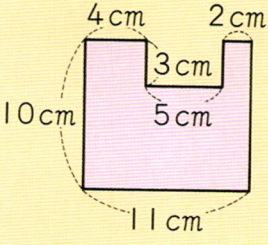
② Using a 20cm long wire, make a rectangle that is 3cm long. How many cm^2 is the area?



Challenge

Which Math Sentence?

Which figure below corresponds best to each of the following math sentences?

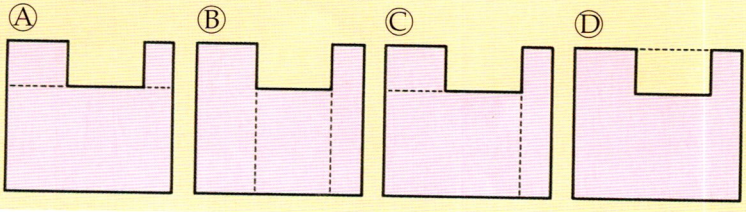


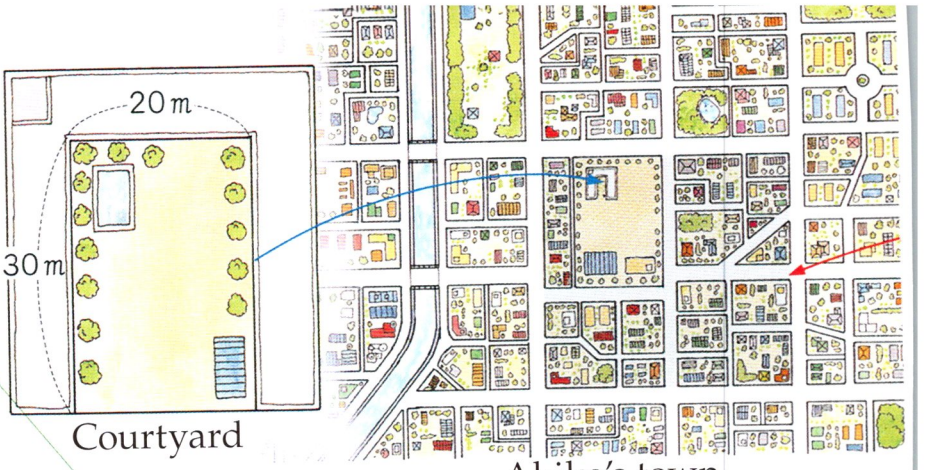
① $10 \times 4 + 7 \times 5 + 10 \times 2$ ☐

② $3 \times 4 + 7 \times 9 + 10 \times 2$ ☐

③ $10 \times 11 - 3 \times 5$ ☐

④ $3 \times 4 + 7 \times 11 + 3 \times 2$ ☐

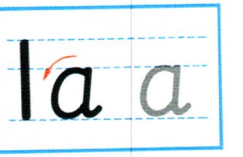




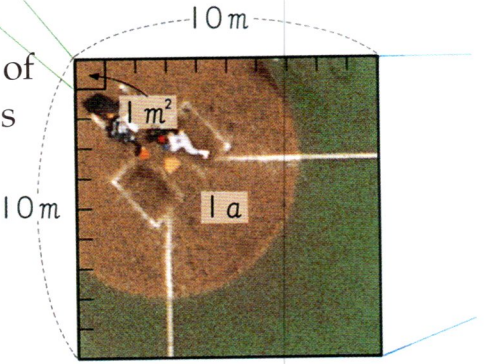
Are

3 Let's look into the area of the rectangular courtyard at Akiko's school shown above!

1 How many m^2 is the area of the courtyard?



An area of $100m^2$ is called 1 **are**, and it is written as 1a. The area of a square with 10m sides is 1a.

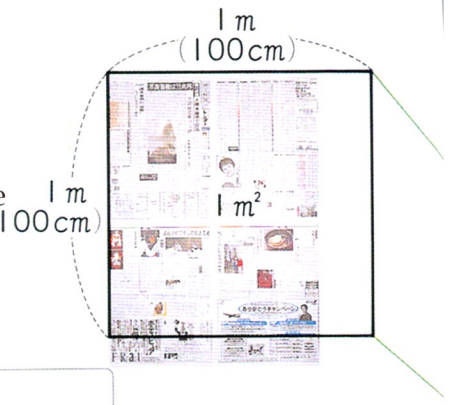


2 How many a is the area of the courtyard?

① There is a square-shaped park with 40m sides. How many a is the area of this park?

2 How many cm^2 is $1m^2$?

1 How many $1cm^2$ squares will be lined up along the length and width of a $1m^2$ square?

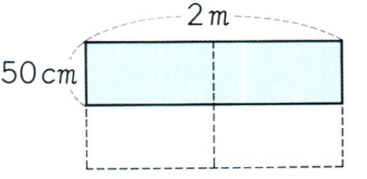


$100 \times 100 =$ ☐

$1m^2 = 10000cm^2$

1 How many cm^2 is the area of a square with $2m$ sides?

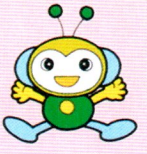
2 How many m^2 is the area of a rectangular flower bed that is $50cm$ long and $2m$ wide?



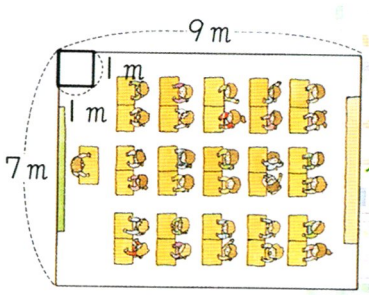
Challenge

How Many People Can Stand on a $1m^2$ Square?

How many people can stand on a $1m^2$ square? Draw a $1m^2$ square on a large sheet of paper, cut it out, then try it!







Classroom



Akiko's town

3 Units for Large Areas

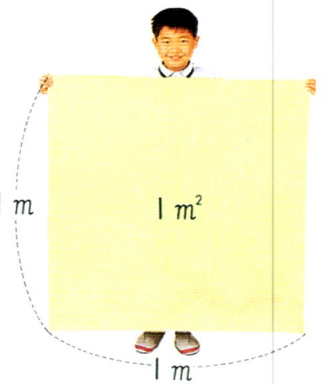
Square meters

1 Let's investigate the area of Akiko's rectangular classroom shown above!

When expressing a large area such as a classroom, we use the area of a square with $1m$ sides as the unit.

1 Inside Akiko's classroom, how many $1m$ squares can be lined up?

The area of a square with $1m$ sides is called **1 square meter**, and it is written as $1m^2$.



2 How many m^2 is the area of Akiko's classroom?

1 How many m^2 is the area of a square-shaped science classroom with $9m$ sides?

Check

1 Fill in the ☐ with the correct unit.

(1) The area of a square with 1cm sides is 1 ☐.

(2) The area of a square with 1m sides is 1 ☐.

(3) The area of a square with 1km sides is 1 ☐.

2 Find the areas of the following rectangles and squares.

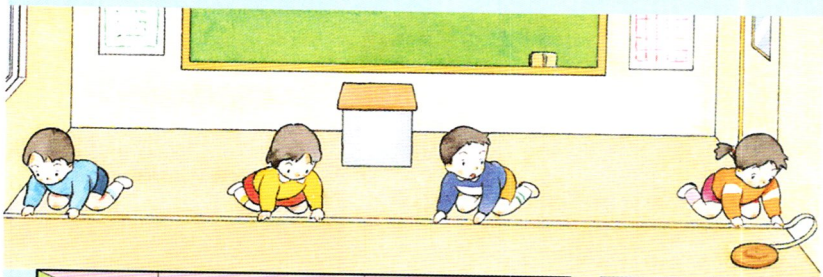
(1) A rectangle that is 8cm long and 12cm wide

(2) A square with 13cm sides

(3) A rectangle that is 3km long and 8km wide

Challenge

Let's Find the Areas



	Estimated Area	Actual Area
Textbook		
Classroom		
Playground		

❶ Estimate the area
❷ Measure the sides
❸ Calculate the area



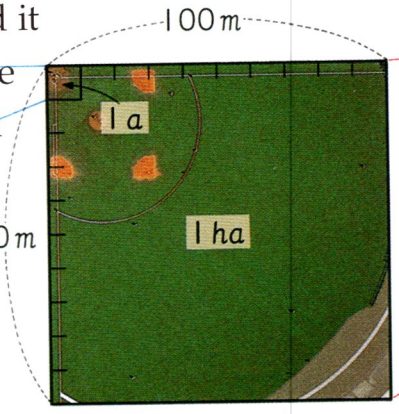
Hectare

4 Let's look into the area of the square-shaped neighborhood where Akiko's school is located, as shown above!

1 How many m^2 is the area of this neighborhood?

1ha ha

An area of $10000 m^2$ is called **1 hectare**, and it is written as 1ha. The area of a square with 100m sides is 1ha.



2 How many ha is the area of this neighborhood?

3 How many a is 1ha?

① There is a rectangular field 400m long and 600m wide. How many ha is the area of this field?

Square kilometers

5 Akiko's town is shaped almost like a rectangle. It is about $3km$ long and about $5km$ wide. Let's think about the area of her town!

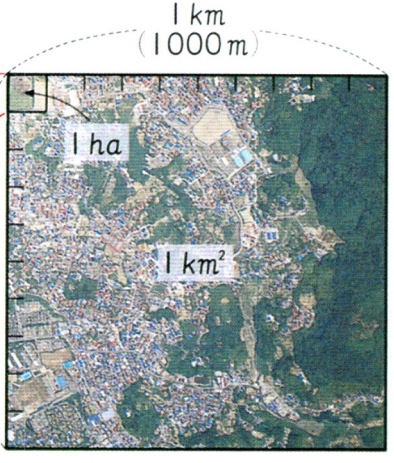
When expressing large areas such as villages, towns, cities, counties, or states, we use the area of a square with $1km$ sides as the unit.

The area of a square with $1km$ sides is called 1 **square kilometer**, and it is written as $1km^2$.

1 About how many km^2 is the area of Akiko's town?

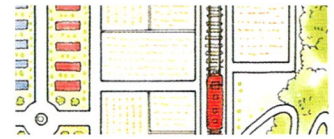
6 How many km^2 is $1km^2$?

2 How many $1m^2$ squares will line up along the length and width of a $1km^2$ square?



$1000 \times 1000 =$ ☐

$1km^2 = 1000000m^2$



Exercise 2

① Find the areas of the following rectangles and squares.

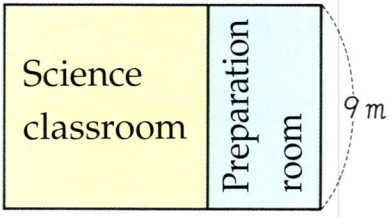
(1) How many m^2 is the area of a rectangle that is $4m$ long and $12m$ wide?

(2) How many a is the area of a square with $50m$ sides?

(3) How many ha is the area of a rectangle that is $600m$ long and $800m$ wide?

(4) How many km^2 is the area of a square with $12km$ sides?

② The combined area of the science classroom and the preparation room to the right is $126m^2$. The science room is a square. How many m is the width of the preparation room?



Math Story

Units for Large Areas



Below is a summary table of units for large areas.

Length of a side	$1m$	$10m$	$100m$	$1km$
Area of square	$1m^2$	$100m^2$ $1a$	$10000m^2$ $1ha$	$1km^2$

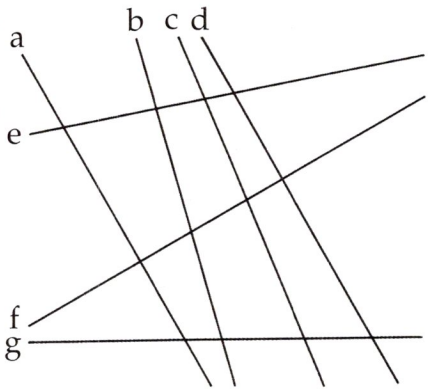
Review ◦ 5

① Calculate the following.

(1) $\begin{array}{r} 3.98 \\ +\ 5.65 \\ \hline \end{array}$ (2) $\begin{array}{r} 0.69 \\ +\ 1.532 \\ \hline \end{array}$ (3) $\begin{array}{r} 0.198 \\ +\ 1.002 \\ \hline \end{array}$ (4) $\begin{array}{r} 15.23 \\ +\ 4.77 \\ \hline \end{array}$

(5) $\begin{array}{r} 5.32 \\ -\ 2.821 \\ \hline \end{array}$ (6) $\begin{array}{r} 57.65 \\ -\ 8.95 \\ \hline \end{array}$ (7) $\begin{array}{r} 6.71 \\ -\ 3.94 \\ \hline \end{array}$ (8) $\begin{array}{r} 5.6 \\ -\ 3.82 \\ \hline \end{array}$

② Which line is parallel to line a? Which line is perpendicular to line a?



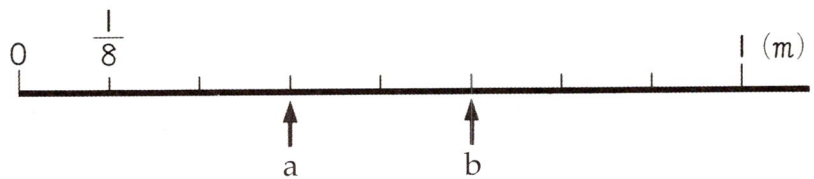
③ What shapes are the following quadrilaterals?

(1) A quadrilateral with one pair of opposite sides that are parallel

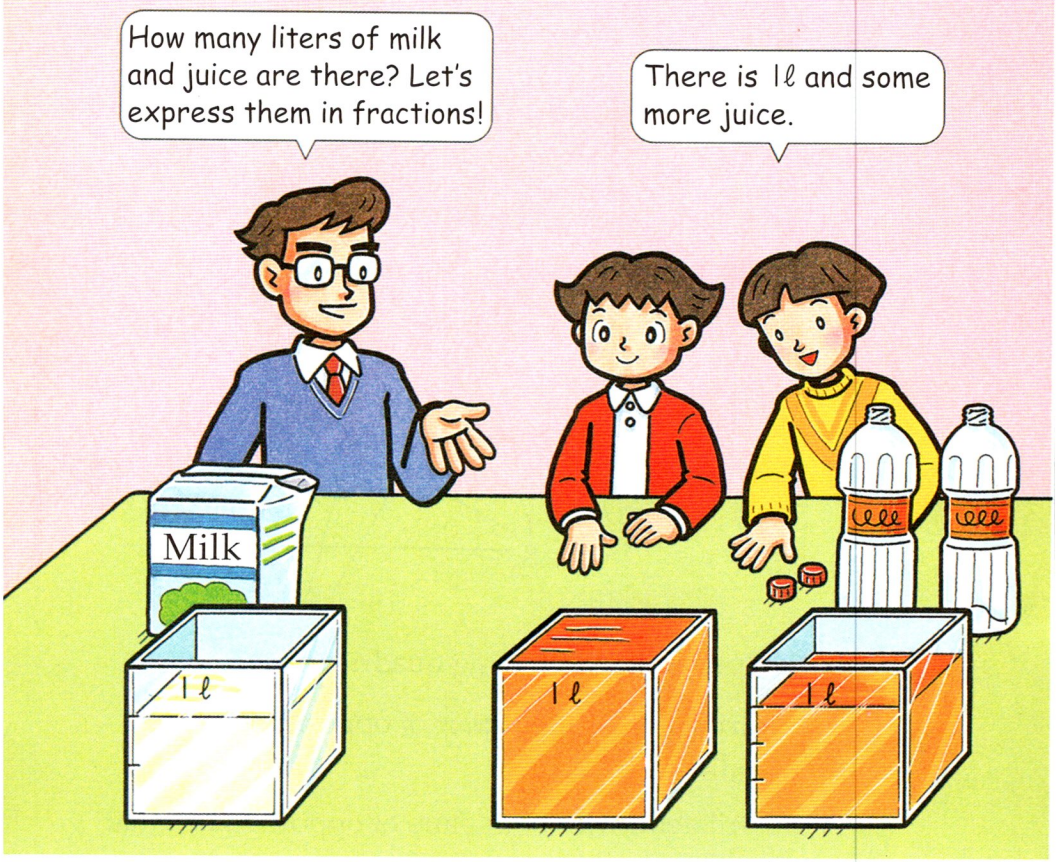
(2) A quadrilateral with both pairs of opposite sides that are parallel

(3) A quadrilateral with all four sides of equal length

④ Express the length at a and b in meters.



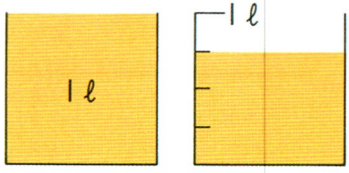
11 Fractions

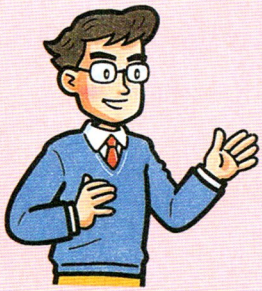


1 How to Express Quantities in Fractions

▶ Fractions that are greater than 1

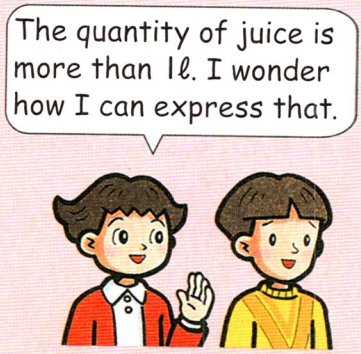
1 How many ℓ is the amount of juice?





The quantity of milk

□ $\frac{1}{3}$ ℓ makes □ ℓ



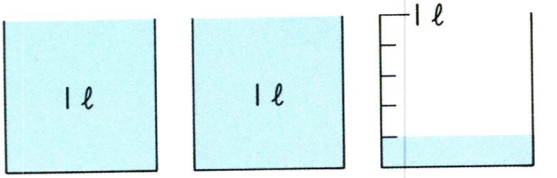
Let's think about various ways to express fractions!

Let's investigate how to express numbers that are greater than 1 in fractions!

1 Let's express the extra part using a fraction!

The combined quantity of 1 ℓ and $\frac{3}{4}$ ℓ is written as $1\frac{3}{4}$ ℓ, and it is read as "one and three fourths liters."

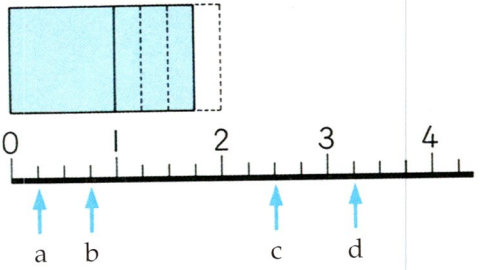
1 How many ℓ of water are shown on the right?



2 How many m is the combined length of $4m$ and $\frac{2}{3}m$?

Proper fractions and mixed numbers

2 Let's show $1\frac{3}{4}$ on the number line.



1 What fractions are indicated by a, b, c, and d?

2 Among the fractions a, b, c, and d above, and $1\frac{3}{4}$, which fractions are less than 1? Which are greater than 1?

A fraction whose numerator is less than the denominator, like $\frac{3}{4}$ or $\frac{1}{5}$, is called **a proper fraction**. A fraction that is made up of a whole number and a proper fraction, like $1\frac{3}{4}$ or $3\frac{1}{5}$, is called a **mixed number**.

Proper fractions are fractions that are less than 1 and mixed numbers are fractions that are greater than 1.

1 Which are mixed numbers?

Ⓐ $\frac{2}{9}$ Ⓑ $1\frac{3}{5}$ Ⓒ $\frac{4}{9}$ Ⓓ $2\frac{5}{6}$ Ⓔ $5\frac{3}{8}$ Ⓕ $\frac{1}{10}$

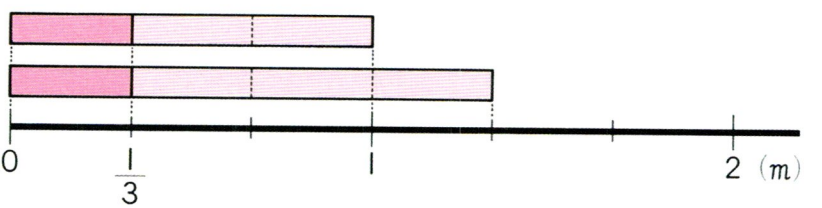
2 Fill in the ☐ with an inequality sign.

(1) $1 \square \frac{1}{8}$ (2) $\frac{3}{4} \square 2\frac{1}{4}$ (3) $2\frac{2}{5} \square 1\frac{4}{5}$

3 Express the fraction that is $\frac{3}{5}$ more than 2.

▶ Improper fractions

3 **2** How many m are three $\frac{1}{3}m$? Four $\frac{1}{3}m$?



Three $\frac{1}{3}m$ are the same as $1m$ and can be expressed as $\frac{3}{3}m$.

Also, four $\frac{1}{3}m$ are the same as $1\frac{1}{3}m$ and can be expressed as $\frac{4}{3}m$.

A fraction whose numerator is the same as or greater than its denominator, like $\frac{3}{3}$ or $\frac{4}{3}$, is called an **improper fraction**.

Improper fractions are fractions that are equal to or greater than 1.

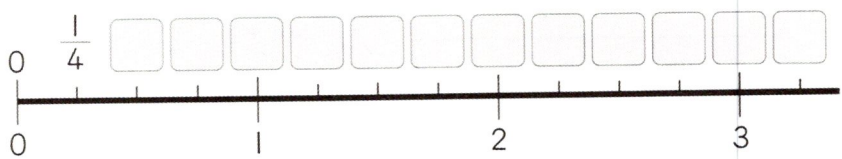
① Let's express the lengths of five $\frac{1}{3}m$ and six $\frac{1}{3}$ m using improper fractions!

② Which of the following are improper fractions?

Ⓐ $\frac{4}{7}$ Ⓑ $\frac{5}{4}$ Ⓒ $3\frac{1}{5}$ Ⓓ $\frac{6}{6}$ Ⓔ $1\frac{7}{9}$ Ⓕ $\frac{11}{8}$ Ⓖ $\frac{5}{12}$

▶ Whole numbers and fractions

4 Let's write proper and improper fractions made up of 2, 3, 4, ... $\frac{1}{4}$'s!



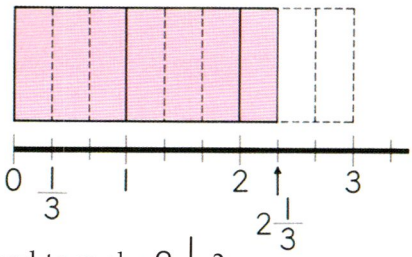
1 Let's compare the denominator and the numerator of the fraction equal to 1! Also, compare the denominators and numerators of the fractions equal to 2 and 3. What did you notice?

① Let's change improper fractions into whole numbers!

(1) $\frac{6}{3}$ (2) $\frac{5}{5}$ (3) $\frac{18}{6}$ (4) $\frac{16}{4}$ (5) $\frac{7}{7}$ (6) $\frac{16}{8}$

Mixed numbers and improper fractions

5 Let's think about how to change $2\frac{1}{3}$ into an improper fraction!



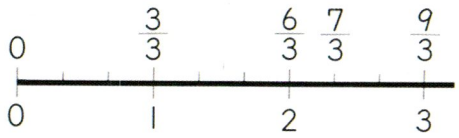
1 How many $\frac{1}{3}$'s do you need to make $2\frac{1}{3}$?

$3 \times 2 + 1 = \square$ $\quad 2\frac{1}{3} = \frac{\square}{3}$

1 Let's change mixed numbers into improper fractions!

(1) $1\frac{1}{3}$ (2) $2\frac{1}{4}$ (3) $3\frac{2}{7}$ (4) $3\frac{4}{5}$ (5) $2\frac{5}{6}$ (6) $4\frac{3}{10}$

6 Let's think about how to change $\frac{7}{3}$ into a mixed number!



1 How many $\frac{3}{3}$'s are in $\frac{7}{3}$?

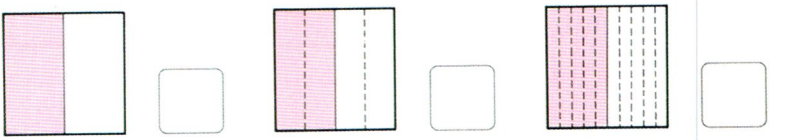
$7 \div 3 = \square$ remainder $\square$ $\quad \frac{7}{3} = \square\frac{\square}{3}$

1 Let's change improper fractions into mixed numbers or whole numbers!

(1) $\frac{9}{2}$ (2) $\frac{21}{7}$ (3) $\frac{13}{3}$ (4) $\frac{25}{5}$ (5) $\frac{33}{8}$ (6) $\frac{30}{6}$

2 Equivalent Fractions

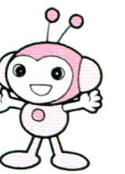
1 Let's express the colored parts below as fractions!



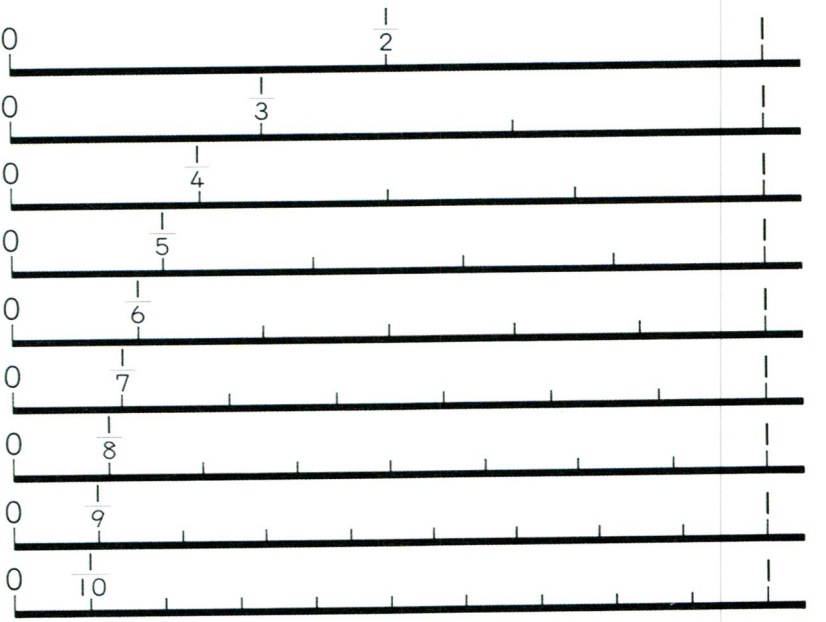
$\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$ are equivalent fractions.

$$\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$$

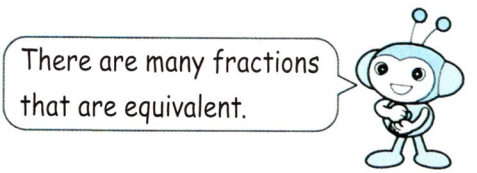
$\frac{1}{2}$ can be expressed as various fractions.



2 Let's find fractions that are equivalent using the number lines below!



1 Let's find fractions that are equivalent to $\frac{1}{3}$ or $\frac{2}{3}$!



Comparing fractions

3 Let's look at the number lines on the previous page, and compare the sizes of the fractions!

1 Which is bigger, $\frac{1}{2}$ or $\frac{1}{4}$?

2 List fractions whose numerators are 1 in decreasing order.

3 Which is bigger, $\frac{2}{3}$ or $\frac{2}{6}$?

4 List fractions whose numerators are 2 in decreasing order!

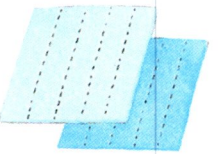
For fractions that have the same numerator, the greater the denominator, the smaller the size of the fraction.

1 Order the fractions from least to greatest.

(1) $\left(\frac{3}{6}, \frac{3}{4}, \frac{3}{5}\right)$ (2) $\left(\frac{4}{5}, \frac{4}{4}, \frac{4}{10}\right)$

3 Addition and Subtraction of Fractions

1 Shiori used $\frac{3}{5}m^2$ of cardboard, and Kiyoshi used $\frac{4}{5}m^2$. How many m^2 of cardboard did they use altogether?

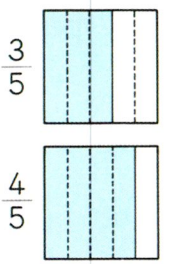


1 Write a math sentence.

Math Sentence ▶ ☐

? Let's think about how to calculate!

2 How many $\frac{1}{5}$'s do you need to make $\frac{3}{5}$ and $\frac{4}{5}$?



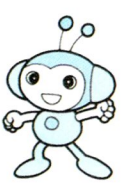
$$\frac{3}{5}+\frac{4}{5}=\frac{\square}{5}$$

$$=\square\frac{\square}{5}$$

Answer: $\square\frac{\square}{5}m^2$

! When adding fractions that have the same denominators, just add the numerators and leave the denominators as they are.

When you get improper fractions as answers, change them into mixed numbers or whole numbers, so that it is easier to figure out the size of the fractions.



1 (1) $\frac{2}{7}+\frac{4}{7}$ (2) $\frac{3}{8}+\frac{1}{8}$ (3) $\frac{4}{9}+\frac{7}{9}$ (4) $\frac{5}{6}+\frac{1}{6}$

2 There is $1\frac{2}{5}$ kg of sugar. If you use $\frac{4}{5}$ kg, how many kg will be left?

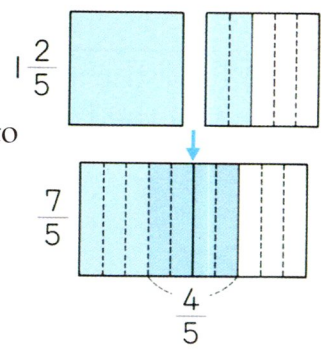


1 Write down a math sentence. Math Sentence

Let's think about how to calculate it!

2 Let's change $1\frac{2}{5}$ into $\frac{7}{5}$ and calculate!

3 How many $\frac{1}{5}$'s do you need to make $\frac{7}{5}$ and $\frac{4}{5}$?



$$1\frac{2}{5}-\frac{4}{5}=\frac{7}{5}-\frac{4}{5}$$

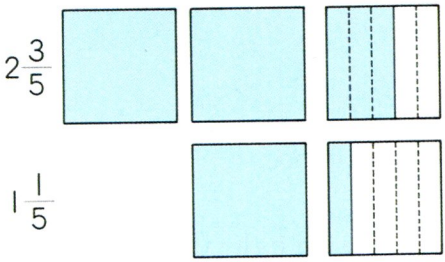
$$=\frac{\square}{5}$$

Answer: $\frac{\square}{5}$ kg

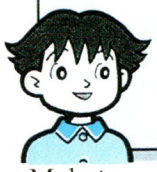
When subtracting fractions that have the same denominators, just subtract the numerators and leave the denominators as they are.

1 (1) $\frac{4}{7}-\frac{2}{7}$ (2) $\frac{7}{9}-\frac{4}{9}$ (3) $1\frac{1}{3}-\frac{2}{3}$ (4) $1\frac{3}{8}-\frac{7}{8}$

3 Let's think about how to calculate $2\frac{3}{5}+1\frac{1}{5}$!



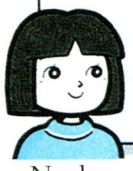
1 Please explain each friend's way of calculating.



Makoto

Find the sum of the whole number parts and the fractional parts, and combine them.

$2\frac{3}{5}+1\frac{1}{5}=\square\frac{\square}{5}$



Naoko

Change the mixed numbers into improper fractions, and calculate.

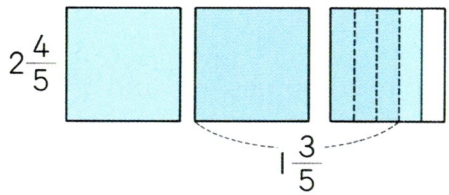
$2\frac{3}{5}+1\frac{1}{5}=\frac{13}{5}+\frac{6}{5}$

$=\frac{19}{5}=\square\frac{\square}{5}$

① (1) $1\frac{1}{3}+3\frac{1}{3}$ (2) $\frac{1}{5}+3\frac{3}{5}$ (3) $2\frac{5}{6}+3$

② (1) $3\frac{4}{7}+1\frac{6}{7}$ (2) $2\frac{3}{4}+\frac{3}{4}$ (3) $\frac{5}{9}+3\frac{8}{9}$

4 Let's think about how to calculate $2\frac{4}{5} - 1\frac{3}{5}$!



1 Please explain each friend's way of calculating.



Mami

Find the difference of the whole number parts and the difference of the fractional parts and combine them.

$2\frac{4}{5} - 1\frac{3}{5} = \square\frac{\square}{5}$



Kazuya

Change the mixed fractions into improper fractions and calculate.

$2\frac{4}{5} - 1\frac{3}{5} = \frac{14}{5} - \frac{8}{5}$

$= \frac{6}{5} = \square\frac{\square}{5}$

1 (1) $3\frac{5}{7} - 1\frac{3}{7}$ (2) $2\frac{3}{4} - \frac{3}{4}$ (3) $6\frac{5}{8} - 4$

2 (1) $4\frac{5}{7} - 2\frac{6}{7}$ (2) $7\frac{1}{5} - 6\frac{4}{5}$ (3) $5 - 1\frac{3}{4}$

Practice

① Fill in the ☐ with the appropriate numbers.

(1) $\frac{7}{9}$ is a number that is made up of ☐ $\frac{1}{9}$'s.

(2) $3\frac{3}{5}$ is a number that combines ☐ and $\frac{3}{5}$.

(3) The fraction whose denominator is 8 and numerator is ☐, is the same as 1.

② Fill in the ☐ with the appropriate numbers.

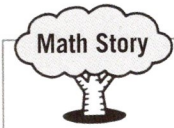
(1) $2 = \frac{\square}{5}$ (2) $3 = 2\frac{\square}{4}$ (3) $2\frac{1}{7} = 1\frac{\square}{7}$

③ Calculate the following.

(1) $\frac{5}{9} + \frac{7}{9}$ (2) $\frac{7}{10} + \frac{9}{10}$ (3) $1\frac{3}{8} - \frac{5}{8}$

(4) $1\frac{1}{10} - \frac{3}{10}$ (5) $2\frac{1}{3} + \frac{1}{3}$ (6) $1\frac{7}{8} + 2\frac{5}{8}$

(7) $4\frac{3}{5} - \frac{1}{5}$ (8) $3\frac{2}{9} - 2\frac{5}{9}$ (9) $1\frac{3}{8} + \frac{7}{8} - \frac{5}{8}$



The Composition of Whole Numbers, Decimal Numbers, and Fractions

Whole numbers, decimal numbers, and fractions all express a quantity with a number of units.

$0.2+0.5$ ··· $(2+5)$ 0.1's

$\frac{2}{9}+\frac{5}{9}$ ··· $(2+5)$ $\frac{1}{9}$'s

7 ··· ☐ 1's

0.7 ··· ☐ 0.1's

$\frac{7}{9}$ ··· ☐ $\frac{1}{9}$'s

Check

1 Answer the questions about the fractions below.

Ⓐ $\frac{11}{3}$ Ⓑ $\frac{2}{7}$ Ⓒ $3\frac{1}{4}$ Ⓓ $\frac{9}{5}$ Ⓔ $1\frac{5}{9}$ Ⓕ $\frac{7}{8}$

(1) Make groups of proper fractions, mixed numbers, and improper fractions.

(2) Change the improper fractions into mixed numbers, and the mixed numbers into improper fractions.

2 Order the following fractions from greatest to least.

(1) $\left(\frac{1}{4}, \frac{1}{9}, \frac{1}{7}\right)$ (2) $\left(\frac{5}{8}, \frac{10}{8}, \frac{8}{8}\right)$

3 Find any errors in the following calculations and recalculate them correctly.

(1) $\frac{4}{5}+\frac{2}{5}=\frac{6}{10}$ (2) $1\frac{2}{7}-\frac{6}{7}=\frac{12}{7}-\frac{6}{7}$
$=\frac{6}{7}$

Challenge

Magic Squares with Fractions

Let's fill in the blank boxes so that every row, column, and diagonal adds up to $2\frac{1}{7}$!



	$\frac{5}{7}$	1
		$\frac{6}{7}$

Review ◦ 6

① Calculate the following.

(1) $46 \times 15 - 18 \times 16$ (2) $144 + 144 \div 12 - 12$

(3) $184 + 75 \times (9 + 6)$ (4) $500 - 96 \div 6 \times 9$

② Calculate the following.

(1) $4.35 + 2.95$ (2) $0.18 + 4.93$ (3) $3.7 + 1.87$

(4) $3.38 - 3.25$ (5) $7.05 - 0.28$ (6) $6 - 0.279$

③ Answer the questions about 5.847.

(1) The number is made up of 5 and what else?

(2) What is the place value of the digit 4?

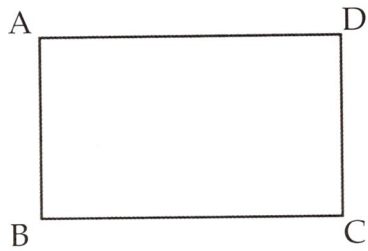
(3) In this number, how many 0.1's does the digit 8 show? Also the digit 7 indicates 7 of what?

④ Please find the areas of the following.

(1) A rectangular paper that is 26cm long and 36cm wide.

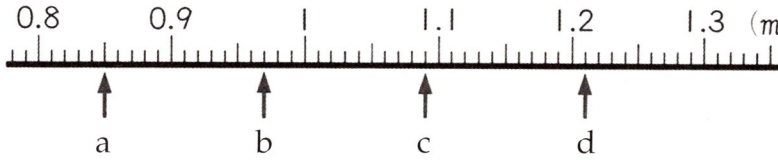
(2) A rectangular flower bed that is 90cm long and 2m wide.

(3) A rectangular garden that is 4m long and 15m wide.

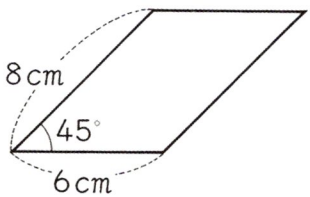


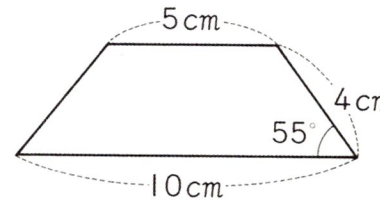
⑤ Which side is parallel to side AB of the rectangle? Which side is perpendicular to side BC?

⑥ Express the lengths at each mark a, b, c, and d in meters.

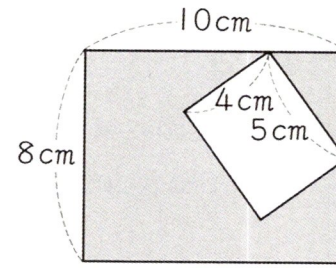


⑦ Draw a parallelogram and a trapezoid as shown below.

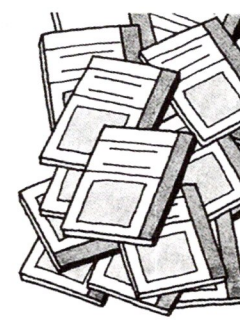




⑧ The drawing on the right is made of two overlapping rectangles. How many cm^2 is the area of the shaded region?

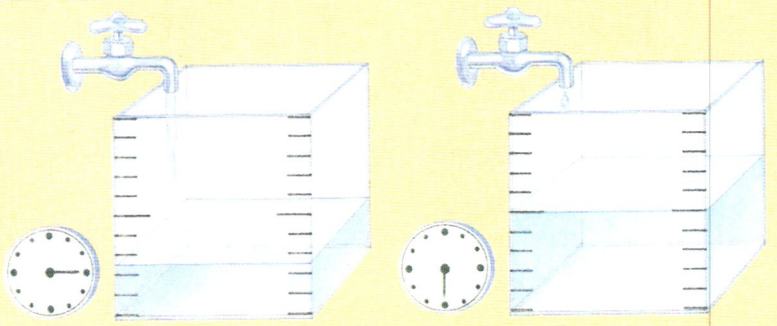


⑨ I bought 18 notebooks at 165 yen each and 18 notebooks at 135 yen each. What was the total price?



12 Investigating Changes in Quantities

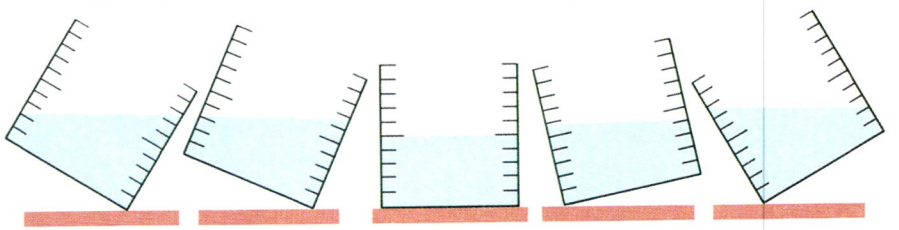
A graduated container is being filled up to the fifth mark.



As time passes, the quantity of water [increases].

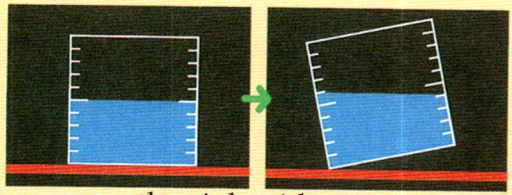
As the quantity of water increases, the depth of water [ ].

1 Let's investigate how numbers on the scales on the sides change as you tilt the container!



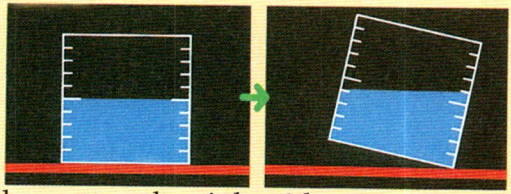
1 In the table on the next page, fill in the numbers on the right-side scales as the numbers on the left-side scales increase.

Make the container tilt to the left.



As the left-side scale increases, the right-side scale ☐.

Make the container tilt to the right.



As the left-side scale decreases, the right-side scale ☐.

Let's investigate how two quantities change together!

Number on Left-Side Scale	0	1	2	3	4	5	6	7	8
Number on Right-Side Scale									

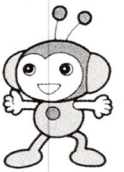
2 As the numbers on the left-side scale increase one by one, how do the numbers on the right-side scale change?

3 What are the sums of the numbers of the left and the right-side scales?

4 Regarding the number on the left-side scale as ☐, and the number on the right-side scale as ○, express the relationship between ☐ and ○ in a math sentence.

Number on the left	+	Number on the right	= 10

□ + ○ = 10



5 In the above math sentence, what is ○ when □ is 10? And what is □ when ○ is 1?



(1) As shown below, cardboard triangles with 1 cm sides are lined up. Let's investigate how the perimeter changes as the number of triangles increases!





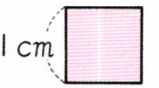
(1) Complete the table below by filling in the perimeters.

Number of Triangles	1	2	3	4	5	6
Perimeter (cm)						

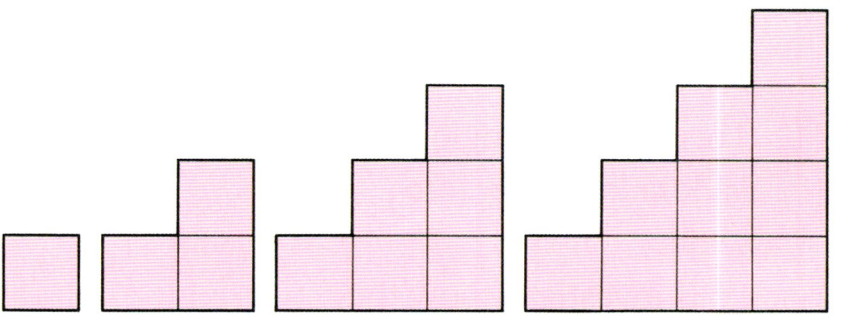
(2) Regarding the number of triangles as □, and the perimeter as ○cm, please express the relationship between □ and ○ in a math sentence.

2

As shown below, cardboard squares with 1cm sides are used to make stairs.



Let's investigate how the perimeter changes as the number of steps increases!

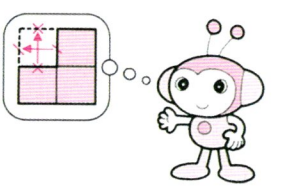


1 Complete the table below by filling in the lengths of the perimeters.

Number of Steps	1	2	3	4	5	6
Perimeter (cm)						

2 As the steps increase one by one, how do the perimeters change?

3 How many times the number of steps is the perimeter?



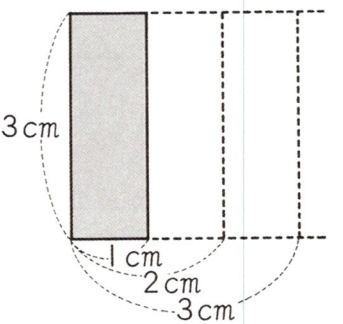
4 Regarding the number of steps as □ and the perimeter as ○ cm, please express the relationship between □ and ○ in a math sentence.

Number of Steps	× 4 =	Perimeter

□ × 4 = ○

5 In the above expression, what is ○ when □ is 10? What is □ when ○ is 28?

① There is a rectangle that is 3cm long and 1cm wide.
Let's investigate how the area changes when the width increases to 2cm, 3cm, . . .



(1) Complete the table below by filling in the areas.

Width (cm)	1	2	3	4	5	6	7
Area (cm^2)							

(2) Regarding the widths as □ and the areas as ○ cm^2, please express the relationship between □ and ○ in a math sentence.

Check

1 We are going to draw a rectangle whose perimeter is 18cm.

(1) Complete the table below by filling in the lengths and widths.

Length (cm)	1	2	3	4	5			
Width (cm)	8							

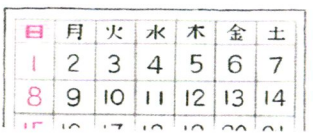
(2) As the length increases 1cm at a time, how does the width change?

(3) Regarding lengths as □ cm and widths as ○ cm, let's express the relationship between □ and ○ in a math sentence.

Challenge

Let's Write Math Sentences Using □ and ○!

Using □ and ○, express the relationships between two quantities in math sentences.



日	月	火	水	木	金	土
1	2	3	4	5	6	7
8	9	10	11	12	13	14

When I buy □ packs of 90 yen gum, the price is ○:

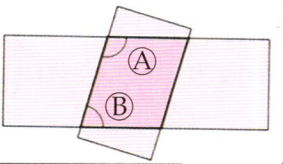
$90 \times \square = \bigcirc$





On a calendar, ○ is immediately below □:

$\square + 7 = \bigcirc$

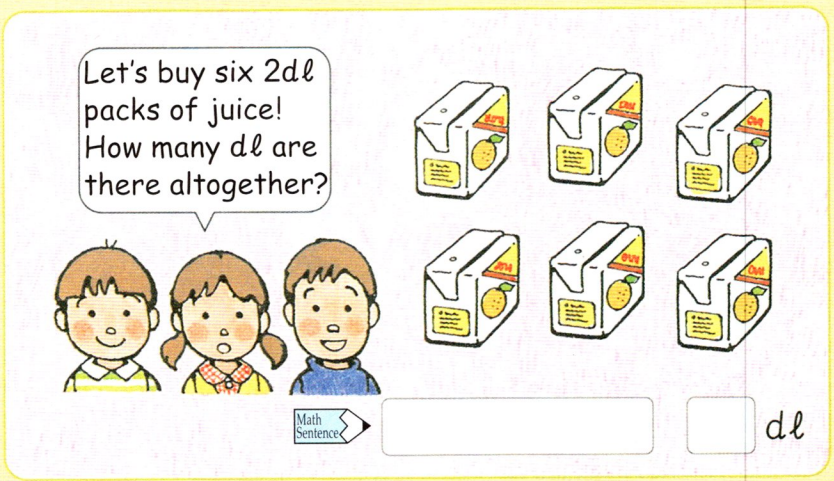


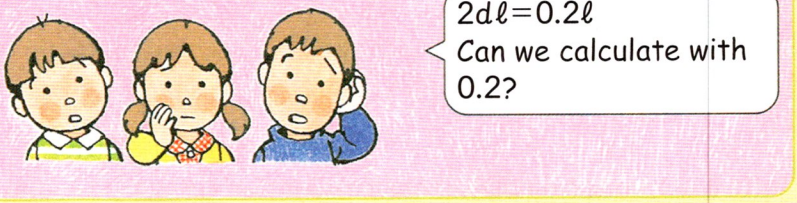


Two rectangles overlap like this. The measure of angle Ⓐ is □° and angle Ⓑ is ○°: $\square + \bigcirc = 180$



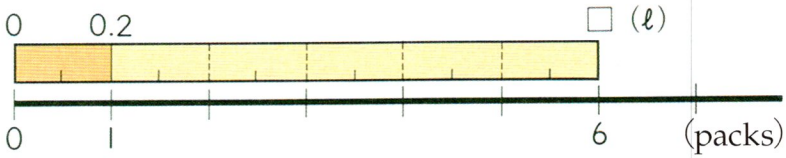
13 Multiplication and Division of Decimal Numbers

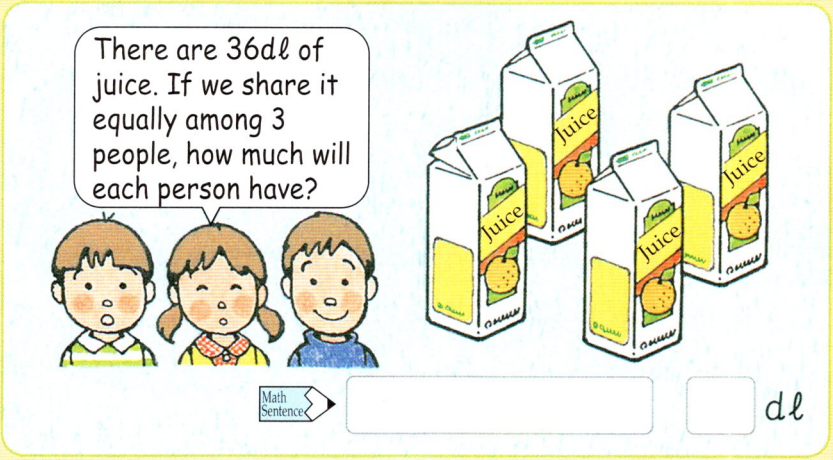


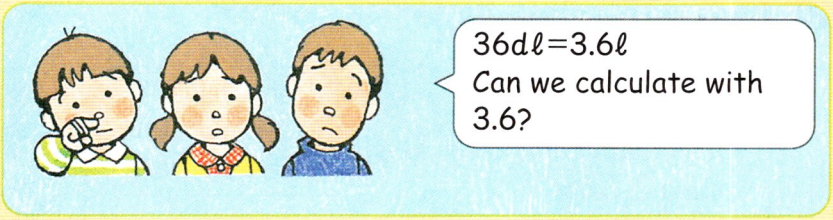


1 Multiplication of Decimal Numbers

1 They bought six 0.2ℓ packs of juice.
How much is the total amount of juice?







Let's think about how to multiply and divide decimal numbers by whole numbers!

1 What kind of math sentence should we write?

Math Sentence

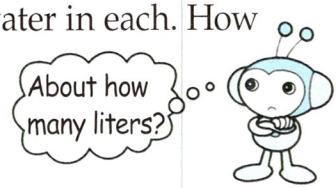
Let's think about how to calculate!

1 How many 0.1ℓ do you need to make 0.2ℓ?

2 How many 0.1ℓ are there in the amount that is 6 times as much as 0.2ℓ?

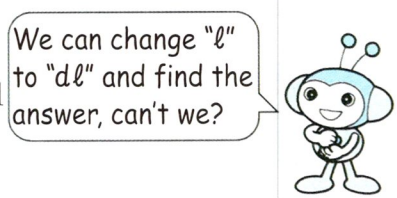
$0.2 \times 6 =$ ☐ Answer: ☐ ℓ

2 There are 7 buckets with 3.6ℓ of water in each. How much water is there altogether?



Math Sentence ☐

Let's think about how to calculate!



1 How many 0.1ℓ do you need to make 3.6ℓ?

2 How many 0.1ℓ are there in the amount that is 7 times as much as 3.6ℓ?

$3.6 \times 7 =$ ☐ Answer: ☐ ℓ

How to calculate 3.6×7

❶
$$\begin{array}{r} 3.6 \\ \times\ \ 7 \end{array}$$

Write 7 below 6.

❷
$$\begin{array}{r} 3.6 \\ \times\ \ 7 \\ \hline 252 \end{array}$$

Calculate as if they were whole numbers.

❸
$$\begin{array}{r} 3.6 \\ \times\ \ 7 \\ \hline 25.2 \end{array}$$

Write the decimal point in the same place as in the multiplicand.

3 Let's think about how to calculate 0.2×4 and 0.8×5!

In (2), the product 4.0 is the same as 4.

$$\begin{array}{r} 0.2 \\ \times\ 4 \\ \hline 0.8 \end{array} \qquad \begin{array}{r} 0.8 \\ \times\ 5 \\ \hline 4.0 \end{array}$$

(1) (2) (3) (4)

$$\begin{array}{r} 3.7 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 0.3 \\ \times\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 17.6 \\ \times\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 8.5 \\ \times\ 6 \\ \hline \end{array}$$

4 Let's think about how to calculate 1.8×34!

1 72 and 54 are the results of what calculation?

2 How can you decide where to place the decimal point of the product?

$$\begin{array}{r} 1.8 \\ \times\ 34 \\ \hline 72 \\ 54 \\ \hline 61.2 \end{array}$$

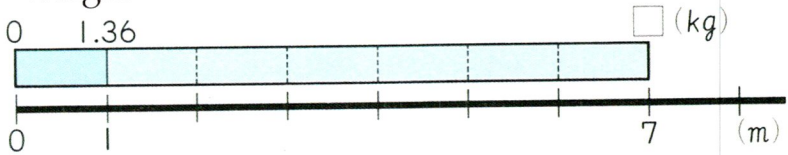
(1) (2) (3) (4)

$$\begin{array}{r} 7.8 \\ \times\ 32 \\ \hline \end{array} \qquad \begin{array}{r} 1.3 \\ \times\ 64 \\ \hline \end{array} \qquad \begin{array}{r} 0.9 \\ \times\ 57 \\ \hline \end{array} \qquad \begin{array}{r} 6.5 \\ \times\ 48 \\ \hline \end{array}$$

(5) (6) (7) (8)

$$\begin{array}{r} 12.3 \\ \times\ 31 \\ \hline \end{array} \qquad \begin{array}{r} 75.8 \\ \times\ 80 \\ \hline \end{array} \qquad \begin{array}{r} 43.6 \\ \times\ 623 \\ \hline \end{array} \qquad \begin{array}{r} 50.8 \\ \times\ 725 \\ \hline \end{array}$$

5 1m of an iron bar weighs 1.36kg. How much does 7m of the same iron bar weigh?



1 Please write a math sentence. Math Sentence

Let's think about how to calculate!

2 How many 0.01kg are there in 1.36kg?

3 How many 0.01kg are there in the amount that is 7 times as much as 1.36?

$1.36 \times 7 =$ ☐ Answer: ☐ kg

4 How do you decide where to place the decimal point of the product?

$$\begin{array}{r} 1.36 \\ \times \quad 7 \\ \hline 9.52 \end{array}$$

6 Let's think about how to calculate 0.025×3 and 1.325×4!

(1) $\begin{array}{r} 0.025 \\ \times \quad 3 \\ \hline 0.075 \end{array}$ (2) $\begin{array}{r} 1.325 \\ \times \quad 4 \\ \hline 5.300 \end{array}$

1 (1) $\begin{array}{r} 4.28 \\ \times \quad 2 \\ \hline \end{array}$ (2) $\begin{array}{r} 0.012 \\ \times \quad 6 \\ \hline \end{array}$ (3) $\begin{array}{r} 3.26 \\ \times \quad 43 \\ \hline \end{array}$ (4) $\begin{array}{r} 0.336 \\ \times \quad 625 \\ \hline \end{array}$

Practice 1

① Please calculate the following.

(1) 2.25×8 (2) 78.5×5 (3) 0.014×7

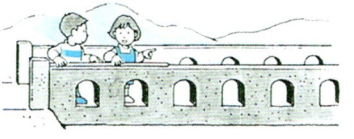
(4) 0.588×37 (5) 0.018×40 (6) 3.75×64

(7) 2.07×918 (8) 0.625×728 (9) 0.96×463

② If you keep water running while brushing your teeth, you waste a lot of water. Suppose the amount of water you waste each time is 11.4ℓ. If you brush your teeth three times a day, how much water would you waste in one week?



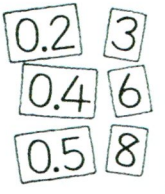
③ Two children measured the length of a bridge and found that it is 25 times as long as a 1.82m stick.
How long is the bridge?



Challenge

Let's Make Multiplication Sentences with Decimal Numbers!

Let's use the cards on the right and make math sentences!

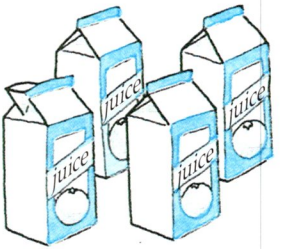


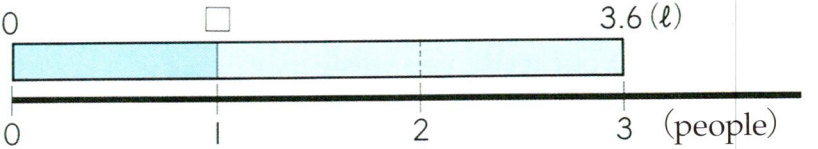
□ × □ = 0.6 □ × □ = 2.4

□ × □ = 3.2 □ × □ = 4

2 Division of Decimal Numbers

1 If you share 3.6ℓ of juice among 3 people, how much will each person get?

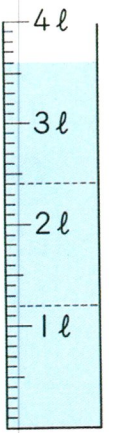




1 Can you write a math sentence?

Math Sentence ▶ ☐

Let's think about how to calculate!



2 How many 0.1ℓ does it take to make 3.6ℓ?

3 If you share 3.6ℓ among 3 people, how many 0.1ℓ will each person get?

$3.6 \div 3 =$ ☐

Answer: ☐ ℓ

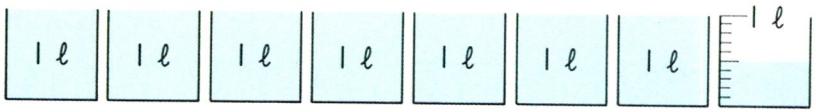
4 Please check your answer this way: Split 3.6ℓ into 3ℓ and 0.6ℓ and share each amount among 3 people.

2 There are 7.5 ℓ of milk. If you share it among 3 people, how much will each person get?

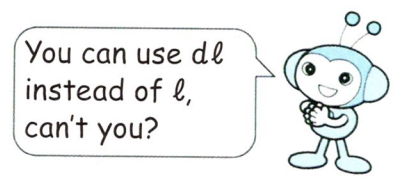


Math Sentence

Let's think about how to calculate!

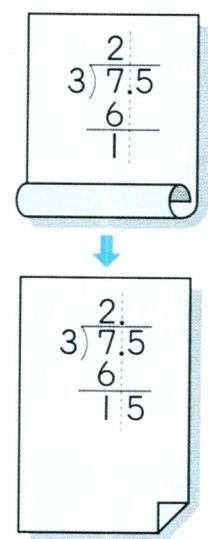


1 If you share the 1 ℓ measuring cups, how many cups will each person get?



2 If you put the rest of the milk in 0.1 ℓ measuring cups, how many cups will you need?

3 How many 0.1 ℓ cups will each person get from the rest of milk?



$7.5 \div 3 =$ ☐

Answer: ☐ ℓ

How to Calculate 7.5÷3

❶
$$\begin{array}{r} 2 \\ 3\overline{)7.5} \\ 6 \\ \hline 1 \end{array}$$

First you divide the whole number part, 7, by 3.

❷
$$\begin{array}{r} 2. \\ 3\overline{)7.5} \\ 6 \\ \hline 15 \end{array}$$

Write the decimal point in the quotient right above the decimal point of the dividend.

❸
$$\begin{array}{r} 2.5 \\ 3\overline{)7.5} \\ 6 \\ \hline 15 \\ 15 \\ \hline 0 \end{array}$$

Complete the rest of the division as you would with whole numbers.

1 (1) $7\overline{)8.4}$ (2) $4\overline{)9.6}$ (3) $6\overline{)73.8}$ (4) $9\overline{)14.4}$

3 Let's think about how to calculate 6.3÷7!

The whole number part, 6, is smaller than 7, so write zero above 6. Write the decimal point of the quotient next to the zero. Complete the rest of the division as before.

$$\begin{array}{r} 0.9 \\ 7\overline{)6.3} \\ 63 \\ \hline 0 \end{array}$$

1 (1) $4\overline{)3.2}$ (2) $8\overline{)5.6}$ (3) $9\overline{)3.6}$ (4) $5\overline{)4.5}$

4 Let's think about how to calculate 95.2÷28!

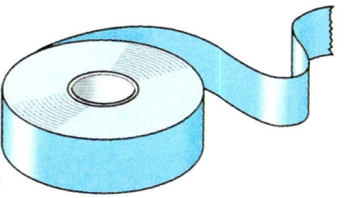
$$\begin{array}{r} 3.4 \\ 28\overline{)95.2} \\ 84 \\ \hline 112 \\ 112 \\ \hline 0 \end{array}$$

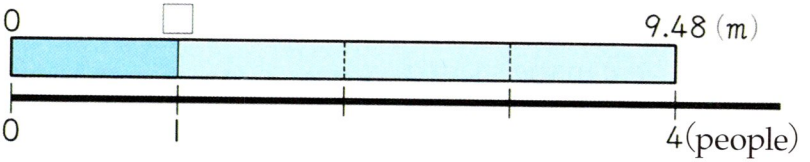
1 (1) $36\overline{)64.8}$ (2) $62\overline{)303.8}$

(3) $42\overline{)25.2}$ (4) $27\overline{)18.9}$

5

We have 9.48m of ribbon. If we share this ribbon among 4 people, how many meters will each person get?





1 Let's write a math sentence! Math Sentence

Let's think about how to calculate!

2 The 28 in the shaded box on the right shows 28 what?

$$\begin{array}{r} 2.37 \\ 4\overline{)9.48} \\ 8 \\ \hline 14 \\ 12 \\ \hline 28 \\ 28 \\ \hline 0 \end{array}$$

1 (1) $7\overline{)8.96}$ (2) $6\overline{)4.02}$

(3) $4\overline{)5.488}$ (4) $34\overline{)86.02}$

6

Let's think about how to calculate 0.24÷6!

$$\begin{array}{r} 0.04 \\ 6\overline{)0.24} \\ 24 \\ \hline 0 \end{array}$$

1 (1) $7\overline{)0.56}$ (2) $9\overline{)0.612}$ (3) $77\overline{)5.621}$

Division with remainders

7 Let's calculate $8.6 \div 3$, and find the quotient up to the tenths place!

1 The 2 in the shaded box at the bottom shows 2 what?

❶
$$\begin{array}{r} 2.8 \\ 3\overline{)\,8.6} \\ 6 \\ \hline 26 \\ 24 \\ \hline 2 \end{array}$$

❷
$$\begin{array}{r} 2.8 \\ 3\overline{)\,8.6} \\ 6 \\ \hline 26 \\ 24 \\ \hline 0.2 \end{array}$$

$8.6 \div 3 = 2.8 \quad \text{R } 0.2$

2 Please check the answer.

$2.8 \times 3 + 0.2 =$ ☐

! When there is a remainder, the decimal point of the remainder should be lined up with the decimal point of the dividend.

1 Please find quotients up to the tenths place. Also, find the remainders. Then check your answers.

(1) $7\overline{)38.4}$ (2) $9\overline{)5.9}$ (3) $12\overline{)45.2}$

▶ Continuing calculation



8 We have 1.7ℓ of juice. If we share it among 5 people, how much will each person get?

? Let's think about how to calculate 1.7÷5!

Think of 1.7 as 1.70, then continue your calculation. This results in no remainder.

❶
$$\begin{array}{r} 0.3 \\ 5\overline{)1.7} \\ 15 \\ \hline 2 \end{array}$$

❷
$$\begin{array}{r} 0.34 \\ 5\overline{)1.70} \\ 15 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

① Please divide each one completely.

(1) $4\overline{)9.4}$ (2) $5\overline{)0.3}$ (3) $8\overline{)3.4}$ (4) $36\overline{)2.7}$

9 ? Let's calculate 6÷4 completely!

Think of 6 as 6.0, then continue your calculation. This results in no remainder.

❶
$$\begin{array}{r} 1 \\ 4\overline{)6} \\ 4 \\ \hline 2 \end{array}$$

❷
$$\begin{array}{r} 1.5 \\ 4\overline{)6.0} \\ 4 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

① Please divide completely.

(1) $6\overline{)21}$ (2) $4\overline{)19}$ (3) $8\overline{)6}$ (4) $48\overline{)36}$ (5) $25\overline{)2}$

10 How much soup will each peason get if 13dℓ of soup are shared among 3 people? Please round the quotient to the first decimal place.

Math Sentence

1 In which place should you round the quotient?

Answer : about 4.3 dℓ

$$\begin{array}{r} 4.33 \\ 3\overline{)13} \\ 12 \\ \hline 10 \\ 9 \\ \hline 10 \\ 9 \\ \hline 1 \end{array}$$

1 Please find an approximate answer to the first decimal place.

(1) $7\overline{)24.3}$ (2) $82\overline{)117}$ (3) $47\overline{)96.6}$ (4) $19\overline{)54.5}$

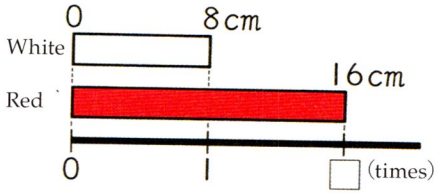
Times as much

11 There are tapes of different lengths. Please compare the length of each tape to the length of the white tape.

Length of Tapes

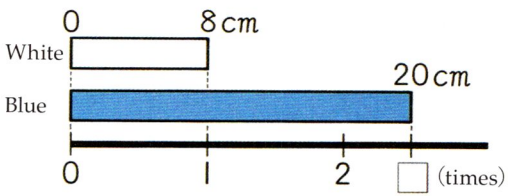
	length (cm)
white	8
red	16
blue	20
yellow	12

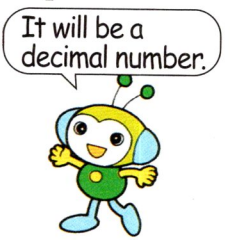
1 How many times as long is the red tape as the white one?



$16 \div 8 =$ ☐ (times)

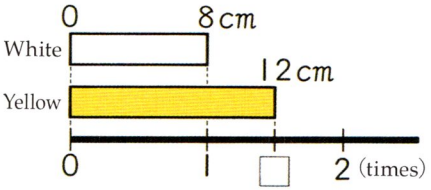
2 How many times as long is the blue tape compared to the white one?





$20 \div 8 =$ ☐ (times)

3 How many times as long is the yellow tape compared to the white one?



$12 \div 8 =$ ☐ (times)

! You can also use decimal numbers to express a "times as much" relationship.

1 A book costs 700 yen and a magazine costs 250 yen. How many times as much does the book cost compared to the magazine?

Practice 2

① Please divide completely.

(1) $75.9 \div 3$ (2) $14.7 \div 6$ (3) $0.26 \div 4$

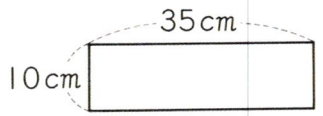
(4) $52.8 \div 75$ (5) $9 \div 24$ (6) $310.7 \div 65$

② Please round the quotients to the nearest hundredths place.

(1) $68\overline{)73.2}$ (2) $78\overline{)5.72}$ (3) $57\overline{)928.4}$

③ 10.8m of white tape is divided into 12 equal segments and 12.8m of red tape is divided into 16 equal segments. Which segments are now longer? How many meters is the difference?

④ How many times as long is the width of the rectangle on the right as its length?



Challenge

Let's Make Division Sentences with Decimal Numbers!

Please use the cards on the right to make math sentences.



□ ÷ □ = 0.3 □ ÷ □ = 0.8

□ ÷ □ = 1.2 □ ÷ □ = 1.6

Check

1 Please find the errors and correct them.

(1) $\begin{array}{r} 1.25 \\ \times\ 18 \\ \hline 1000 \\ 125 \\ \hline 2.250 \end{array}$ (2) $\begin{array}{r} 0.042 \\ \times\ 17 \\ \hline 294 \\ 42 \\ \hline 7.14 \end{array}$ (3) $5\overline{)\begin{array}{r} 12 \\ 6 \\ 5 \\ \hline 10 \\ 10 \\ \hline 0 \end{array}}$ (4) $8\overline{)\begin{array}{r} 4.5 \\ 3.6 \\ 32 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}}$

2 Please calculate.

(1) $\begin{array}{r} 62.9 \\ \times\ 6 \\ \hline \end{array}$ (2) $\begin{array}{r} 34.5 \\ \times\ 8 \\ \hline \end{array}$ (3) $\begin{array}{r} 5.74 \\ \times\ 35 \\ \hline \end{array}$ (4) $\begin{array}{r} 0.36 \\ \times\ 92 \\ \hline \end{array}$

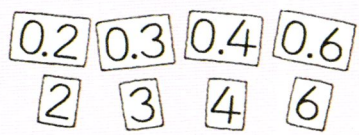
3 Please divide completely.

(1) $7\overline{)60.2}$ (2) $26\overline{)18.2}$ (3) $75\overline{)5.1}$ (4) $8\overline{)22}$

Challenge

Let's Make Math Sentences!

Please use the cards on the right to make math sentences.



$\square \div (\square + \square) = 0.1$ $(\square + \square) \div \square = 0.2$

$\square \div (\square - \square) = 0.3$ $\square \div (\square - \square) = 0.4$

$(\square + \square) \div \square = 0.5$ $\square \div (\square - \square) = 0.6$

What Kind of Calculations Do We Need to Use?



The Weight and Price of Ingredients for Making Curry (450 servings)

ingredient	weight (kg)	price for 1kg (yen)	cost (yen)
pork	13	985	
vegetable	53	385	
curry paste	12	256	
other	34.5	—	4053
total		—	

One serving of rice 0.15kg
One serving of soup.....0.18ℓ

① Let's fill in the empty cells in the table above!

② What is the weight of one serving of curry?

③ What is the cost of one serving of curry?

④ What is the total weight of 450 servings of rice?

⑤ What is the total weight of 450 servings of soup?

Review ◦ 7

① Please calculate the following.

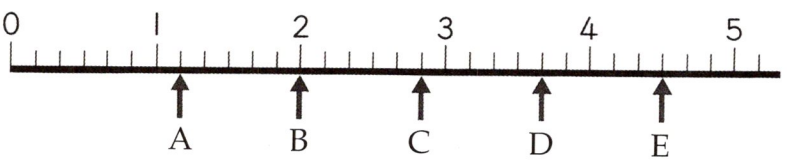
(1) $\frac{3}{5}+\frac{3}{5}$ (2) $\frac{3}{4}+2\frac{3}{4}$ (3) $2\frac{5}{7}+\frac{3}{7}$

(4) $1\frac{6}{7}-\frac{3}{7}$ (5) $1\frac{5}{8}-\frac{7}{8}$ (6) $2\frac{2}{9}-1\frac{5}{9}$

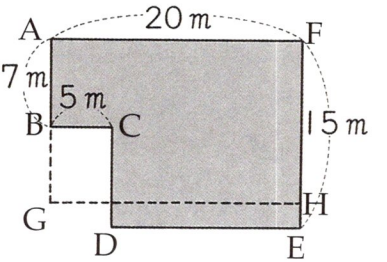
② The following math sentence shows the relationship between one side of an equilateral triangle and its perimeter. What do □ and ○ indicate?

$$□\times 3=○$$

③ Write the numbers indicated by A to E as improper fractions with 6 as their denominators.
Also, write them as mixed numbers or whole numbers.



④ The diagram of a lot is shown on the right.
If we want to make the lot into the rectangle AGHF, how long should the lengh of FH be?



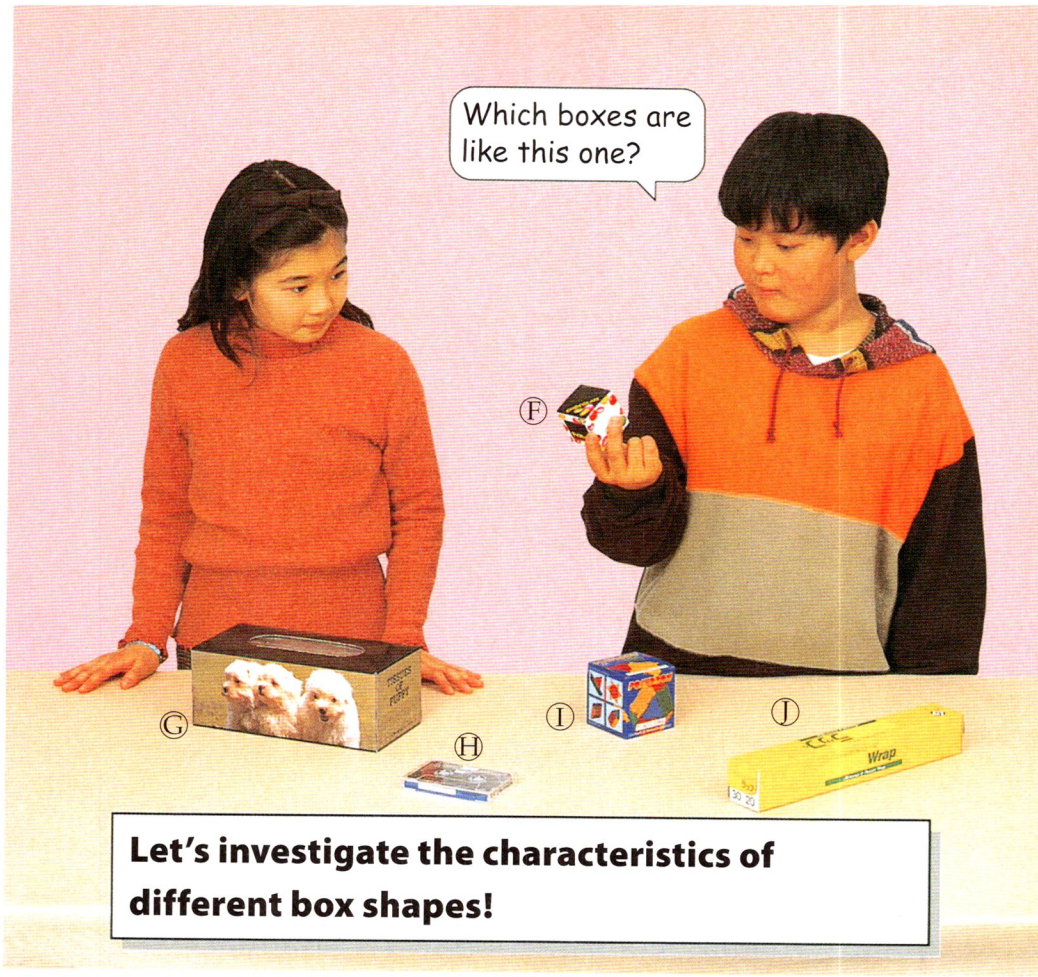
14 Rectangular Prisms and Cubes



1 Rectangular Prisms and Cubes

1 Let's investigate the characteristics of the boxes shown above!

1 Sort the boxes Ⓐ, Ⓑ, …, Ⓙ into two groups according to the shapes of the faces!



Let's investigate the characteristics of different box shapes!

(1)

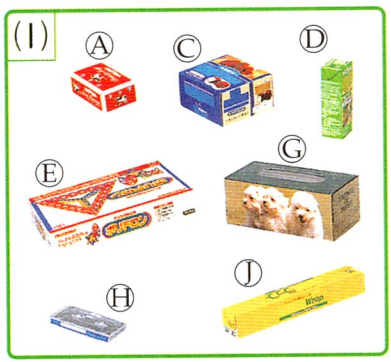
Ⓐ

(2)

Ⓕ



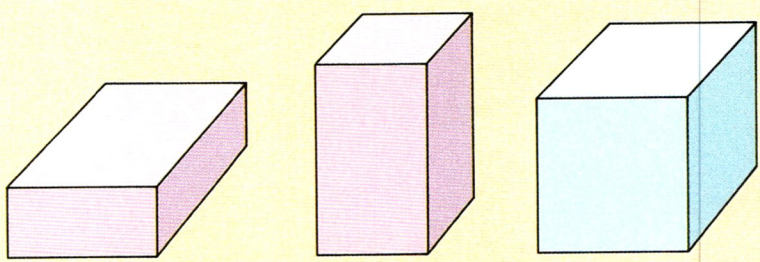
Masaya made two groups of boxes according to the shapes of their faces.



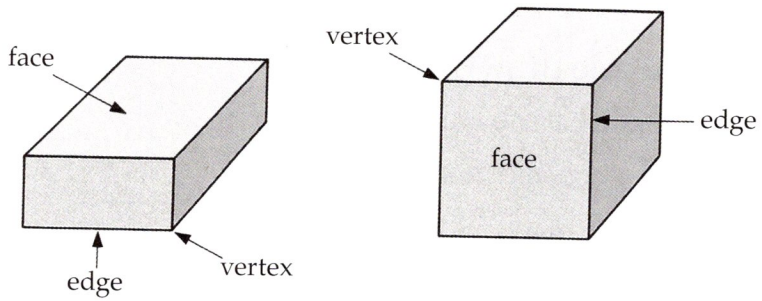
2 What kinds of shapes are the faces of the boxes in group (1)?

3 What kinds of shapes are the faces of the boxes in group (2)?

Boxes that are made up of only rectangular faces or rectangular and square faces are called **rectangular prisms**. Boxes that are made up of only square faces are called **cubes**.



2 Let's investigate the faces, edges and vertices of rectangular prisms and cubes!



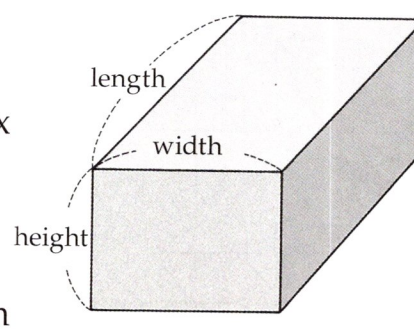
Rectangular prisms and cubes are all made up of flat faces. Flat faces are called **planes**.

1 How many faces, edges, and vertices are there?

2 How many sets of faces that are exactly the same size and shape are there? Also, how many faces are there in each set?

3 How many sets of edges that are the same length are there? Also, how many edges are there in each set?

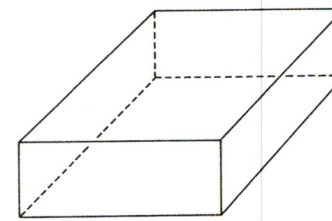
When the lengths of three edges that are connected by a vertex (namely, the length, width, and height) are fixed, the size of the rectangular prism is also fixed.



2 Nets

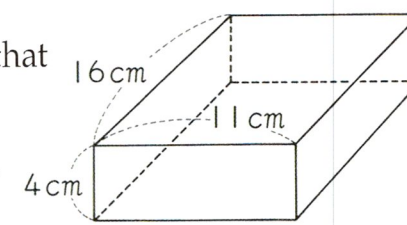
1 Let's make a box in the shape of a recatngular prism to store post cards!

A simple drawing of a shape like the one on the right is called **a sketch**.

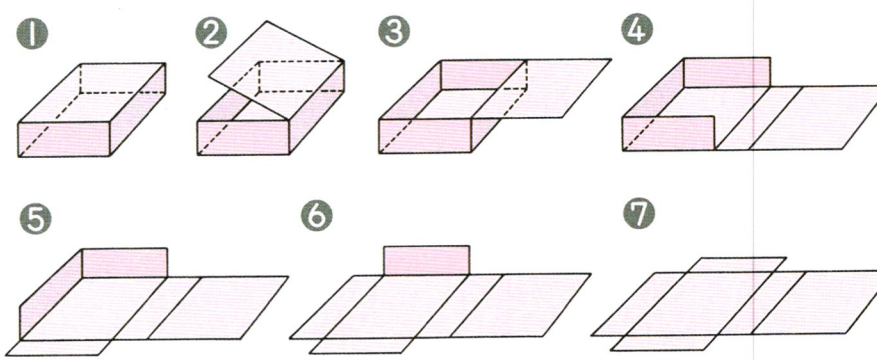


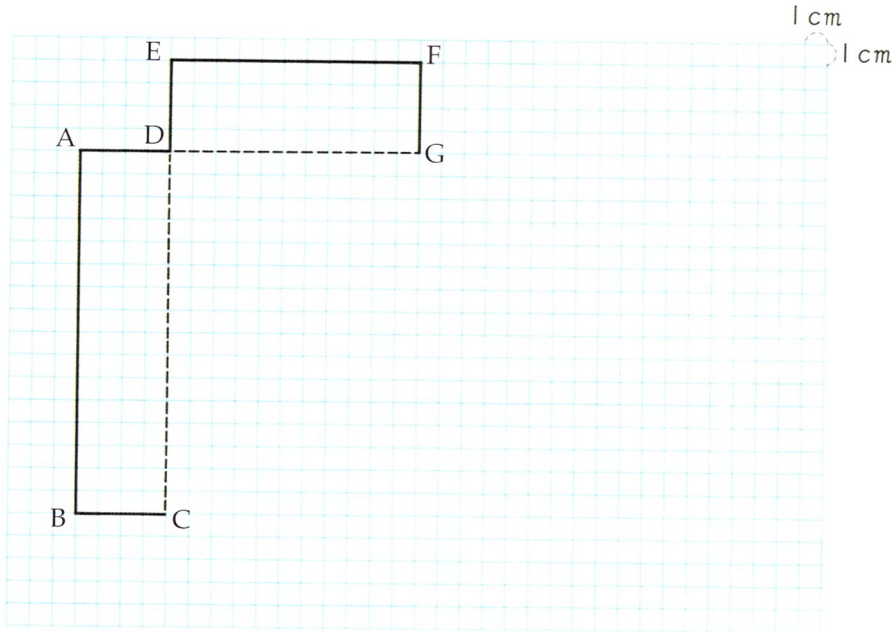
1 Which edges do we need to know the lengths of in order to make the rectangular prism in the sketch above?

We will make a box that is 16cm long, 11cm wide, and 4cm high.



? Let's draw a figure to show what the box looks like when it is cut open completely!



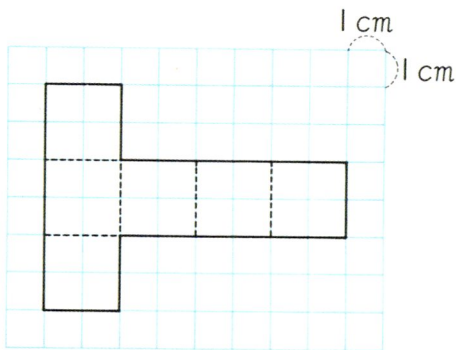


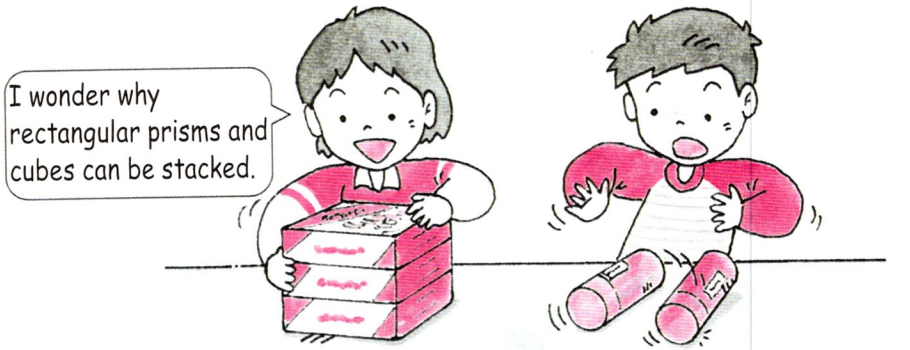
A figure like the one above which shows how a shape such as a box may look when it is cut open on a plane is called **a net**.

2 If we assemble the above net into a box, which edge will match up with edge AD?

3 Please cut out the net, and assemble it into a rectangular prism.

1 If you cut out the net on the right and assemble it, what kind of shape will you get?



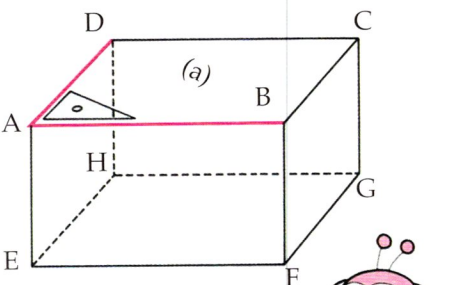


3 Perpendicular and Parallel Faces and Edges

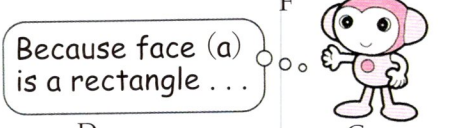
Perpendicular and parallel edges

1 Let's investigate how the edges of a rectangular prism are related to each other!

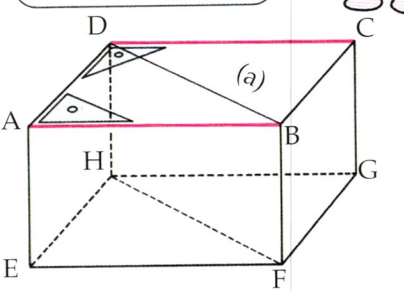
Edges AB and AD are perpendicular to each other.



1 Which edges are perpendicular to edge BF at vertex B?



Edges AB and DC are parallel.

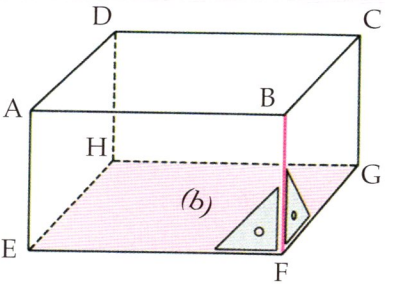


2 Which edges are parallel to edge BF?

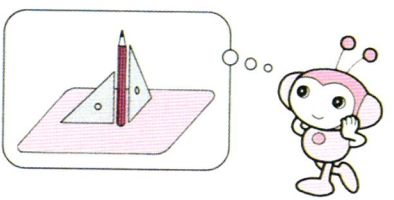
Perpendicular edges and faces

2 Let's investigate how the edges and faces of a rectangular prism are related to each other!

Edge BF and face (b) are perpendicular to each other.



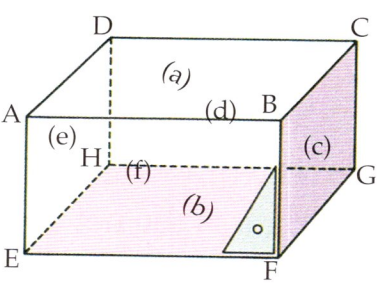
1 Which edges besides edge BF are perpendicular to face (b)?



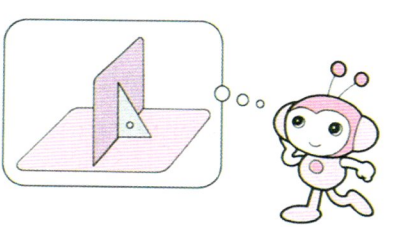
Perpendicular and parallel faces

3 Let's investigate how the faces of rectangular prism are related to each other!

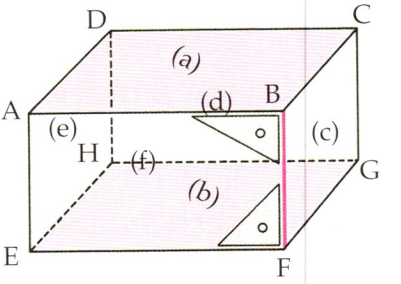
The two adjacent faces, (b) and (c), are perpendicular to each other.



1 Which faces are perpendicular to face (a)?



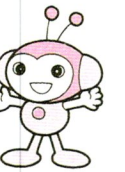
The two opposite faces, (a) and (b), are parallel to each other.
And both faces, (a) and (b), are perpendicular to edge BF.



2 Which face is parallel to face (f)?

3 How many pairs of parallel faces are there in a rectangular prism?

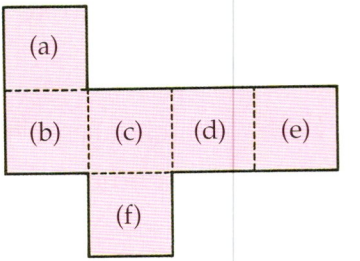
Because the edges and faces of rectangular prisms are either perpendicular or parallel, many boxes tend to be rectangular prisms.

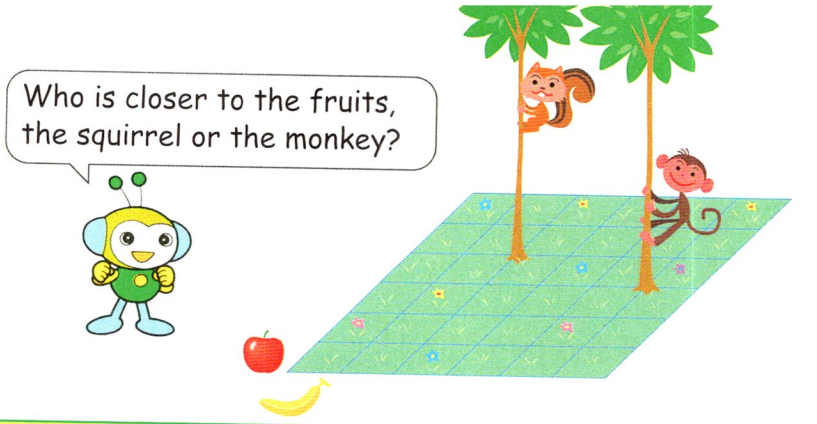


1 Let's find any edges or faces that are perpendicular or parallel to each other in your surroundings!



2 The drawing on the right is the net of a cube. When assembed, which face is parallel to face (a)? Which faces are perpendicular to face (b)?

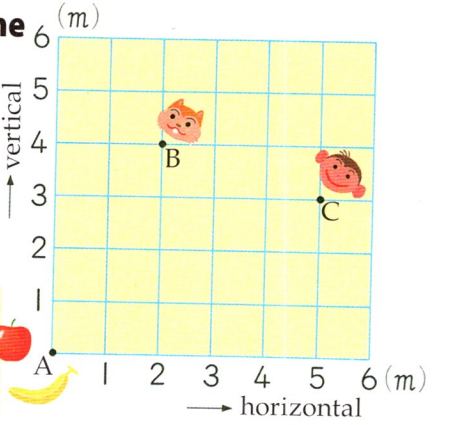




4 How to Represent Location

Location of points on a plane

1 Based on the location of the fruits, let's represent the location of the squirrel and the monkey!



? Let's think of a way to represent the location of points on a plane!

1 Based on the location of point A, let's represent the location of point B with horizontal and vertical distance.

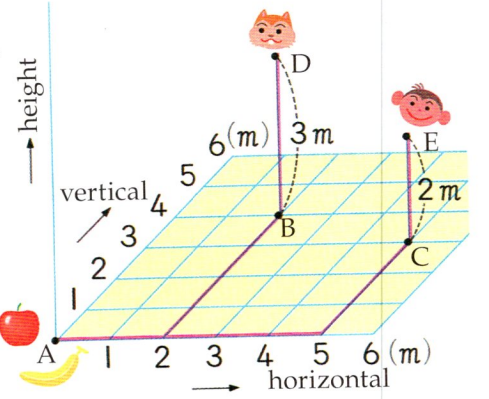
Based on the location of point A, the location of point B can be represented as (2*m* horizontal, 4*m* vertical).

2 Based on the location of point A, let's represent the location of point C.

! The location of a point on a plane can be represented with a set of two distances.

Location of points in space

2 Based on the location of the fruits, let's represent the location of the squirrel and the monkey!



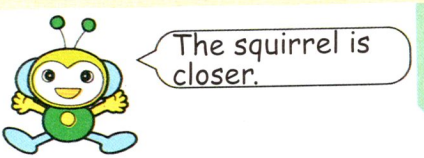
? Let's think about how to represent the location of points in space!

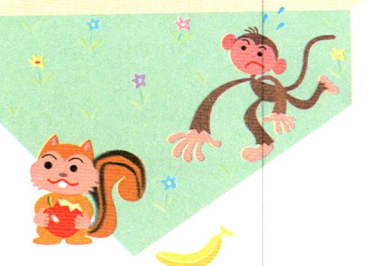
1 Based on the location of point A, let's represent the location of point D with horizontal and vertical distance, and height.

Based on the location of point A, the location of point D can be represented as (2m horizontal, 4m vertical, 3m height)

2 Based on the location of point A, please represent the location of point E.

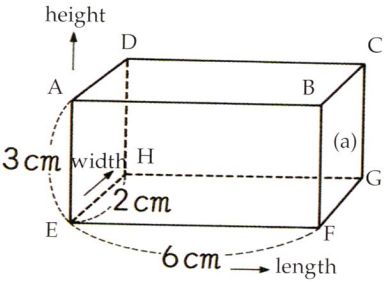
! The location of a point in space can be represented by a set of three distances.





Check

1 The shape on the right is composed only of rectangular faces.



(1) What shape is this?

(2) How many faces are there?

(3) Which edges are perpendicular to face (a)?

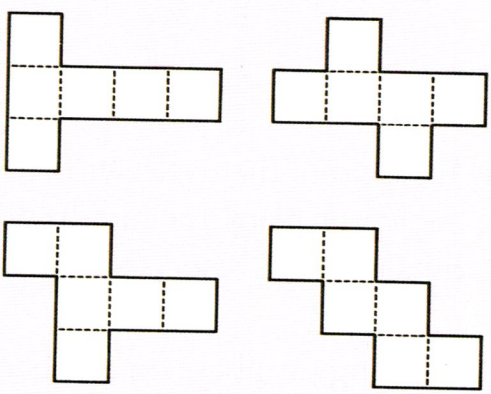
(4) Based on the location of point E, represent the location of points G and C.

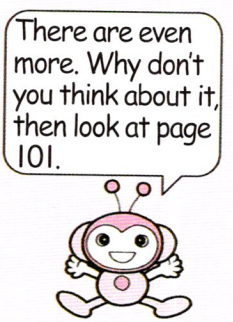
Challenge

Let's Investigate the Nets of a Cube!

There are various nets for a cube. Let's think about them!

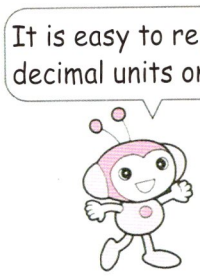






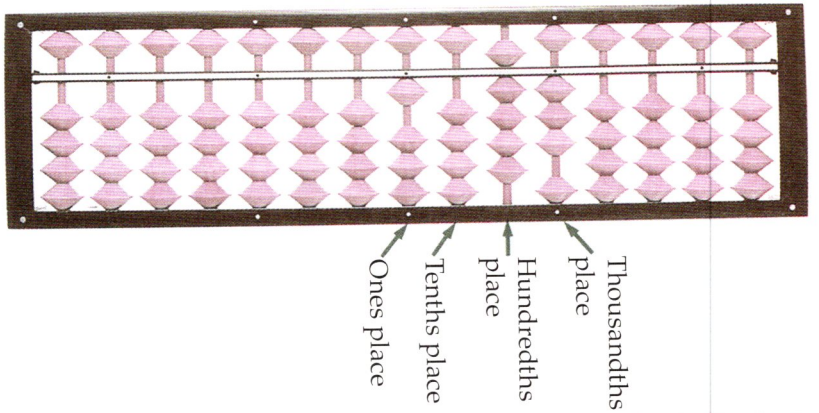
Abacus





How to express decimals

1 Please set 1.093 on an abacus.



To the right of the fixed point that we refer to as the first digit, are the tenths place, the hundredths place, the thousandths place , ... respectively.

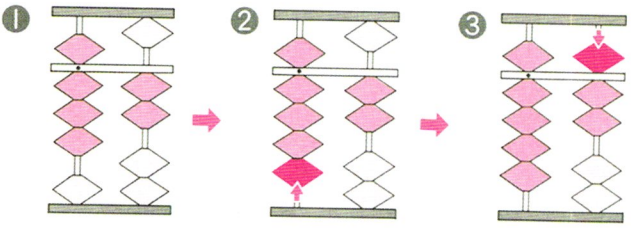
1 Please set the following numbers on an abacus.

(1) 5.73 (2) 40.35 (3) 2.408 (4) 60.046

Addition and subtraction

2 Let's think about how to calculate 8.2+1.5!

● How to calculate 8.2+1.5.

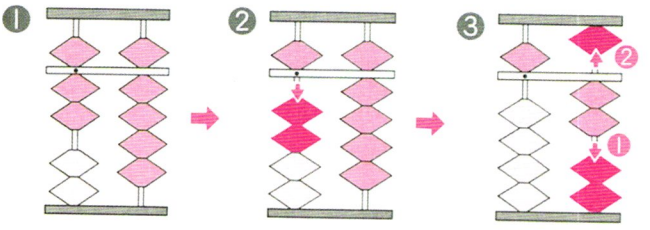


1 (1) 3.3+5.6 (2) 6.3+2.7 (3) 1.25+24.6

(4) 0.43+1.24 (5) 4.67+3.543

3 Let's think about how to calculate 7.9−2.7!

● How to calculate 7.9−2.7.



1 (1) 9.8−3.7 (2) 8.6−4.3 (3) 6.92−6.7

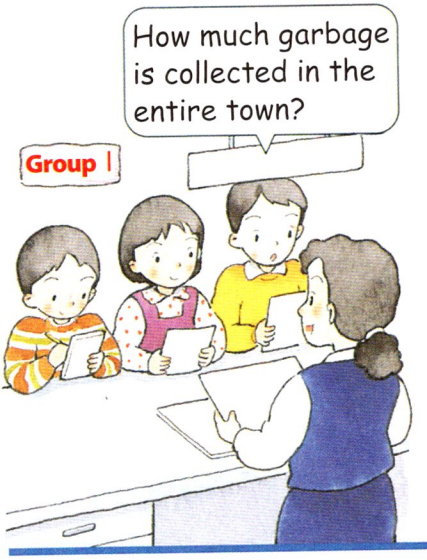
(4) 7.56−6.42 (5) 0.754−0.467

2 (1) 9174−4862 (2) 609−85+520−807

Life and Math

Town Environment — Let's Create a Better Neighborhood!

Emiko's classmates decided to investigate the environment of their town by visiting the town hall.



Weight of Garbage
(Jan. ~ May, 1997)

month	Weight of Garbage (kg)
1	143524
2	152608
3	167590
4	171345
5	164098

1 **Answer the following problems about garbage.**

(1) Approximate the weight of the garbage each month from January through May by rounding the digit in the hundreds place.

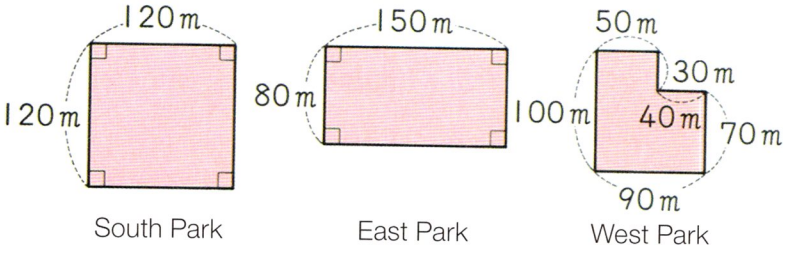
(2) Make a line graph of the rounded figures from exercise (1) showing the weight of the garbage each month from January through May.

2 **Answer the following problems about parks.**

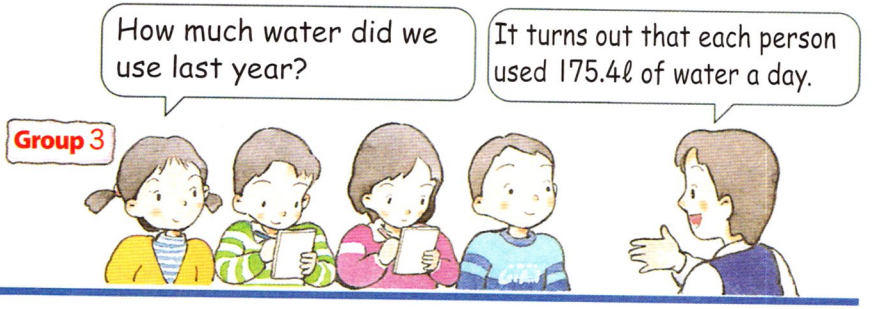
(1) How many m^2 are the areas of South Park and East Park?

(2) How many times as large is the area of South Park as the area of East Park?





Group 3 investigated about water usage.



3 **Answer the following problems about water usage.**

(1) How many liters of water a day were used by the 12 members of group 3 altogether?

(2) About how many 4ℓ buckets is the quantity of water you calculated in exercise (1)? Round your answer to the second highest place.

① Using the information gathered by the three groups, create some problems on your own and answer them.

4th Grade **R e v i e w**

▶ Numbers and calculations

① Please read the following numbers.

(1) 352904718561 (2) 4128370495245

② In 2135698013298, what is the place value of the left-most digit, 2?
The fourth digit from the left, 5, indicates 5 of what?

③ Please round off the following numbers to the nearest one *man* (10 thousand).

(1) 28493 (2) 604712 (3) 8597213

④ Please calculate the following. For the division problems, find whole number quotients and remainders if there are any.

(1) 563×478 (2) 889×675 (3) 603×124

(4) 294×850 (5) 786×509 (6) 931×700

(7) $184 \div 27$ (8) $742 \div 14$ (9) $1792 \div 56$

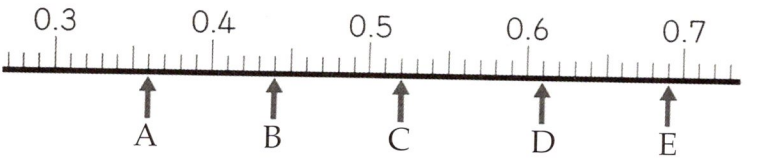
(10) $3825 \div 19$ (11) $5312 \div 64$ (12) $2818 \div 36$

⑤ Please calculate the following.

(1) $500 - (75 + 40)$ (2) $70 + 30 \times 5$

(3) $3766 \div (15 - 8)$ (4) $66 \times 29 - 273 \div 3$

⑥ Please express the decimals at each mark, A, B, C, D, and E.



⑦ Please calculate the following. For the division problems, divide completely.

(1) $4.97 + 7.56$ (2) $0.064 + 0.016$ (3) $0.78 + 49.3$

(4) $8.12 - 2.35$ (5) $6.342 - 5.74$ (6) $30 - 4.02$

(7) 1.87×36 (8) 4.48×85 (9) 0.75×68

(10) $7.04 \div 4$ (11) $5.53 \div 7$ (12) $15.1 \div 25$

⑧ Please change the mixed numbers into improper fractions and the improper fractions into mixed numbers.

(1) $1\frac{1}{7}$ (2) $\frac{13}{4}$ (3) $3\frac{1}{9}$ (4) $\frac{14}{5}$ (5) $\frac{25}{6}$ (6) $4\frac{2}{3}$

⑨ Please calculate the following.

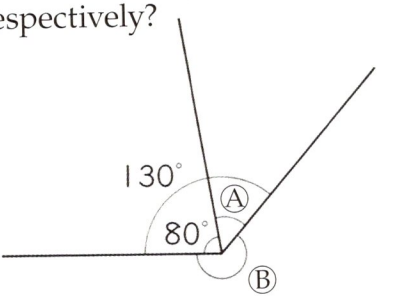
(1) $\frac{3}{7} + \frac{6}{7}$ (2) $1\frac{8}{9} + \frac{5}{9}$ (3) $3\frac{7}{8} + \frac{3}{8}$

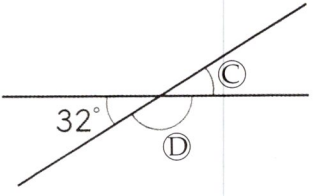
(4) $2\frac{5}{6} + 4\frac{2}{6}$ (5) $1\frac{2}{7} - \frac{6}{7}$ (6) $1\frac{5}{10} - \frac{8}{10}$

(7) $2\frac{1}{4} - \frac{3}{4}$ (8) $2\frac{2}{5} - 1\frac{3}{5}$ (9) $3 - 1\frac{1}{4}$

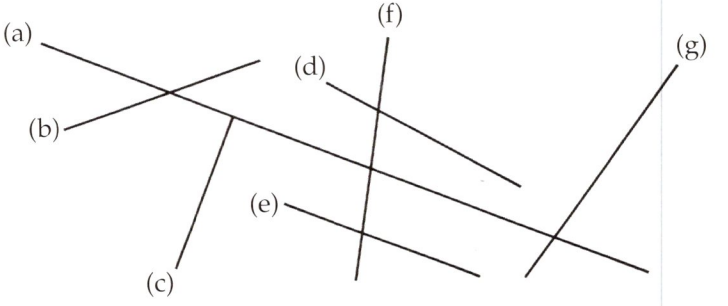
▶ Shape and size

① How many degrees are angles Ⓐ,Ⓑ,Ⓒ,and Ⓓ, respectively?

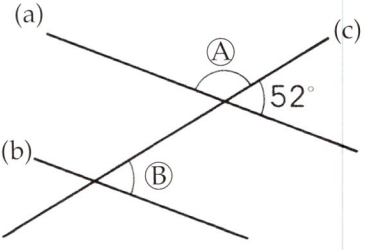




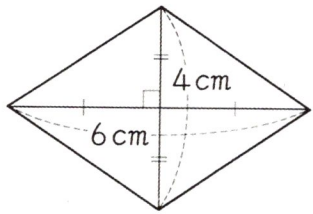
② Which line segment is perpendicular to line (a)? Also which line segment is parallel to line (a)?

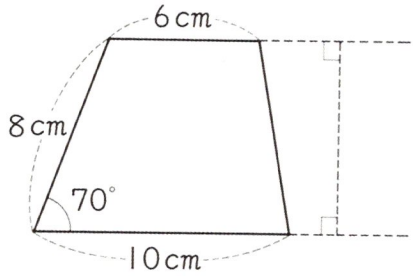


③ Lines (a) and (b) to the right are parallel. How many degrees are angles Ⓐ and Ⓑ, respectively?

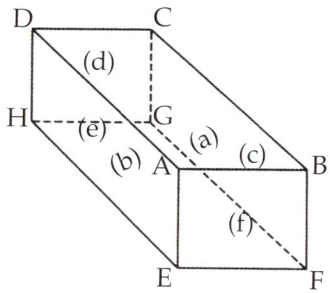


④ Please draw the quadrilaterals shown below. What are the names of these shapes?

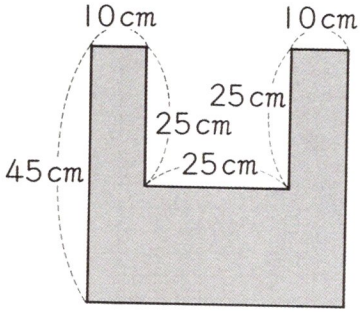


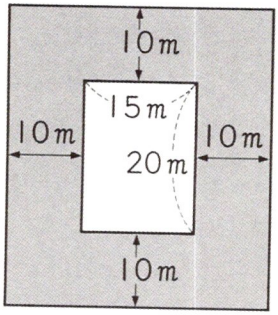


⑤ For the rectangular prism on the right, which edges are perpendicular to edge EF? Which faces are perpendicular to face (a)?



⑥ Please find the area of the shaded parts below.





▶ Tables and graphs

① Using the table to the right, please find the number of students who satisfy the given conditions.

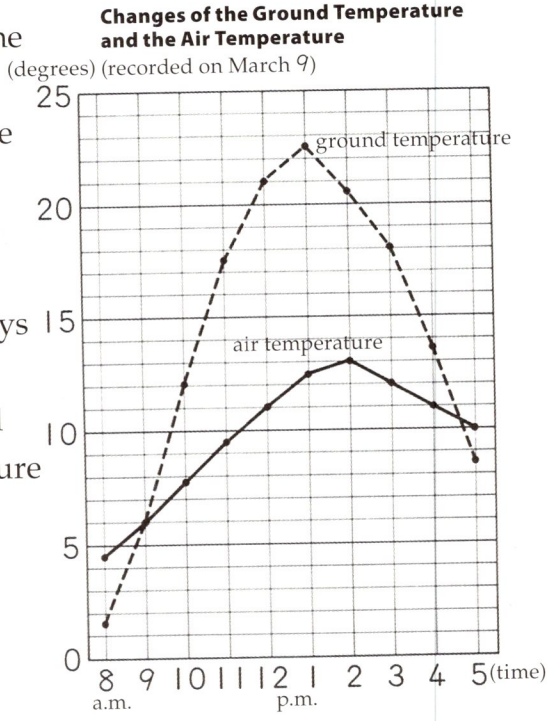
(1) The number of students who do not have a grandmother

(2) The number of students who have a grandfather

(3) The number of students who have only a grandfather

		Grandfather	
		Have	Have Not
Grandmother	Have	2	5
	Have Not	3	26

② In the graph to the right, from what time to what time did the ground temperature rise most rapidly? Compare the ways the ground temperature and the air temperature changed during the day.



Calculation Practice

① (1) $4.92 + 7.6$ (2) $5.83 + 9.78$ (3) $3.54 + 8.38$

(4) $6.2 + 13.89$ (5) $3.68 + 14.02$ (6) $12.5 + 0.49$

(7) $8.15 - 2.3$ (8) $7.08 - 4.12$ (9) $6.34 - 4.91$

(10) $11.2 - 1.53$ (11) $5 - 3.03$ (12) $21 - 11.57$

② (1) $\frac{5}{8} + \frac{7}{8}$ (2) $3\frac{7}{9} + \frac{4}{9}$ (3) $1\frac{8}{10} + 1\frac{3}{10}$

(4) $1\frac{1}{8} - \frac{7}{8}$ (5) $2\frac{1}{9} - \frac{7}{9}$ (6) $4\frac{7}{10} - 2\frac{9}{10}$

③ (1) 7.4×4 (2) 1.6×9 (3) 3.8×7

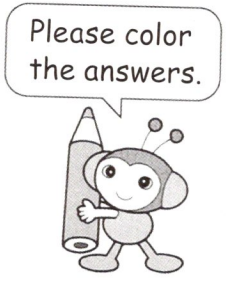
(4) 4.32×6 (5) 3.25×16 (6) 5.36×25

④ Please divide completely.

(1) $90.6 \div 6$ (2) $78.4 \div 7$ (3) $157.5 \div 15$

(4) $614.4 \div 48$ (5) $139.5 \div 45$ (6) $81 \div 18$

①	17.7	11.92	12.99	20.09	2.96	1.43
	15.61	5.85	1.97	12.52	9.43	9.67
②	$3\frac{1}{10}$	$1\frac{3}{9}$	$4\frac{2}{9}$	$1\frac{8}{10}$	$1\frac{4}{8}$	$\frac{2}{8}$
③	26.6	14.4	25.92			
	134	29.6	52			
④	10.5	3.1	15.1			
	12.8	11.2	4.5			



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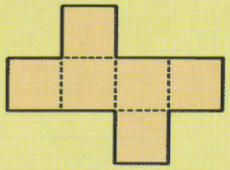
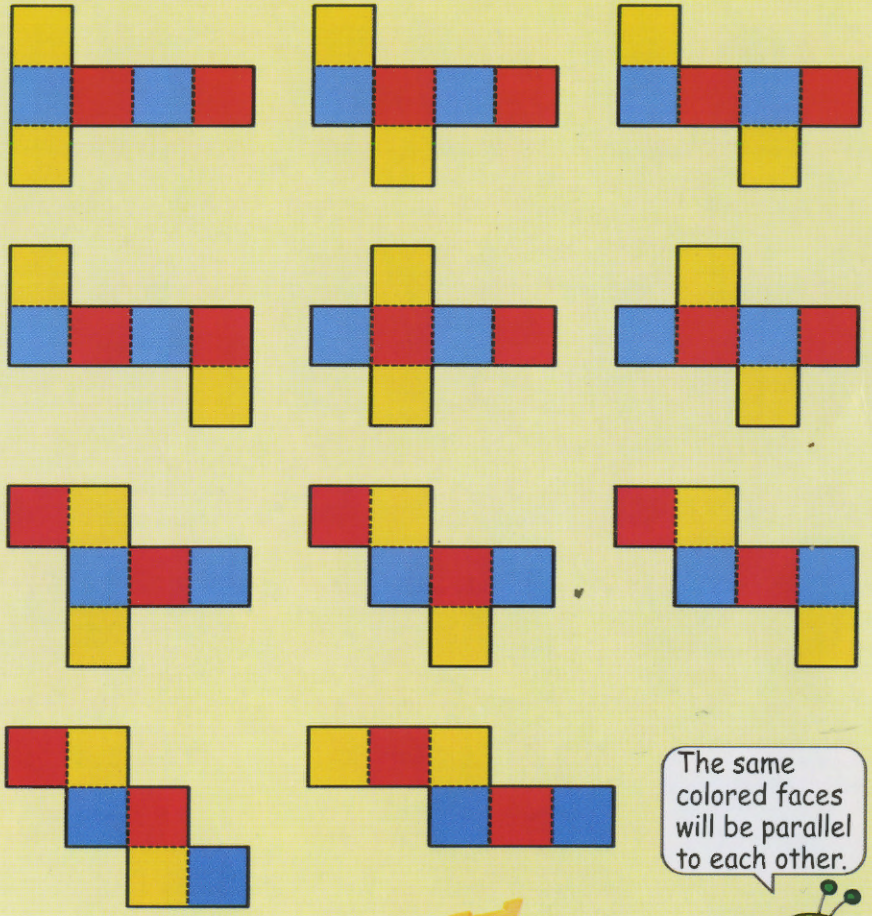
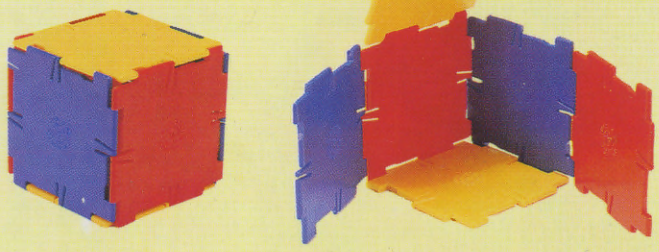
Nets of a Cube







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TOKYO SHOSEKI

Mathematics 5A

for Elementary School



SUBMARINE

TOKYO SHOSEKI

There are many children who think that studying mathematics is interesting. That is probably because, in mathematics, you can usually find the answer if you think persistently and diligently. It is like climbing up stairs one by one, although it may take time.

In mathematics, there is not only one way to find an answer. You can think of many ways to solve a problem. So it is important to listen to your friends' ideas and discuss the ideas with them.

Let's work hard and do challenging mathematics together!



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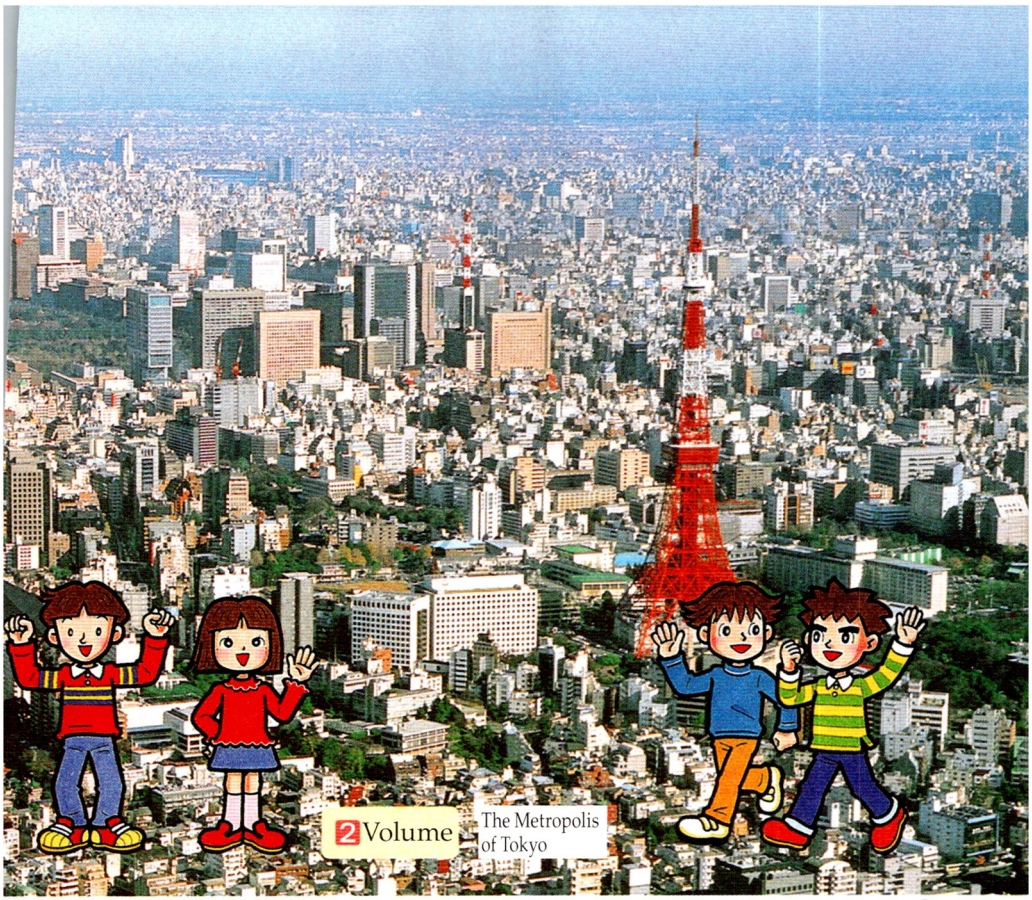

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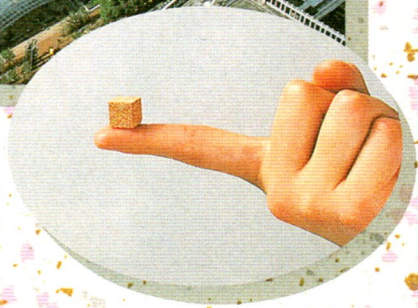
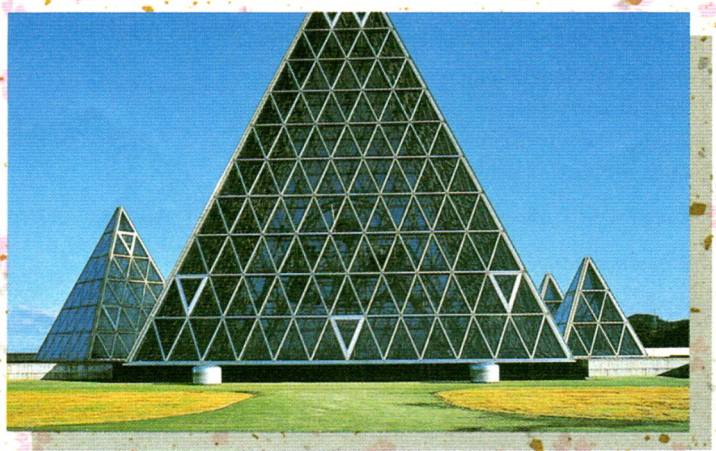
- ..... New Problem
- ..... Aims of Learning
- ..... Hints for Thinking
- ..... Summary of Learning
- ..... Practice Problems

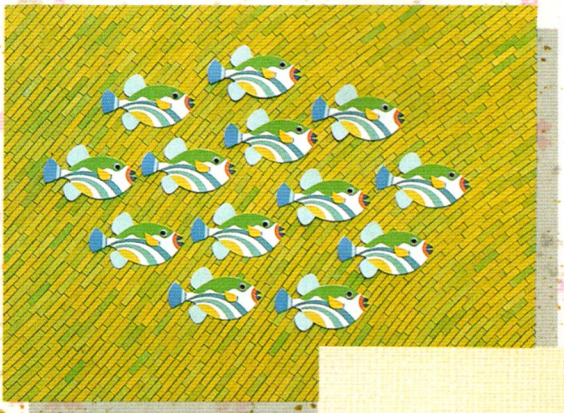


2 Volume Tokyo

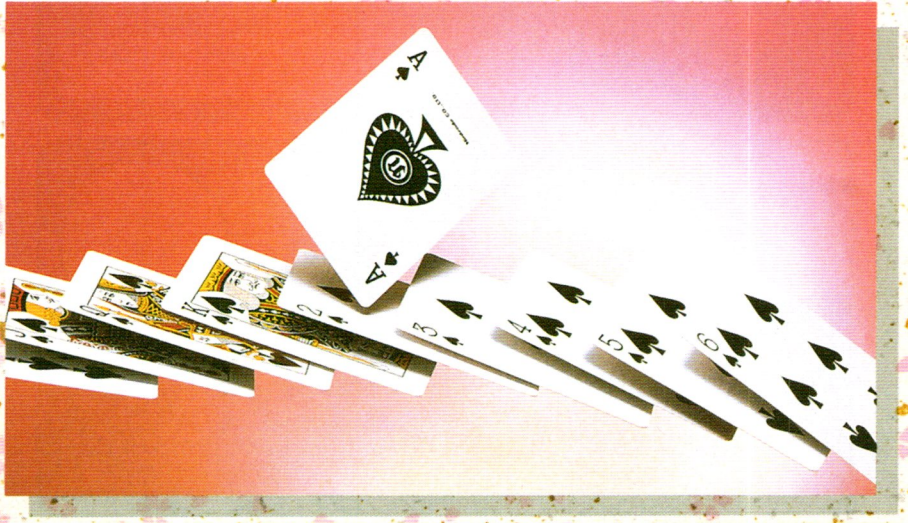




5 Congruent Shapes Nima Town, Shimane Prefecture

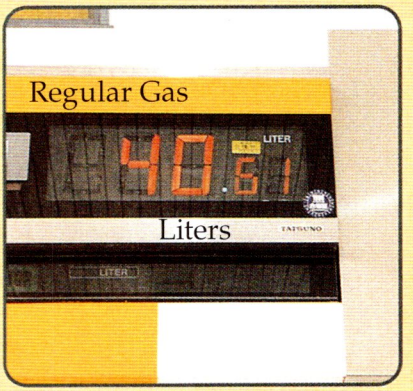


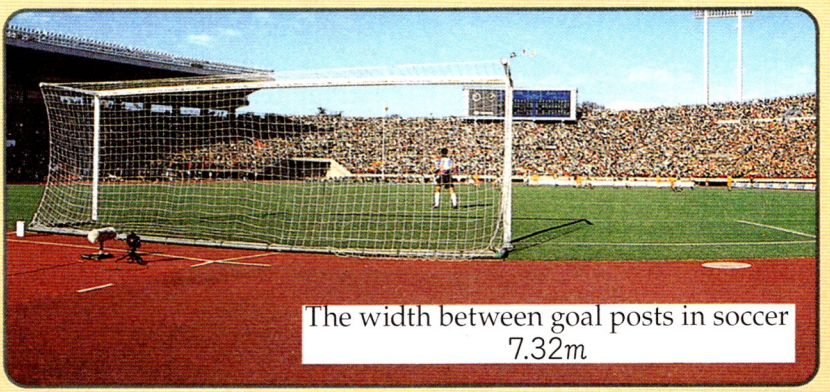




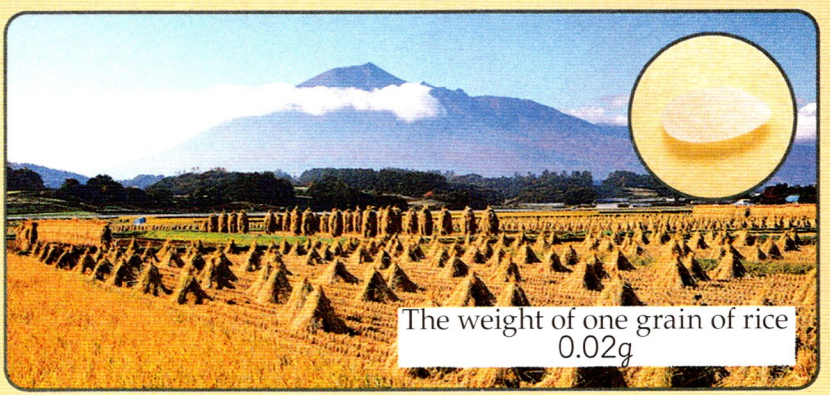
1 Whole Numbers and Decimal Numbers



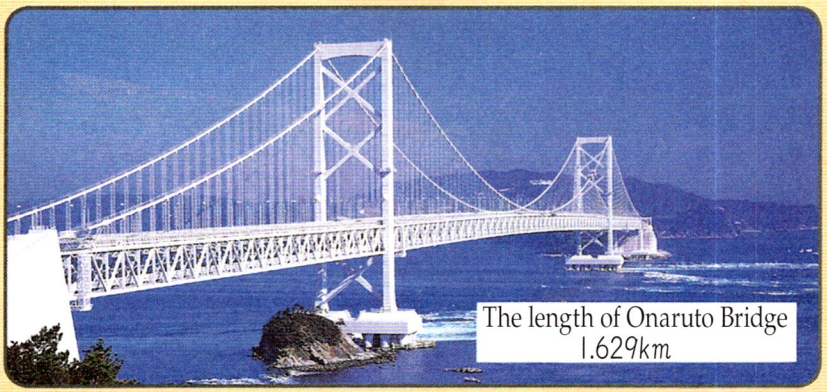




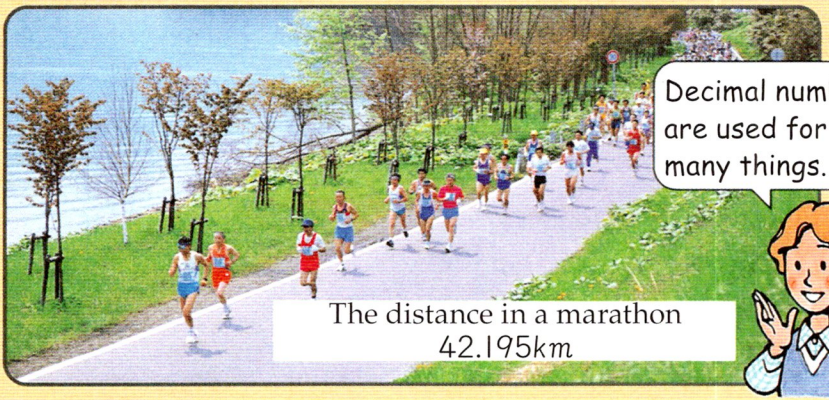
The width between goal posts in soccer
7.32m



The weight of one grain of rice
0.02g



The length of Onaruto Bridge
1.629km

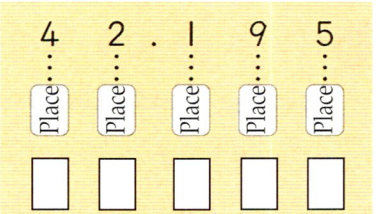


The distance in a marathon
42.195km

Let's learn the structure for expressing whole numbers and decimal numbers!

1 Structure of the Numeration System

1 Let's find out the structure of the number 42.195!



1 What does the 4 in the tens place mean? How many of what does this digit represent? Also, let's find out about other numbers!

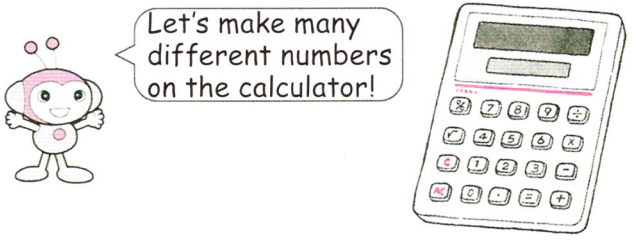
42.195 has the following structure:

$$
\begin{aligned}
42.195 = \quad & 10 \quad \times 4 \cdots\cdots 40 \\
+ \ & 1 \quad \times 2 \cdots\cdots 2 \\
+ \ & 0.1 \quad \times 1 \cdots\cdots 0.1 \\
+ \ & 0.01 \quad \times 9 \cdots\cdots 0.09 \\
+ \ & 0.001 \times 5 \cdots\cdots 0.005 \\
& \qquad\qquad\qquad\qquad 42.195
\end{aligned}
$$

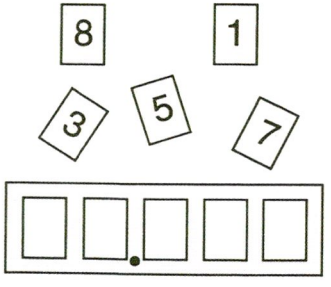
2 Fill in the $\square$ for the following equation.

$605.9 = 100 \times \square + 10 \times \square + 1 \times \square + 0.1 \times \square$

By using the numbers from 0 to 9 and a decimal point, you can express all kinds of whole numbers and decimal numbers.



1 Please place the cards on the right in the boxes to make the greatest number. Also, make the smallest number.

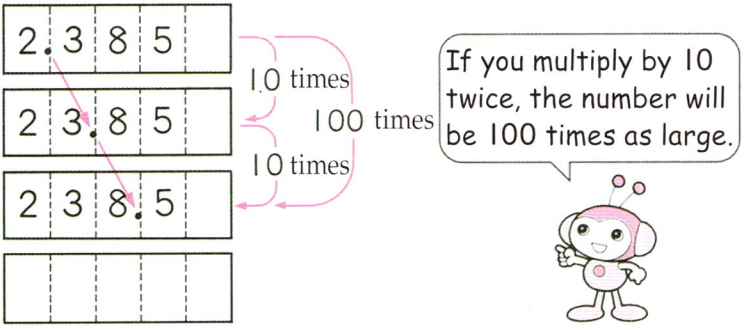


Numbers that are 10 times, 100 times, $\frac{1}{10}$ and $\frac{1}{100}$ as large, and the place of their decimal points

2 Please find how the position of the decimal point changes when you multiply 2.385 by 10 and 100.

$2.385 \times 10 =$ ☐

$2.385 \times 100 =$ ☐



1 What number is 1000 times as large as 2.385? How does the decimal point move?

When you multiply a whole number or decimal number by 10, 100, ... , the decimal point moves 1, 2, ... places to the right.

1 Find the numbers that are 10 and 100 times as large as the following numbers.

(1) 4.072 (2) 0.54 (3) 83

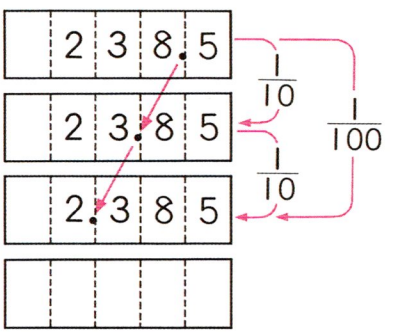
2 What number do you multiply 2.78 by to get the following numbers?

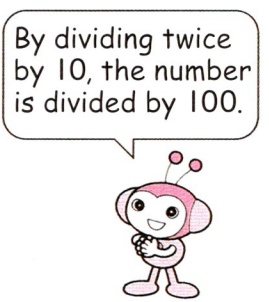
(1) 278 (2) 27.8 (3) 2780

3 Find out how the decimal point moves if you make a number that is $\frac{1}{10}$ and $\frac{1}{100}$ of 238.5.

$238.5 \div 10 =$ ☐

$238.5 \div 100 =$ ☐





1 Find the number that is $\frac{1}{1000}$ of 238.5. How will the decimal point move?

If you make a number that is $\frac{1}{10}$, $\frac{1}{100}$, ... of a whole number or decimal number, the decimal point will move 1, 2, ... places to the left.

1 Find $\frac{1}{10}$ and $\frac{1}{100}$ of the following numbers.

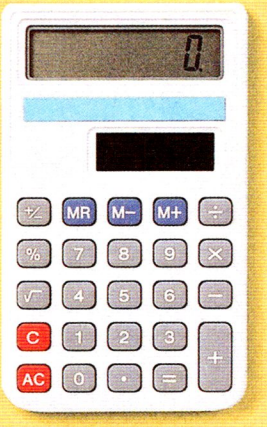
(1) 341.9 (2) 9.8 (3) 710

2 What fraction do you multiply 39.7 by to make the following numbers?

(1) 0.397 (2) 3.97 (3) 0.0397

Story of Math

Let's Figure Out the Following Interesting Calculations Using a Calculator!



$1 \times 9 + 2 =$ ☐

$12 \times 9 + 3 =$ ☐

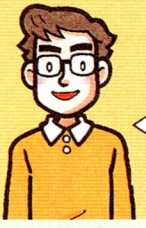
$123 \times 9 + 4 =$ ☐

$1234 \times 9 + 5 =$ ☐

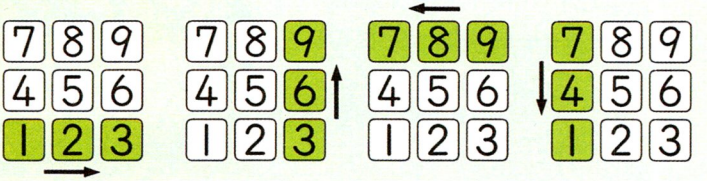
$12345 \times 9 + 6 =$ ☐

$123456 \times 9 + 7 =$ ☐

$1234567 \times 9 + 8 =$ ☐



Start with 1 and press 3 numbers on your keypad counter-clockwise to make a 3-digit number. Then add the next 3-digits to the first one. Repeat by going around the keypad.



$123 + 369 + 987 + 741 =$ ☐



Try starting from 2 and going clockwise.

2 Estimating Products and Quotients

1 183 fifth and sixth grade students went to a nature park by bus. Each student paid 735 yen for the bus. What is the total cost of the bus?



1 Estimate the product by rounding 735 and 183 to the highest place of each number.



2 Please compare the estimated product with the actual value of 735×183 by using a calculator.

2 In the nature park, there is a rectangular field with an area of $1034m^2$. One of its sides is $47m$ long. How many is the estimated length of the other side of the field?



1 Estimate the quotient by rounding 1034 and 47 to the highest place of each number.

2 Use a calculator to find the actual value of $1034 \div 47$, and compare it with the estimated quotient.

To estimate a product or a quotient, you usually round the numbers to their highest places.



1 Find the approximate values of the following calculations, then use the calculator to find the actual values.

(1) 8130×698 (2) 2870×9210 (3) 83.7×196

(4) $97310 \div 185$ (5) $581064 \div 3124$ (6) $870.4 \div 512$

Check

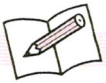
1 Fill in the ☐ for the following equations.

(1) $8723 = 1000 \times \square + 100 \times \square + 10 \times \square + 1 \times \square$

(2) $4.605 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

2 What numbers are 10 times and 100 times as large as 0.162?

3 What are the numbers $\frac{1}{10}$ and $\frac{1}{100}$ of 5.23?



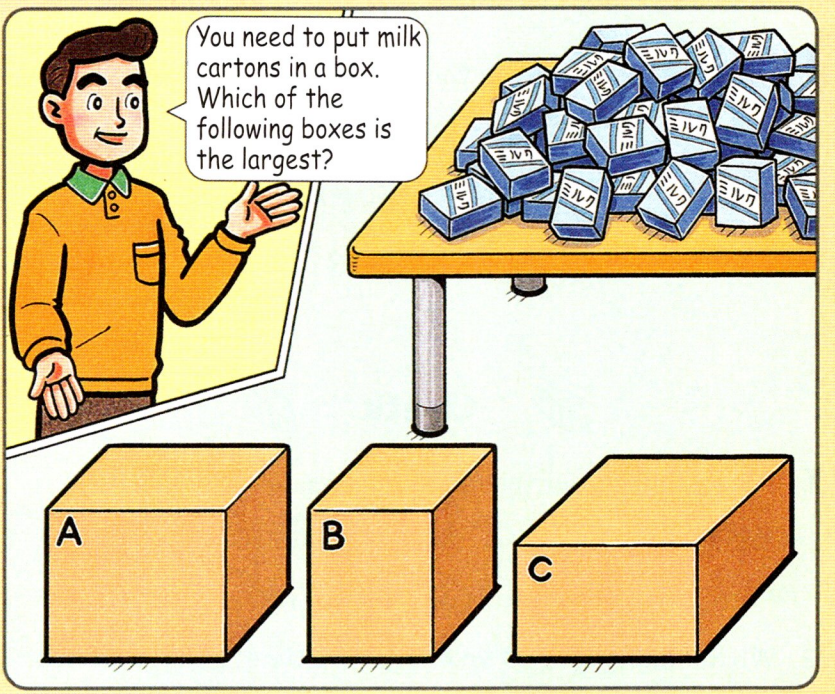
At the end of the unit,
write down the following in your notebook.

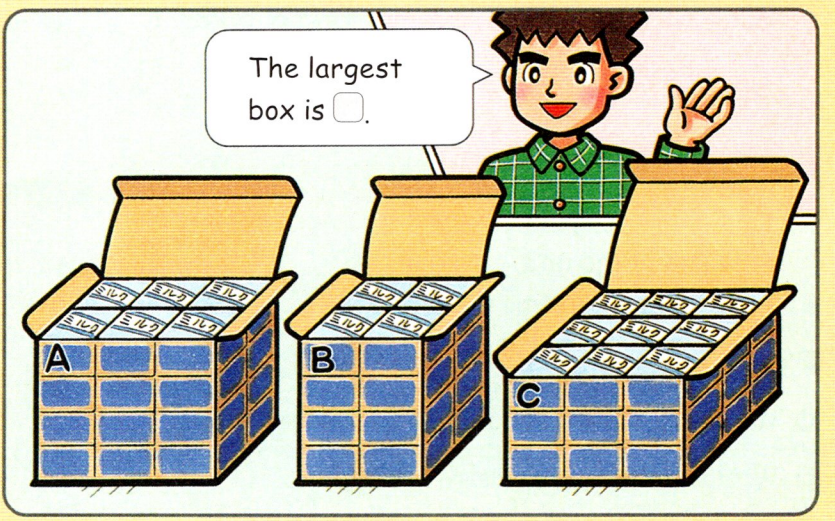
- ◎ What you understood and felt good about
- ◎ What things are similar to what you learned before
- ◎ What you would like to learn in the future

It will be useful if you write down what you thought about after each unit.



2 Volume

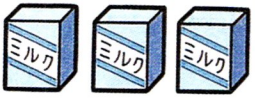

You need to put milk cartons in a box. Which of the following boxes is the largest?
A
B
C


The largest box is ☐.
A
B
C



How much larger is box C than box A?

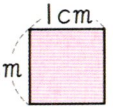
If you compare them by the number of cartons of milk, box C is ☐ packs larger than box A.





Sayuri

We could measure the area by using $1cm^2$ squares. I wonder if we can do something similar for the size of a box.



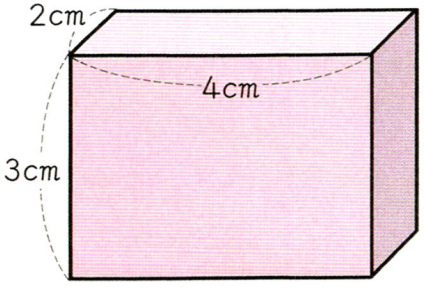


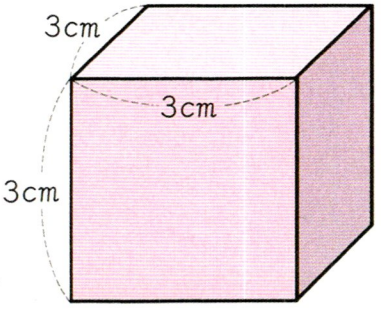
Makoto

Let's learn how to measure the size of rectangular prisms and cubes!

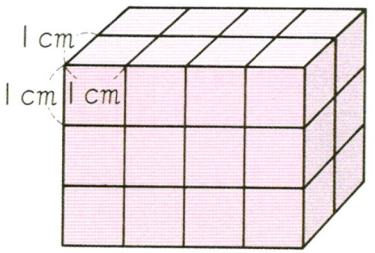
1 Volume

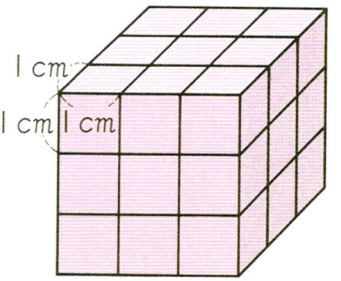
1 Which of the following boxes is larger and by how much? Think about how to show the size of each box.





1 Use cubes with 1 cm edges to build the same shaped rectangular prism and cube that appeared on the previous page.

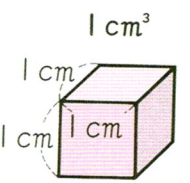




2 How many cubes did you use for each box?

The size of rectangular prisms and cubes can be expressed as the number of cubes with 1 cm edges.

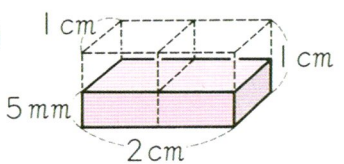
The amount of space an object takes up is called **volume**.
The volume of a cube with a 1 cm edge is 1 **cubic centimeter** and is written as $1\,cm^3$.



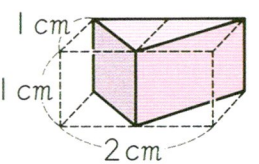
3 What are the volumes of the rectangular prism and the cube on the previous page? Which shape is larger and by how much?

1 What are the volumes of the following shapes?

(1)

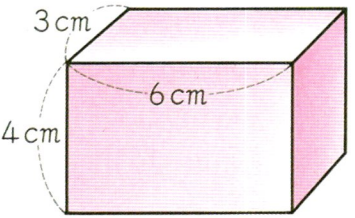


(2)



2 Volume of Rectangular Prisms and Cubes

1 Let's think about how to calculate the volume of the rectangular prism on the right!

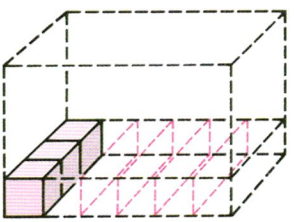




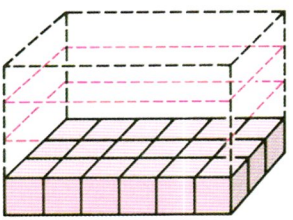
Mami

We could find the area using length x width . . .

1 Find how many $1cm^3$ cubes are in the rectangular prism.

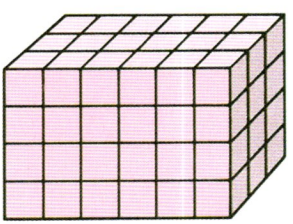


❶ How many cubes fit into the length of the first layer?



❷ How many cubes fit into the width of the first layer?

❸ How many layers are there?



2 Find how many cubes there are in the rectangular prism by calculation.

The number of $1cm^3$ cubes in the rectangular prism above is:

$$3 \times 6 \times 4 = 72$$

Because there are seventy-two $1cm^3$ cubes, its volume is $72cm^3$.

3 Find the volumes of the rectangular prism and the cube on page 13 by calculation.

To calculate the volume of a rectangular prism or a cube, follow the steps below:

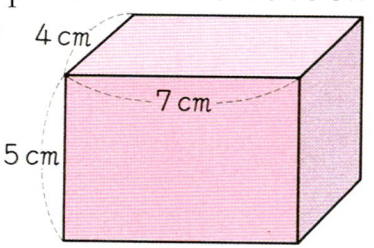
1. Measure the length, width, and height of the rectangular prism or cube.
2. Multiply the three values above.

You can find the volumes of rectangular prisms and cubes with the following formulas.

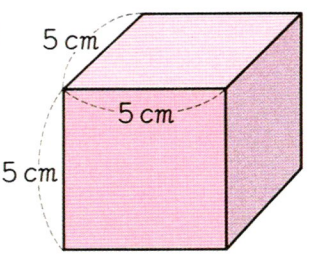
Volume of a rectangular prism = **Length** × **Width** × **Height**
Volume of a cube = **1 edge** × **1 edge** × **1 edge**

1 How many cm^3 are the volumes of the rectangular prism and the cube below?

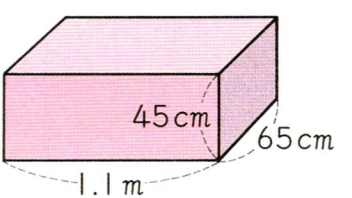
(1)



(2)



2 Find ways to calculate the volume of the rectangular prism on the right.



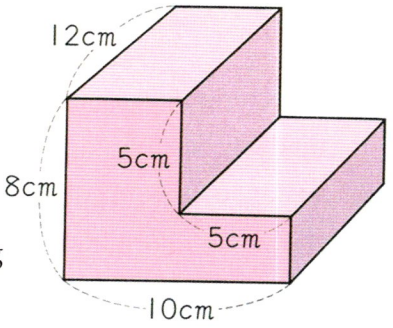


Naoko

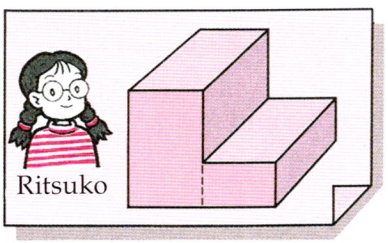
When the units for each edge are different . . .

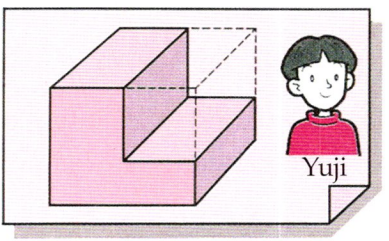
Ideas for finding volume

2 Let's think about how to find the volume of the shape on the right!



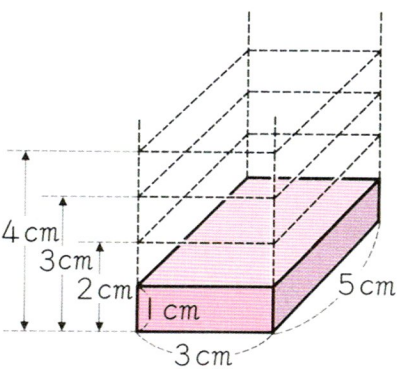
1 Please explain the following two people's solutions.





2 Find the volume using the two methods above.

3 Let's investigate how the volume changes when the height changes from 1cm to 2, 3, or 4cm without changing the width and length of the figure on the right!



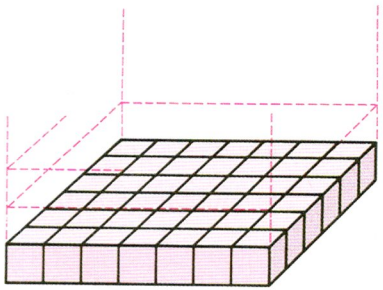
1 Complete the table below and investigate the results.

Height(cm)	1	2	3	4
Volume(cm^3)				

2 When the height becomes 2 times, 3 times, and 4 times as much as 1cm, how does the volume change?

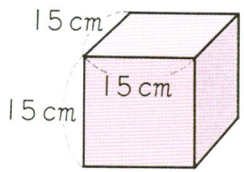
Practice 1

① A rectangular prism was made by putting together six layers of $1cm$ cubes as shown on the right. What is the volume of the rectangular prism in cm^3?

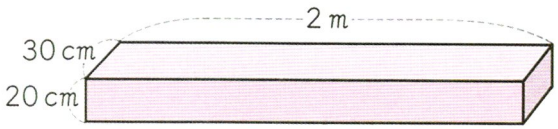


② Please find the volumes of the cube and the rectangular prism below.

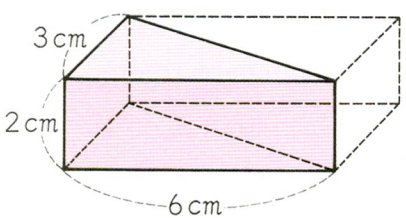
(1)



(2)

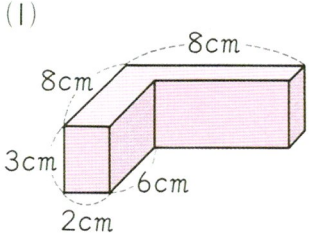


③ Please find the volume of the figure on the right.

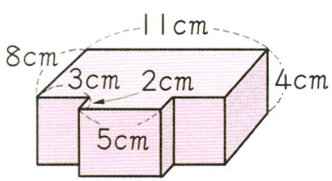


④ Please find the volumes of the figures below.

(1)

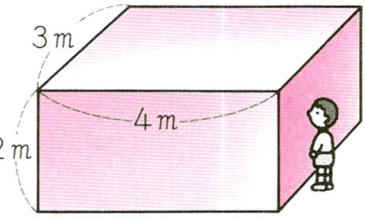


(2)



3 Units for Large Volumes

1 Let's think about how to show the volume of the rectangular prism on the right!

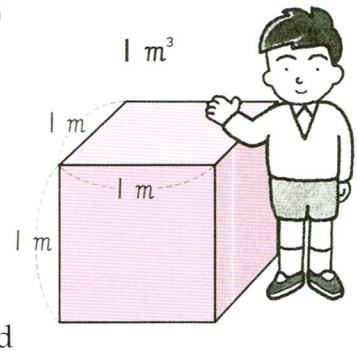




Kazuya

For large areas, we use an area unit made out of a square with 1 meter sides . . .

To show a large volume, you use a unit made out of a cube with $1m$ edges.



The volume of a cube with $1m$ edges is called **1 cubic meter** and is written as $1m^3$.

1 What is the volume of the rectangular prism above?

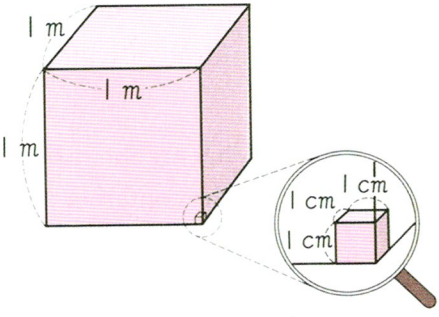
1 What is the volume of a cube which has an edge of $2m$ in cm^3?

2 What is $1m^3$ in cm^3?

1 How many $1cm^3$ cubes are there in a $1m^3$ cube?

$100 \times 100 \times 100$

$= 1000000$

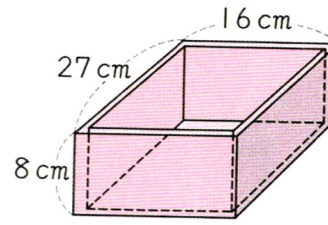


$1m^3 = 1000000cm^3$

1 How many m^3 is the volume of a rectangular prism with a length of $3m$, a width of $3m$, and a height of $200cm$? How many cm^3 is it?

4 Capacity

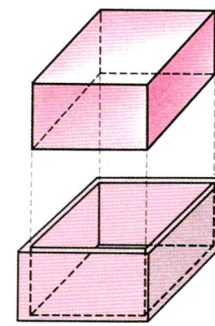
1 We made a container with a $1cm$ thick board that looks like the rectangular prism on the right. Let's find the volume of water that fits in the container!



1 To find the volume of water that fits in the container, what dimensions do you need to know?

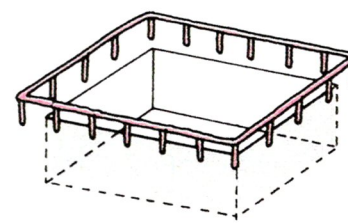
Capacity

The dimensions of the inside of a container are called the **inside measure**. The height of the inside of a container is called the depth. The size of the inside of a container is called **capacity** and it shows the volume of water needed to fill it.



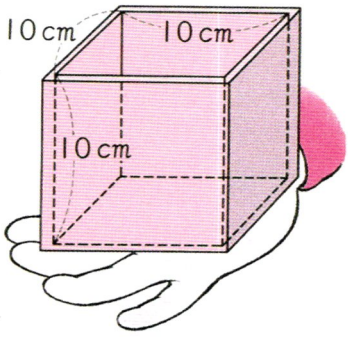
2 What are the inside measures (length, width, and depth) of the container above? How many cm^3 is the capacity?

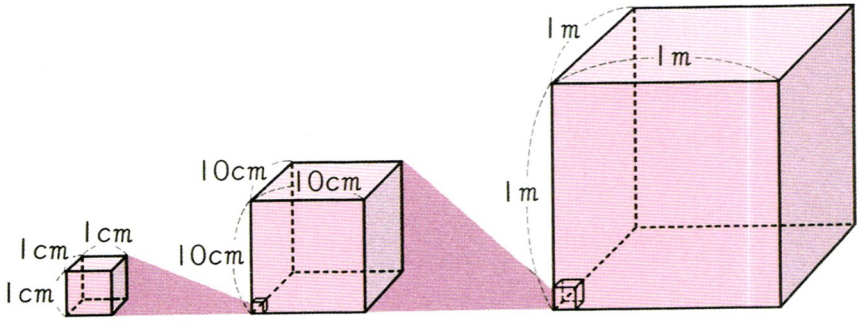
1 What is the capacity of a water tank with an inside measure of $2m$ length, $3m$ width, and $1m$ depth?



2

The capacity of a container that has an inside measure of **10cm** each for length, width, and depth is **1ℓ**. Let's look at the figure below and find the relationship between the units for volume and capacity!





1 How many cm^3 is 1ℓ? Also, how many $m\ell$ is 1ℓ?

$$1\ell = 1000\,cm^3$$

$$1\ell = 1000\,m\ell$$



Mami

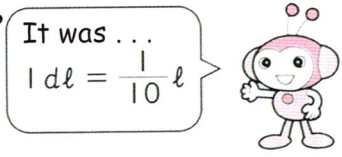
2 How many ℓ is $1m^3$?

$$1\,m^3 = 1000\ell$$



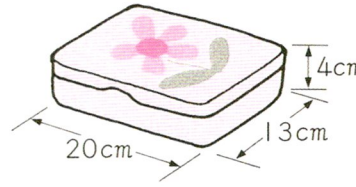
Minoru

1 How many cm^3 is $1d\ell$?

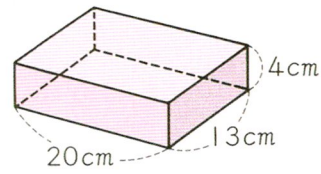


▶ Approximate capacity

3 A lunch box looks like the one on the right. Let's find the approximate capacity of this lunch box!



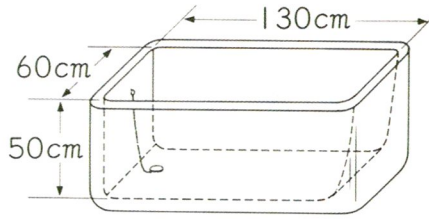
1 What does the shape of this lunch box look like?



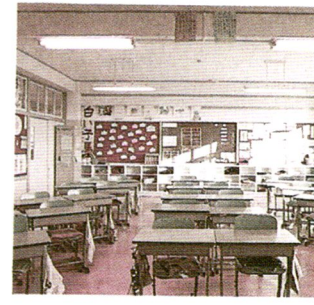
$13 \times 20 \times 4 =$ ☐

Answer: about ☐ cm^3

1 A bathtub looks like the one on the right. What is the approximate capacity of this bathtub in ℓ?



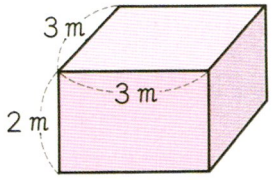
2 Let's estimate the capacity of your classroom! Also, find the approximate capacity of many different things around you.



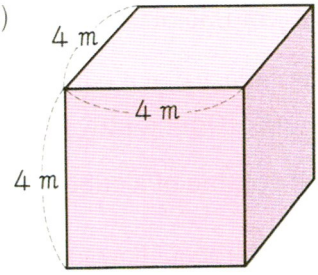
Practice 2

① What are the volumes of the following rectangular prism and cube?

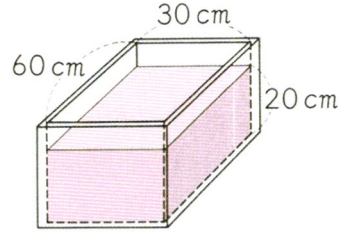
(1)



(2)

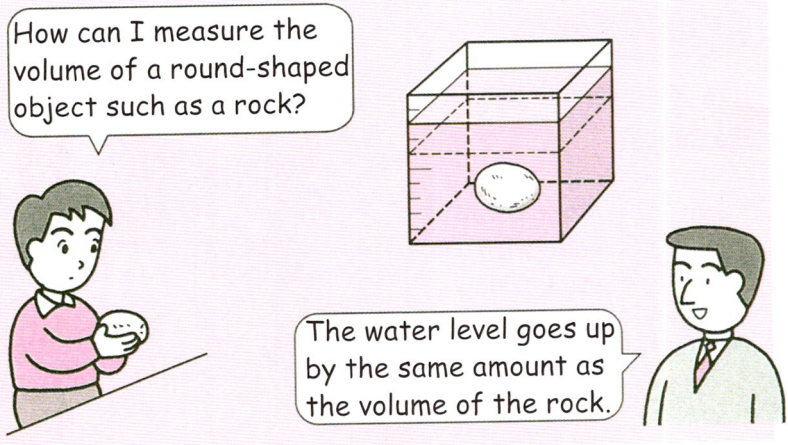


② The inside measures of the length and width of the water tank on the right are as indicated. We filled it with water up to a depth of 20cm. How many ℓ of water were put in it?



Challenge

Can You Measure the Volume of a Round-shaped Rock?



Check

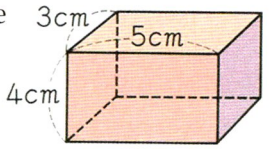
1 Please fill in the boxes with the appropriate units.

(1) The volume of a cube with 1cm edges is 1☐.

(2) The capacity of a cube with inside measures of 10cm length, 10cm width, and 10cm depth is 1☐.

(3) The volume of a cube with 1m edges is 1☐.

2 Please think about how to find the volume of the rectangular prism on the right. Then fill in the boxes.

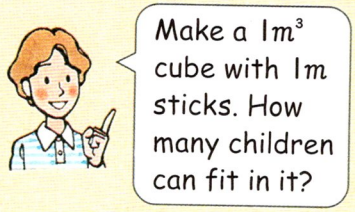


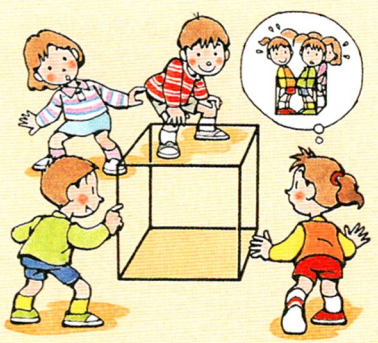
(1) 3 cubes with 1cm edges fit in the length, and 5 cubes with 1cm edges in the width. So the first layer will have ☐ cubes. The height is 4cm so you need to make ☐ layers.

(2) The total number of 1cm^3 cubes can be found by the caluculation of ☐×☐×☐. So the volume of this rectangular prism is ☐ cm^3.

Challenge

How Many People Can Fit in the Space?





Review 1

① Please calculate.

(1) $0.49 + 0.839$ (2) $7.53 + 2.47$ (3) $4.345 + 1.7$

(4) $0.93 - 0.631$ (5) $1.3 - 0.39$ (6) $7 - 2.87$

(7) $\frac{7}{9} + \frac{5}{9}$ (8) $\frac{8}{11} + 2\frac{6}{11}$ (9) $2\frac{3}{5} - \frac{1}{5}$ (10) $1\frac{1}{3} - \frac{2}{3}$

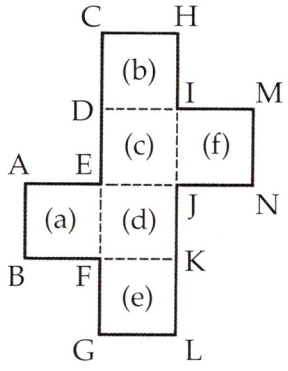
② Please find the estimated value of the following calculations. Then use a calculator to find the actual value.

(1) 748×521 (2) 840×326 (3) 461×304

(4) $5742 \div 319$ (5) $6678 \div 106$ (6) $3395 \div 485$

③ Using the digits 1, 3, 5, and 9 only once, and a decimal point, make the largest possible number and the smallest possible number.

④ The figure on the right is the net of a cube. When you fold it back into a cube, which faces will be parallel to each other? Which faces will be perpendicular to edge AB?

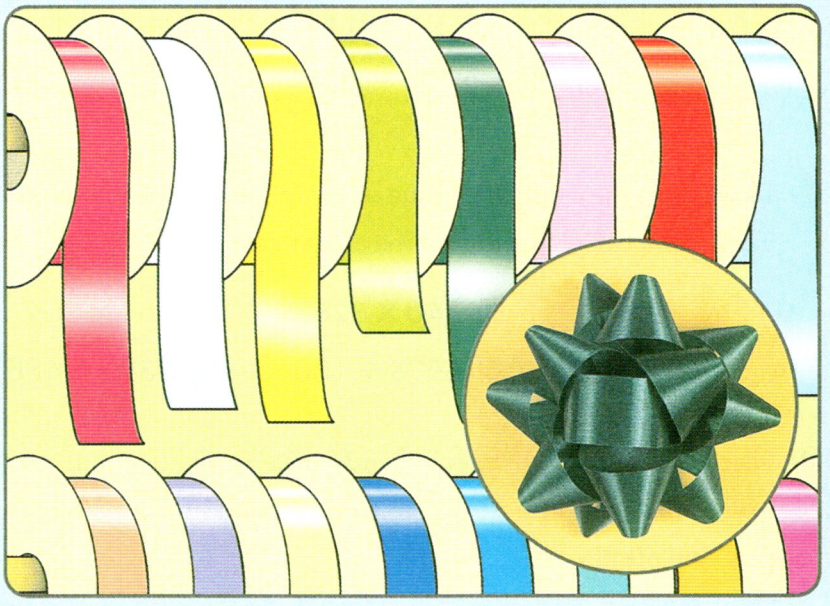


⑤ Please calculate. For division calculations, divide completely.

(1) 0.2×3 (2) 3.6×2 (3) 13.6×6 (4) 2.3×54

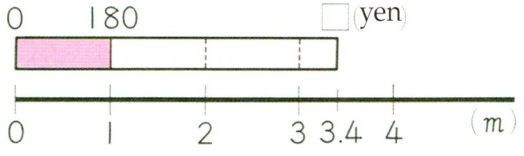
(5) $3.2 \div 4$ (6) $6.5 \div 5$ (7) $15.4 \div 7$ (8) $51.6 \div 43$

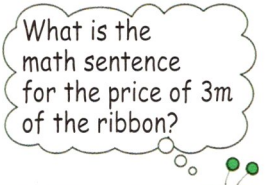
3 Multiplication of Decimal Numbers



1 Multiplying by a Decimal Number

1 We bought 3.4 meters of ribbon that cost 180 yen per meter. What was the price of the ribbon?





? How should we write the math sentence?



Price for 1m	×	Length we bought (Unit: m)	=	Price we have to pay

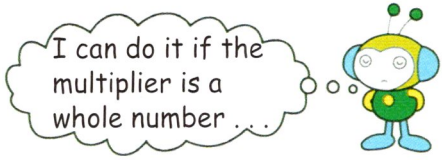


Let's think about multiplication of decimal numbers!

Even if the length of the ribbon is shown as a decimal number, we can construct a multiplication sentence just like we do with whole numbers to find the price of the ribbon.

180×3.4

Let's think about how to calculate 180×3.4!

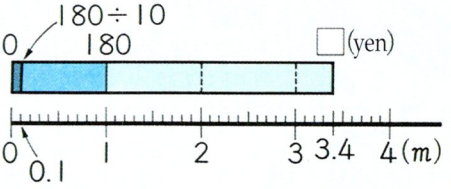


1 Let's explain these two friends' methods!



Because 3.4m is made out of 34 pieces of 0.1m...

Sayuri



Price for 0.1m ·····180 ÷ 10

Total for 34 pieces ·····(180 ÷ 10) × 34

180×3.4 = 180 ÷10×34 = □ Answer: □ yen



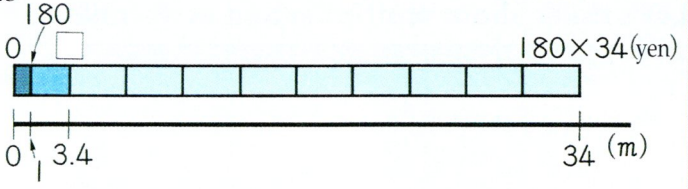
If the length of the ribbon is 10 times longer, the price becomes 10 times as much.

Makoto

180 × 3.4 = □

10 times ↓ ↓ 10 times, $\frac{1}{10}$

180 × 34 = 6120



Price for 34m ·····180 × 34

Because 3.4m is $\frac{1}{10}$ of 34m ····(180 × 34) ÷ 10

180×3.4 = 180×34÷10 = □ Answer: □ yen

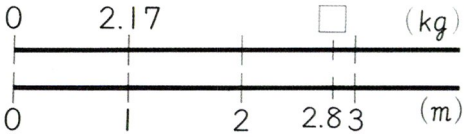
Both of them came up with the idea of using whole numbers to do the calculation.

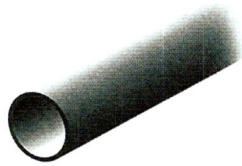


1 If we buy 2.7m of ribbon that costs 240 yen per meter, what will the price be?

Algorithm for decimal multiplication

2 There is a 1m pipe that weighs 2.17kg. How much would 2.8m of the pipe weigh?





Math Sentence

1 Let's think about how to do the calculation!



Mami

$2.17 \times 2.8 = \square$

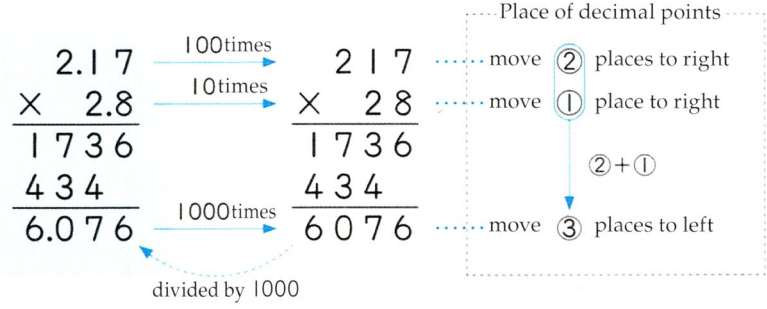
100 times ↓ 10 times ↓ 1000 times ↓ $\frac{1}{1000}$

$217 \times 28 = 6076$

The product of 2.17×2.8 can be found by multiplying two numbers that are 100 times 2.17 and 10 times 2.8, and then dividing the product by 1000.

$2.17 \times 2.8 = 6.076$ Answer: 6.076 kg

? Let's think about how to calculate 2.17×2.8 with the multiplication algorithm!



How to carry out multiplication when the multiplier is a decimal number

1. Carry out the calculation as if there were no decimal points.
2. To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand and the multiplier. Then move the decimal point of the product from right to left the same number of places.

$$\begin{array}{r} 2.17 \\ \times\ \ 2.8 \\ \hline 1736 \\ 434 \\ \hline 6.076 \end{array}$$

2.17 ····· ② places
2.8 ····· ① place
② + ①
6.076 ····· ③ places

1 (1) 4.37×5.6 (2) 3.81×7.4 (3) 48.7×3.1

(4) 9.6×9.52 (5) 4.8×2.64 (6) 0.41×6.83

3 Let's think about how to carry out the following two calculations!

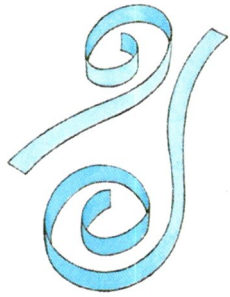
(1) $\begin{array}{r} 4.92 \\ \times\ \ 7.5 \\ \hline 2460 \\ 3444 \\ \hline 36.900 \end{array}$ (2) $\begin{array}{r} 0.018 \\ \times\ \ 3.4 \\ \hline 72 \\ 54 \\ \hline 0.0612 \end{array}$

1 (1) 2.35×5.6 (2) 5.48×3.65 (3) 0.37×2.7

(4) 0.36×1.25 (5) 54×6.8 (6) 875×5.12

Multiplier and the size of the product

4 There is a ribbon that costs 180 yen for 1m. We want to buy two strips of that ribbon. One strip is 1.8m and the other one is 0.8m. Which strip of ribbon costs less than 180 yen?





Let's think about the relationship between the multiplier and the size of the product!

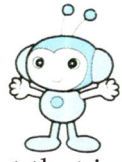
(1) $180 \times 1.8 =$ ☐

(2) $180 \times 0.8 =$ ☐

1 Which one of the products becomes smaller than 180?

In multiplication of decimal numbers, if the multiplier is less than 1, the product will be less than the multiplicand.

If multiplier > 1, then the product > multiplicand
If multiplier < 1, then the product < multiplicand



1 Which of the following produces a product that is smaller than 6?

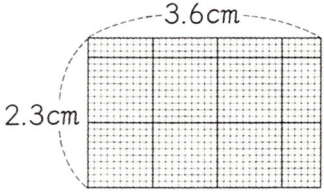
Ⓐ 6×0.9 Ⓑ 6×1.4 Ⓒ 6×2.08 Ⓓ 6×0.85

2 (1) 83×0.7 (2) 29.3×0.4 (3) 0.98×0.8

▶ Rules of formulas and calculations

5 Let's find out if we can use the formulas for area and volume, even if the lengths of the sides are shown with decimal numbers!

(1) The area of a rectangle that has a length of 2.3cm and a width of 3.6cm

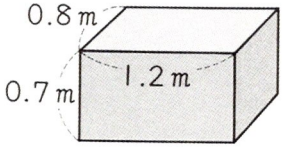


1 How many squares with 1mm sides are in this rectangle?

2 When we have 100 squares with 1mm sides, it becomes 1cm². How many cm² is the area of the rectangle?

3 Please find the area of the rectangle by calculating 2.3×3.6.

(2) The volume of a rectangular prism that has a length of 0.8m, a width of 1.2m, and a height of 0.7m



4 Calculate the volume by using centimeters and meters as units, then compare the results.

Even if the lengths of the sides are shown with decimal numbers, we can find area and volume by carrying out the multiplication calculations indicated by formulas.

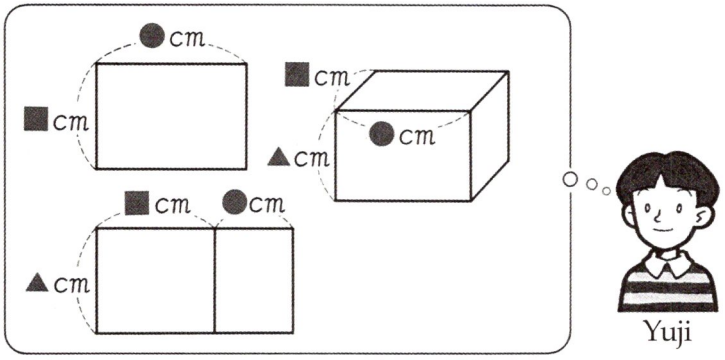
1 What is the area of a postcard that has a length of 10.3cm and a width of 14.8cm?

6 Let's find out if the calculation rules we found for whole number calculations work for calculating decimal numbers!

(1) ■ × ● = ● × ■

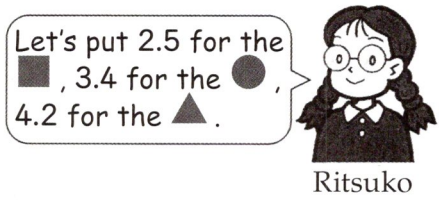
(2) (■ × ●) × ▲ = ■ × (● × ▲)

(3) (■ + ●) × ▲ = ■ × ▲ + ● × ▲



Yuji

1 Please put decimal numbers in the ■, ●, and ▲, and find out if rules (1), (2), and (3) work.



Ritsuko

The calculation rules we found for whole numbers apply to the calculation of decimal numbers.

1 Please carry out the following calculations by thinking about better ways to calculate.

(1) 4.8 × 4 × 2.5 (2) 2.4 × 1.8 + 2.6 × 1.8

Practice

① Please calculate the following problems. Also, please check your work with a calculator.

(1) 7.2×36.5 (2) 1.63×4.9 (3) 0.15×29.7

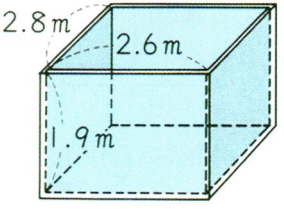
(4) 21.6×16.2 (5) 6.97×18.4 (6) 5.66×1.28

(7) $1.3 \times 2.5 \times 8.4$ (8) $1.57 \times 3.8 \times 4.5$

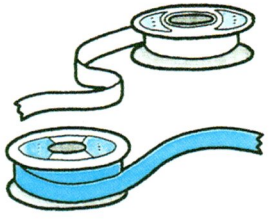
(9) $3.6 \times 1.2 + 1.4 \times 1.2$ (10) $5.8 \times 0.45 - 1.36$

② The weight of 1ℓ of milk is $1.03kg$. If we pour 0.18ℓ of this milk into a cup, what is the weight of the milk in the cup in kg?

③ There is a water tank that looks like the figure on the right. What is the capacity of the tank in m^3?



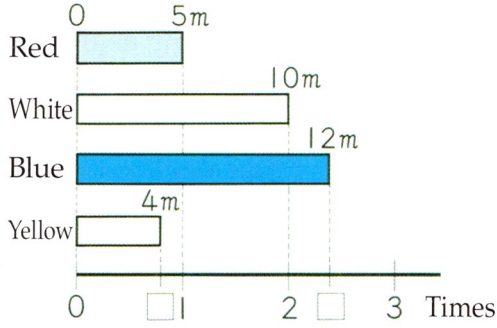
④ We bought two strips of ribbon. Each one is $2.3m$ in length. One of them cost 160 yen for $1m$ and the other one cost 240 yen for $1m$. Then we gave 1000 yen to the cashier. What is the change?

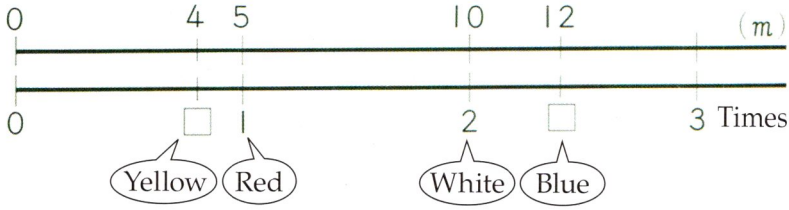


⑤ There is a $1m$ pipe that weighs $1.8kg$. Please write a word problem that has 1.8×0.8 as its math sentence.

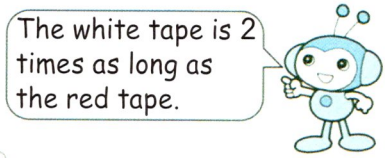
2 Times as Many/Much and Multiplication

1 There are several tapes like the ones shown on the right. If we make the length of the red tape the base unit, how many times as long as the red tape are the other tapes?





1 How many times as long is the blue tape as the red tape?



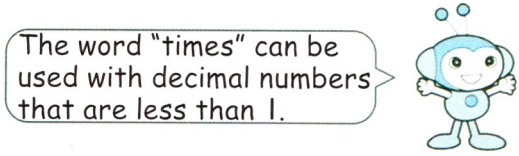
Math Sentence

Answer: ☐ times

2 How many times as long is the yellow tape as the red tape?

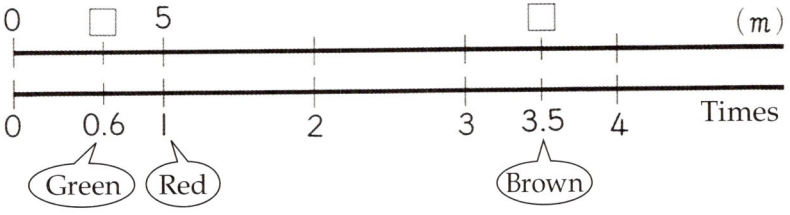
Math Sentence

Answer: ☐ times



2 There is a 5m strip of red tape. There are also a brown tape and a green tape. The length of the brown tape is 3.5 times as long as the red tape. The length of the green tape is 0.6 times as long as the red tape.
Please find the length of the brown tape and the green tape.

? Think about what we need to do to find the size of something when it is expressed as a decimal number of times.



1 What is the length of the brown tape?

The length that is 3.5 times as long as 5m is the length that corresponds to 3.5 when we consider 5m as 1.

$5 \times 3.5 =$ ☐ Answer: ☐ m

2 What is the length of the green tape?

Math Sentence ☐ Answer: ☐ m

! To find something that is 3.5 times or 0.6 times of the given size, you multiply the given size by 3.5 or 0.6.

1 Please find the capacity of a tank that will hold 4.2 times and 0.8 times as much as a 7ℓ tank.

Check

1 Please find the mistakes in the following calculations and make corrections.

(1)
$$\begin{array}{r} 8.1 \\ \times\ 3.4 \\ \hline 324 \\ 243 \\ \hline 275.4 \end{array}$$

(2)
$$\begin{array}{r} 0.07 \\ \times\ 0.14 \\ \hline 28 \\ 7 \\ \hline 0.098 \end{array}$$

(3)
$$\begin{array}{r} 5.74 \\ \times\ 2.5 \\ \hline 2870 \\ 1148 \\ \hline 14350 \end{array}$$

2 Please calculate the following problems.

(1) 1.3×4.8 (2) 0.42×6.5 (3) 0.82×0.72

3 Please find the weight of two bags that are 1.5 times and 0.75 times as heavy as a 5.6kg bag.

Challenge

Let's Try a Mysterious Multiplication Calculation!



❶Select a decimal number
❷Multiply the number by 0.25
❸Multiply the result by 0.625
❹Multiply the result by 12.8
❺Multiply the result by 0.5
What is the result of the calculations?

3.2 ×0.25= □
□ ×0.625= □
□ ×12.8= □
□ ×0.5 = □

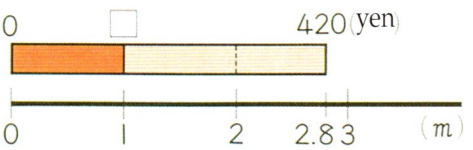


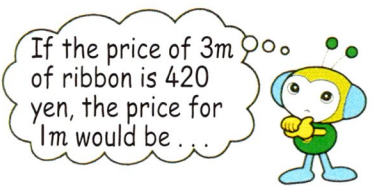
4 Division of Decimal Numbers



1 Dividing by a Decimal Number

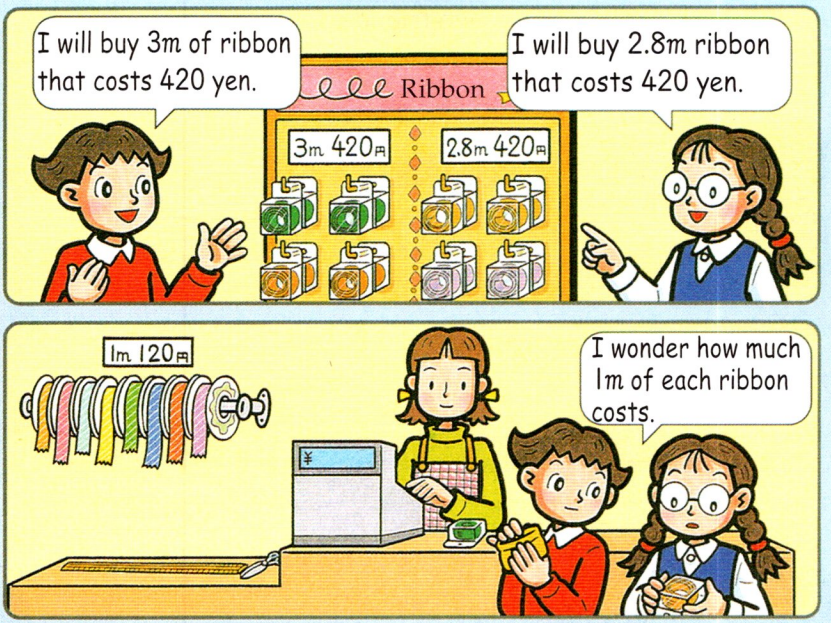
1 When we bought 2.8m of ribbon, the price was 420 yen.
How much was the price of 1m of ribbon?





How should we write the math sentence?

Price we paid ÷ Length we bought (Unit: m) = Price for 1m

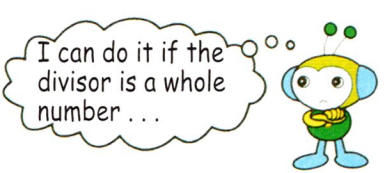


Let's think about how to divide a number by a decimal number!

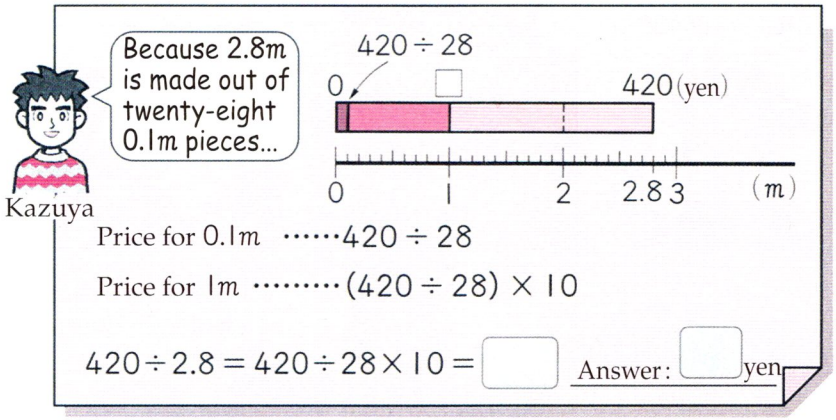
Even if the length of the ribbon is shown with a decimal number, we can construct a division math sentence just like we do with a whole number to find the price of 1m of ribbon.

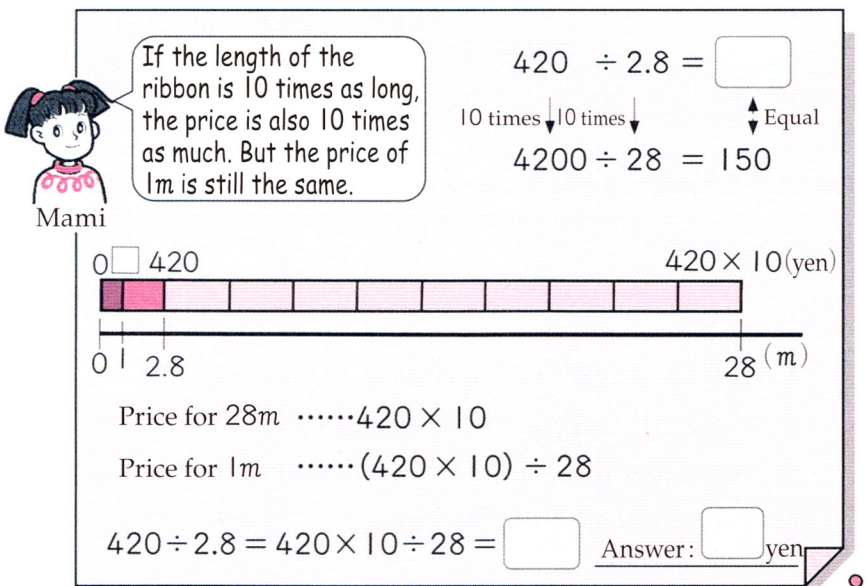
$$420 \div 2.8$$

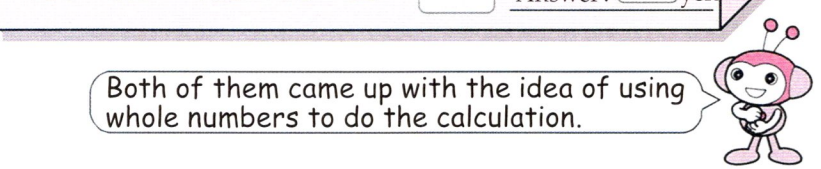
Let's think about how to calculate $420 \div 2.8$!



1 Let's explain these two friends' methods!





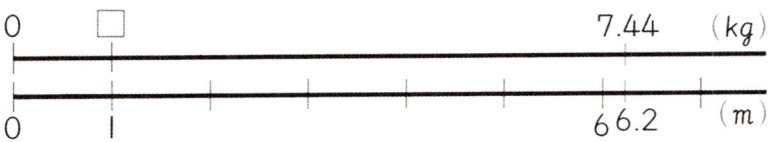


1 The price for 3.5m of ribbon was 630 yen. What would be the price for 1m of ribbon?

▶ Algorithm for decimal division

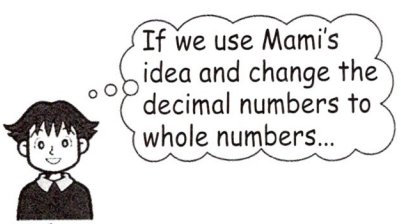
2 There is a 6.2m pipe that weighs 7.44kg.
How much would 1m of pipe weigh?





Math Sentence

1 Let's think about how to do the calculation!



$7.44 \div 6.2 = \square$

10 times ↓ 10 times ↓ ↕ Equal

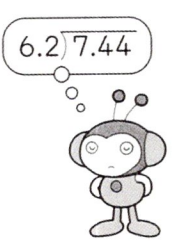
$74.4 \div 62 = 1.2$

The quotient of $7.44 \div 6.2$ is equal to the quotient of $74.4 \div 62$ because both the dividend and divisor were multiplied by 10.

$7.44 \div 6.2 = 1.2$

Answer: 1.2 kg

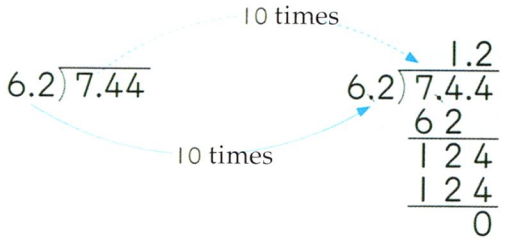
? Let's think about the calculation of $7.44 \div 6.2$ using the algorithm!



10 times

$$6.2\overline{)7.44} \qquad \begin{array}{r} 1.2 \\ 6.2\overline{)7.4.4} \\ \underline{6\,2} \\ 1\,2\,4 \\ \underline{1\,2\,4} \\ 0 \end{array}$$

10 times



How to carry out the division algorithm when the divisor is a decimal number

1. Move the decimal point of the divisor to the right and make it a whole number.
2. Move the decimal point of the dividend to the right the same number that you moved the divisor.
3. Carry out the calculation just as if the divisor were a whole number. Then place the decimal point of the quotient right above the decimal point of the dividend that was moved to the right.

$$\begin{array}{r} 1.2 \\ 6.2\overline{)7.4.4} \\ \underline{6\,2} \\ 1\,2\,4 \\ \underline{1\,2\,4} \\ 0 \end{array}$$

1 (1) $4.2\overline{)7.56}$ (2) $2.8\overline{)8.96}$ (3) $3.7\overline{)9.731}$

(4) $3.84\overline{)9.216}$ (5) $2.37\overline{)9.954}$ (6) $7.3\overline{)58.4}$

3 Let's think about how to carry out the following three calculations!

(1) $2.368 \div 3.7$ (2) $8.4 \div 3.36$ (3) $7 \div 1.25$

$$\begin{array}{r} 0.6\,4 \\ 3.7\overline{)2.3.6\,8} \\ \underline{2\,2\,2} \\ 1\,4\,8 \\ \underline{1\,4\,8} \\ 0 \end{array} \qquad \begin{array}{r} 2.5 \\ 3.36\overline{)8.4\,0} \\ \underline{6\,7\,2} \\ 1\,6\,8\,0 \\ \underline{1\,6\,8\,0} \\ 0 \end{array} \qquad \begin{array}{r} 5.6 \\ 1.25\overline{)7.0\,0} \\ \underline{6\,2\,5} \\ 7\,5\,0 \\ \underline{7\,5\,0} \\ 0 \end{array}$$

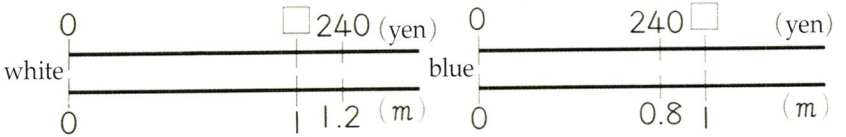
1 (1) $5.04 \div 8.4$ (2) $2.738 \div 3.7$ (3) $5.9 \div 2.36$

(4) $92.3 \div 1.42$ (5) $6 \div 2.5$ (6) $42 \div 5.6$

▶ Divisor and the size of the quotient

4 The white ribbon costs 240 yen for $1.2m$ and the blue ribbon costs 240 yen for $0.8m$.

Which ribbon costs more for $1m$?



Let's find the relationship between the divisor and the size of the quotient!

(1) $240 \div 1.2 =$ □

(2) $240 \div 0.8 =$ □

1 Which quotient becomes larger than 240?

For division of decimal numbers, when you divide by a number smaller than 1, the quotient becomes larger than the dividend.

If divisor > 1, then the quotient < dividend
If divisor < 1, then the quotient > dividend



1 Which of the following produces a quotient larger than 8?

Ⓐ $8 \div 1.25$ Ⓑ $8 \div 0.02$ Ⓒ $8 \div 0.64$ Ⓓ $8 \div 5$

2 (1) $19.8 \div 0.3$ (2) $0.39 \div 0.6$ (3) $69 \div 0.92$

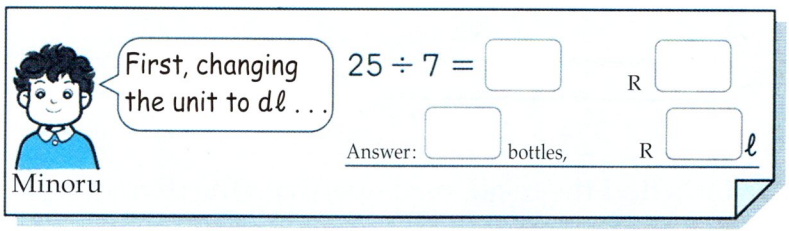
Size of remainder

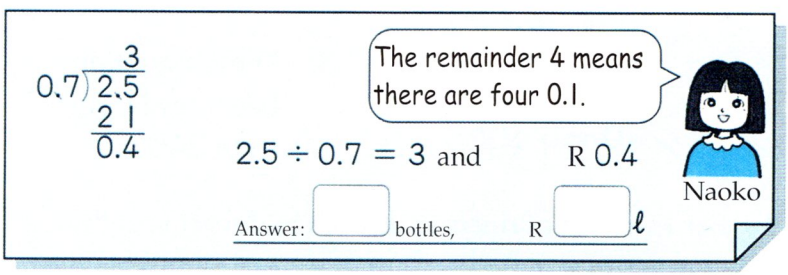
5 We are going to put 2.5ℓ of juice into a 7dℓ capacity bottle.
How many bottles can we fill? How many ℓ of juice will be left over?



? Let's think about division calculations that involve a remainder!

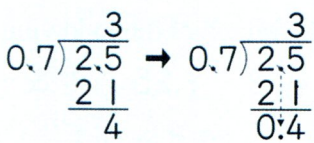
1





2 Please check the answer by changing Naoko's math sentence into "divisor × quotient + remainder = dividend."

! The decimal point of the remainder should line up with the original decimal point of the dividend.

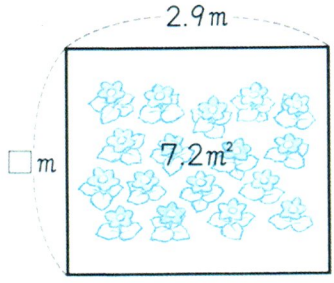


1 Please calculate and find the quotient to the $\frac{1}{10}$ place.
Also, please find the remainder.

(1) $17.58 \div 9.6$ (2) $46.37 \div 2.45$ (3) $34 \div 72.5$

▶ Quotient of rounded numbers

6 We are planning to build a flower garden that has an area of $7.2m^2$.
If we make the width $2.9m$, then how many m should the length be?
Please round off the number to the second highest place.



1 Please make the length $\square$ m and construct a multiplication math sentence. Then find the answer.

$\square \times 2.9 = 7.2$

$7.2 \div 2.9 = 2.48\cdots\cdots$ (5)

Answer: about $2.5\,m$

1 1.5ℓ of sand weighs $2.6kg$.
How many kg does 1ℓ of sand weigh? Please round the answer to the second highest place.

Practice

① Please calculate. For division calculations, divide completely. Also, check the answer using a calculator.

(1) $62.9 \div 3.7$ (2) $5.76 \div 7.2$ (3) $3.24 \div 1.08$

(4) $1.62 \div 2.16$ (5) $0.779 \div 8.2$ (6) $12.21 \div 1.65$

(7) $4.2 \times 3.5 \div 2.8$ (8) $13.6 \times 0.6 \div 1.6$

② 6.5ℓ of oil weighs 5.2kg.

How much does 1ℓ of oil weigh in kg?

③ We bought 32m of rope to make jump ropes.

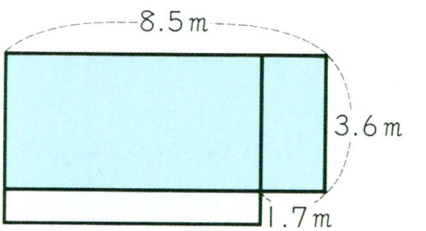
How many jump ropes can we make if the length of each jump rope is 2.5 m?

How many m will be left over?



④ A flower garden has a length of 3.6m and a width of 8.5m.

If we keep the area of the flower garden the same and make the width 1.7m shorter, how many m should the length be?



⑤ A pipe weighs 1.8kg.

Please make up a word problem in which the math sentence would be $1.8 \div 1.2$.

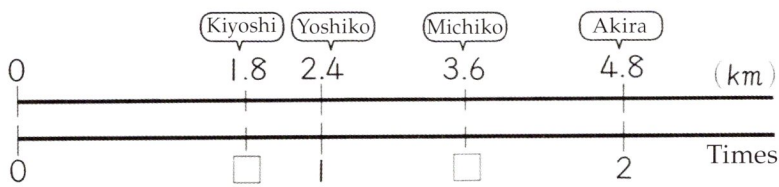
2 Times as Many/Much and Division

1 We have measured the distance from our houses to the train station.
If we make Yoshiko's distance as the base unit, how many times as far are the other people's houses from the station?

Distance from a house to the train station

Name	Distance(km)
Yoshiko	2.4
Akira	4.8
Michiko	3.6
Kiyoshi	1.8

2 Let's think about what kind of calculation we need to find how many times as far!



1 How many times as far from the station are Michiko and Kiyoshi's houses as Yoshiko's house?



Michiko
Math Sentence
Answer: ☐ times

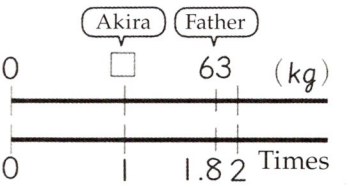
Kiyoshi
Math Sentence
Answer: ☐ times

1.5 times as far means that when we look at 2.4km as the base unit of 1, 3.6km is the same as 1.5.

Even when decimal numbers are involved, we use division to find how many times as much a quantity is, compared to the size of the base unit.

2 Akira's father weighs 63kg. He is 1.8 times as heavy as Akira.
What is Akira's weight in kg?

Let's think about the kind of calculation we need to find the size of the base unit!



1 Let's construct a multiplication sentence by setting Akira's weight to □ kg! Then, please find the number that goes in the □.

Math Sentence

Answer: □ kg

When you find the size of the base unit, it is easier if you construct a multiplication sentence using a □.

1 There is a thermos that has a capacity of 1.5ℓ. It is 0.6 times the capacity of a kettle.
How many ℓ is the capacity of the kettle?

Check

1 Please find the mistakes in the following calculations and make corrections.

(1) $5.41 \div 1.3$

$$\begin{array}{r} 4.1 \\ 1.3\overline{)5.41} \\ 52 \\ \hline 21 \\ 13 \\ \hline 0.8 \end{array}$$

(2) $75.4 \div 1.16$

$$\begin{array}{r} 6.5 \\ 1.16\overline{)75.4} \\ 696 \\ \hline 580 \\ 580 \\ \hline 0 \end{array}$$

(3) $2.1 \div 2.8$

$$\begin{array}{r} 7.5 \\ 2.8\overline{)2.10} \\ 196 \\ \hline 140 \\ 140 \\ \hline 0 \end{array}$$

2 Please divide completely.

(1) $5.74 \div 0.7$ (2) $12 \div 7.5$ (3) $3.6 \div 4.8$

3 How many times as much as $6.4m^2$ are $43.2m^2$ and $1.6m^2$?

Challenge

Let's Try a Mysterious Division Calculation!



❶ Select a decimal number
❷ Divide the number by 1.6
❸ Divide the result by 0.5
❹ Divide the result by 1.25

What is the result of the calculations?

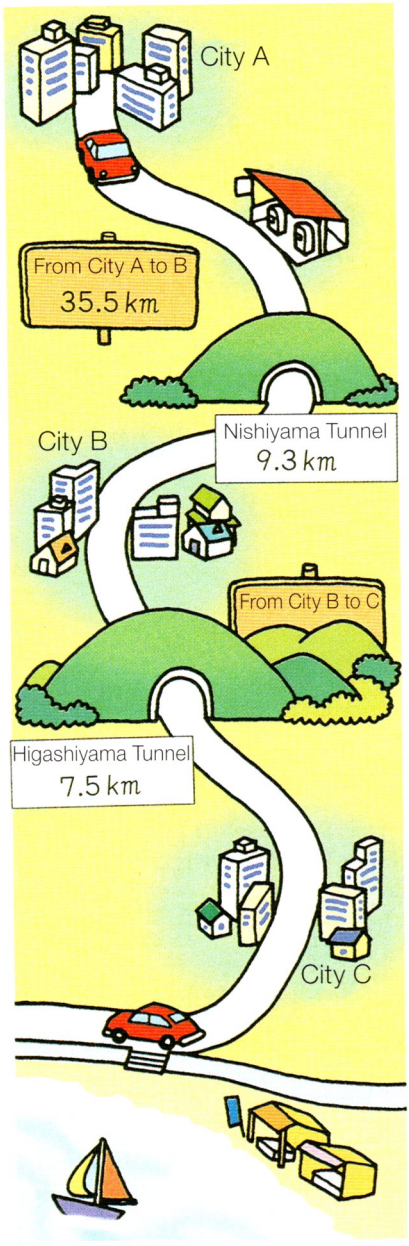
4.2 ÷1.6= □

□ ÷0.5= □

□ ÷1.25= □



What kind of calculation is it going to be?



We bought 37.5ℓ of gasoline that cost 108 yen for 1ℓ.
How much was the total price?

Nishiyama Tunnel is how many times as long as Higashiyama Tunnel?

The distance between City B and City C is 1.4 times as long as the distance between City A and City B.
How many km is the distance between City B and City C?

After driving 90.3km we used 10.5ℓ of gasoline.
How many km can we drive with 1ℓ of gasoline?

We bought a bag of clams as a souvenir for our friend. It cost 450 yen for 1.8kg. How much does 1kg of clams cost?

Review · 2

① Please fill in the ☐ with the appropriate numbers.

$4.385 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

② Please write the numbers that are 10 times, 100 times, $\frac{1}{10}$, and $\frac{1}{100}$ of the numbers below.

(1) 3.208 (2) 0.27 (3) 40.1

③ Please estimate first, then use a calculator to find the answers.

(1) 469×137 (2) 91.1×203 (3) 3.89×695

(4) $9861 \div 519$ (5) $703.5 \div 67$ (6) $960.4 \div 196$

④ Please calculate the following.

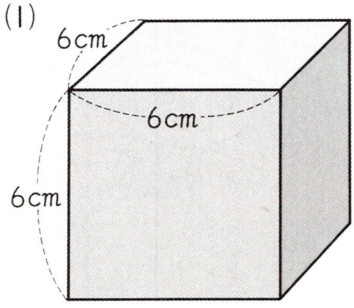
(1) 3.2×4.7 (2) 7.3×0.6 (3) 2.89×5.6

(4) 43.5×7.2 (5) 8.7×7.23 (6) 6.9×27.4

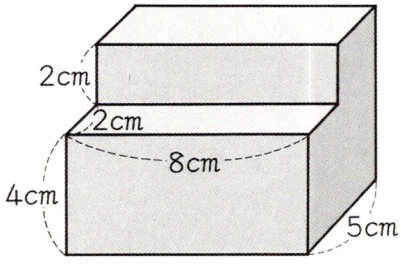
(7) 3.47×5.63 (8) 0.72×3.86 (9) 1.85×0.32

⑤ Please find the volumes of these shapes.

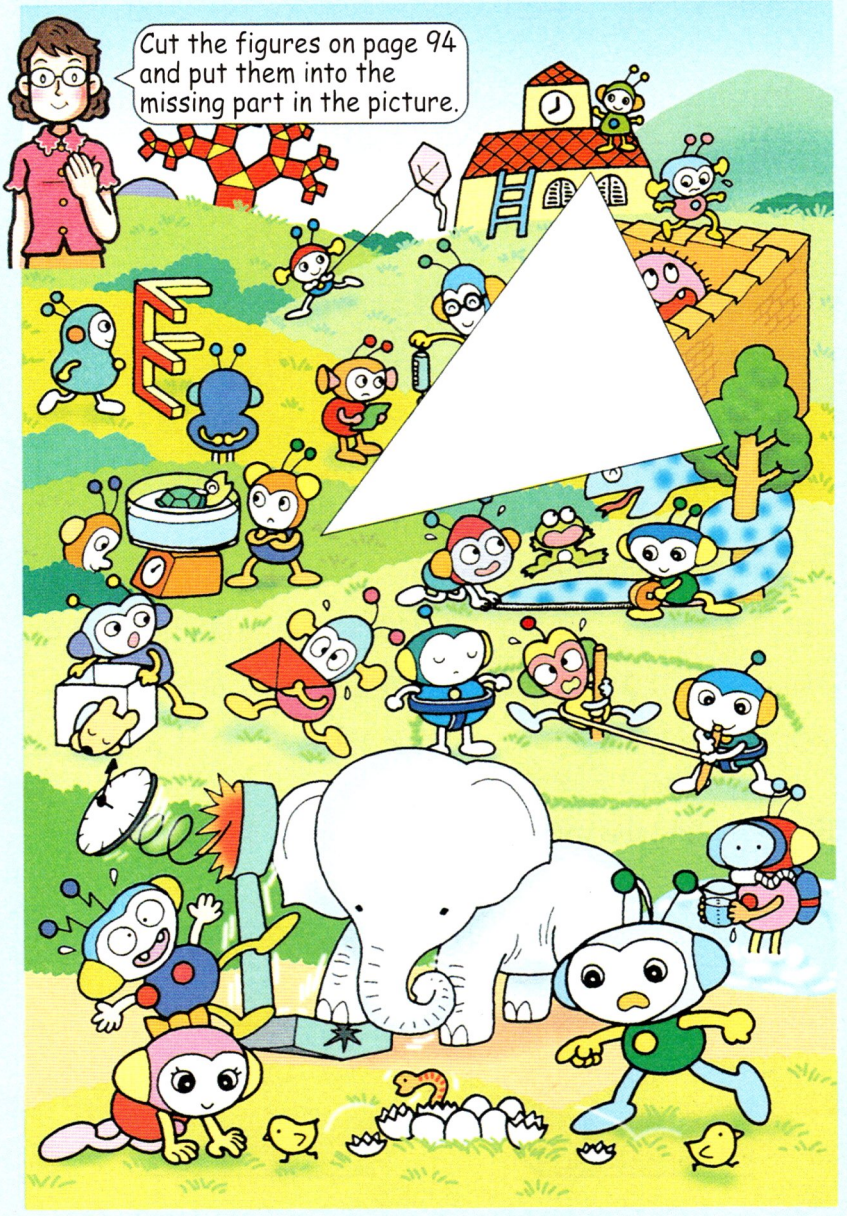
(1)

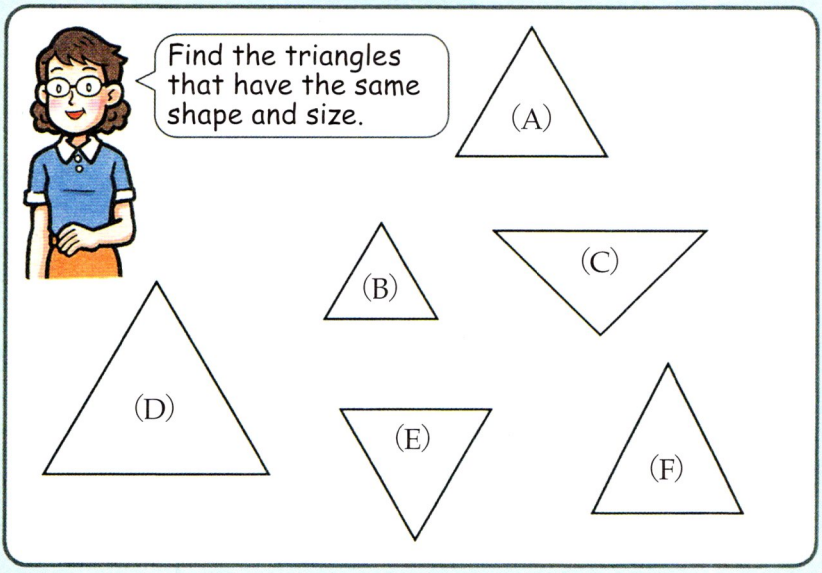


(2)



5 Congruent Shapes

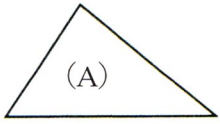


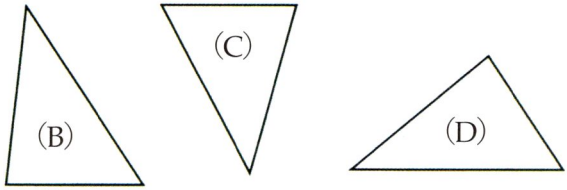


Let's investigate the characteristics of figures that have the same shape and size!

1 Congruent Shapes

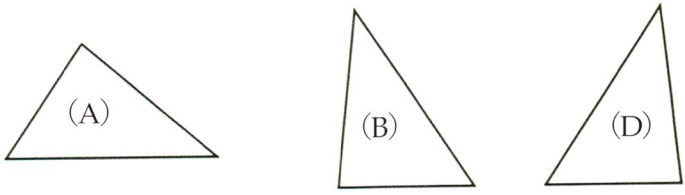
1 From the shapes below, which one has the same shape and size as figure A on the right?



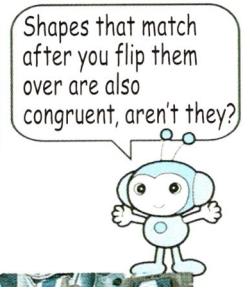




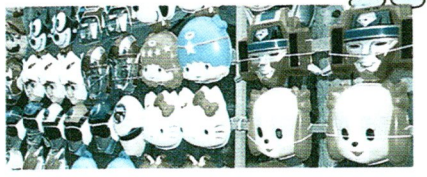
1 Trace triangle A on a thin sheet of paper and lay it over figures B and D. Then, flip the paper and try again.



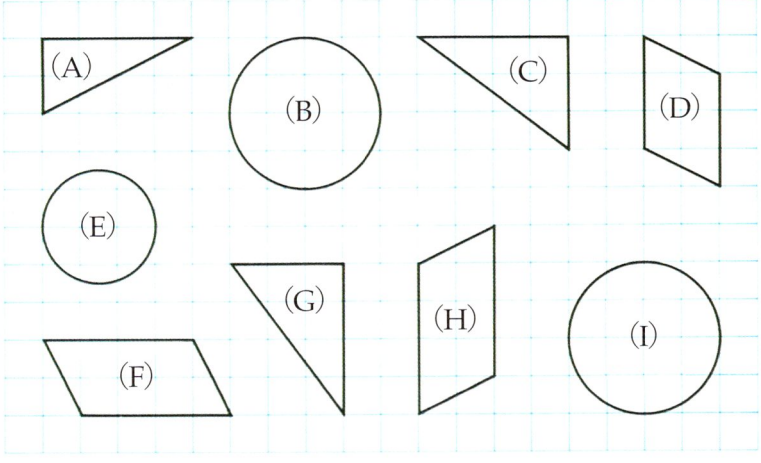
When two shapes can fit on top of each other perfectly, we say they are **congruent**.



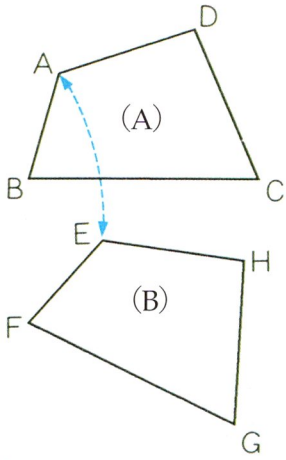
1 Let's find two shapes that are congruent around you!



2 Let's find the congruent shapes!



2 Two quadrilaterals, (A) and (B), are congruent. Let's investigate the vertices, sides, and angles that fit on top of each other!



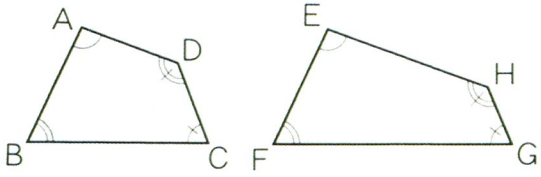
1 With which vertex of quadrilateral (B) does each vertex of quadrilateral (A) match up?

On congruent figures, the vertices, sides, and angles which fit on top of each other are called **corresponding** vertices, corresponding sides, and corresponding angles.

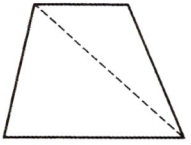
2 On quadrilaterals (A) and (B), please name the sides that correspond to sides BC and GH. Also name the corresponding angles of B and H.

! In congruent shapes, the lengths of the corresponding sides are equal. Also, the measures of the corresponding angles are the same.

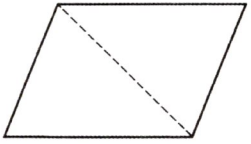
1 On the two quadrilaterals below, angles A and E, angles B and F, angles C and G, and angles D and H are equal. Are they congruent?



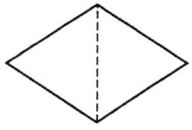
② Of the quadrilaterals below, which will have congruent triangles when divided along a diagonal line?



Trapezoid



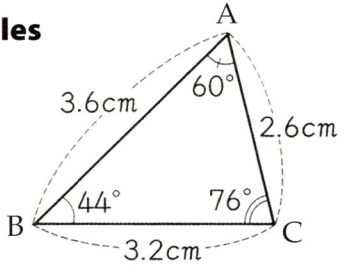
Parallelogram



Rhombus

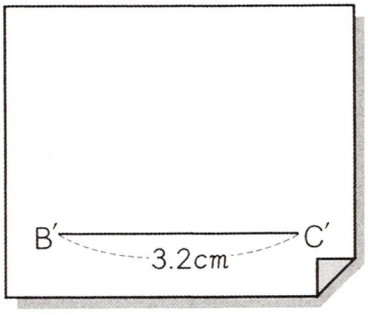
▶ How to draw congruent triangles

3 Let's think about how to draw a triangle A'B'C' that is congruent to triangle ABC on the right!





Congruent triangles have corresponding sides of the same length, as well as corresponding angles of the same size.



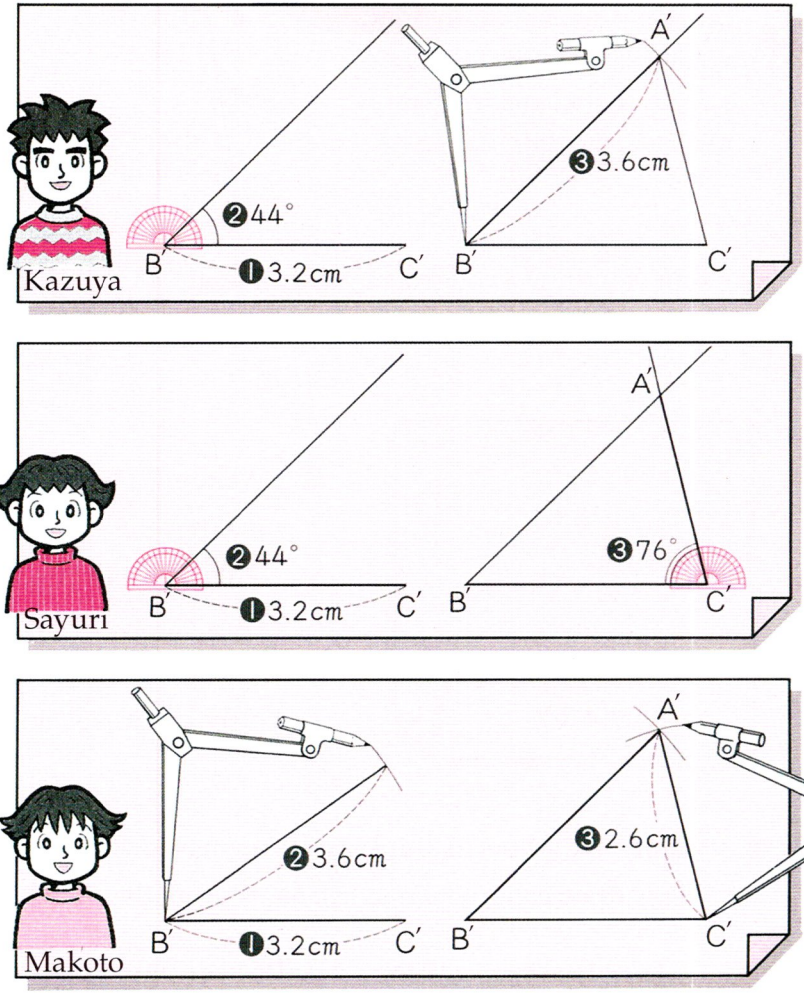
1 Draw a line segment B'C' that has the same length as side BC above. How can we determine where to place vertex A'?

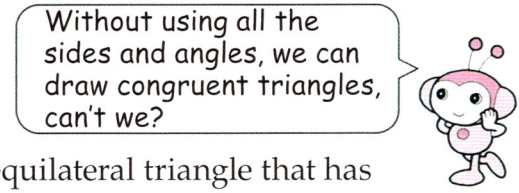


Yuji

Besides side BC, there are 2 sides and 3 angles. Which ones should we use?

2 Using the following three friends' methods, draw congruent triangles.

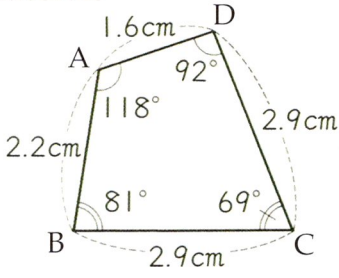




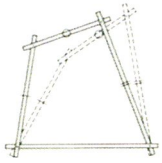
1 Please draw an equilateral triangle that has sides that are 5cm.

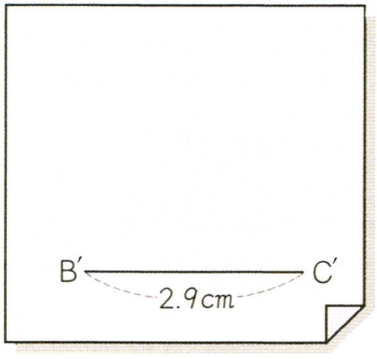
How to draw congruent quadrilaterals

4 Let's think about how to draw a quadrilateral A′B′C′D′ that is congruent to quadrilateral ABCD on the right!



1 Can you draw a congruent guadrilateral by just using the length of the 4 sides?





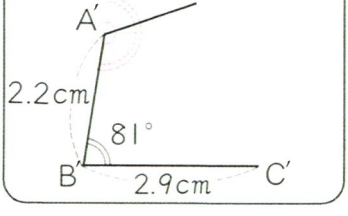
2 We drew a line B′C′ that has the same length as side BC. How can we determine where to place vertices A′ and D′?

3 Mami's and Minoru's thinking is shown below. Please draw congruent quadrilaterals using their methods.

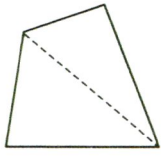


Mami

Use the angles and the lengths of the sides . . .

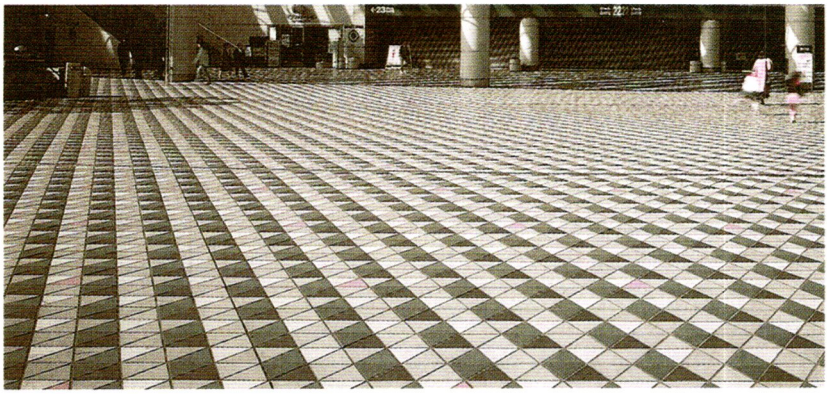


Divide it into 2 triangles and . . .





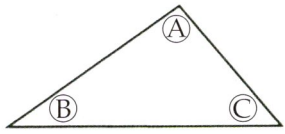
Minoru



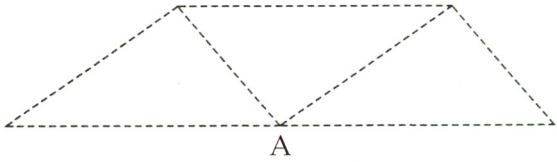
2 Angles of Triangles and Quadrilaterals

The 3 angles of a triangle

1 Let's investigate the 3 angles of a triangle!

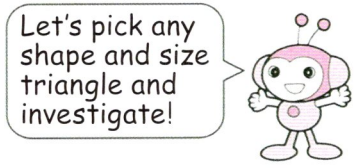


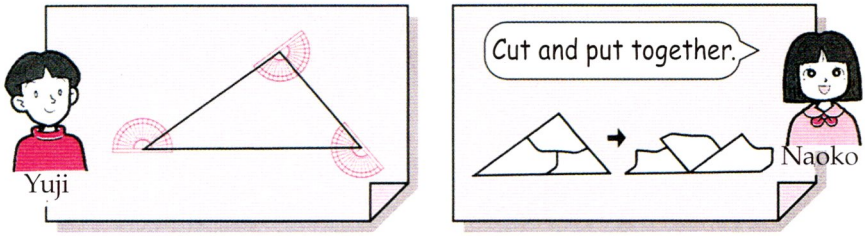
1 Cut out the three congruent triangles on page 94 and place them as below.
Which angles would meet at point A?



2 What is the sum of the angles of those 3 angles?

3 Draw a triangle on paper and find the sum of the angles.



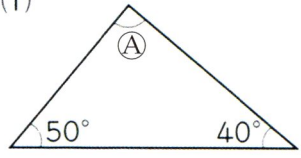


4 Using these two friends' methods, find the sum of the 3 angles of a triangle.

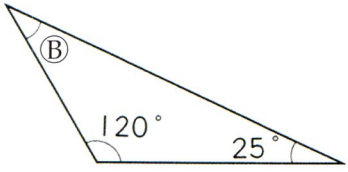
The sum of the 3 angles of a triangle is 180°.

1 What is the size of angles Ⓐ and Ⓑ? Find them by calculation.

(1)

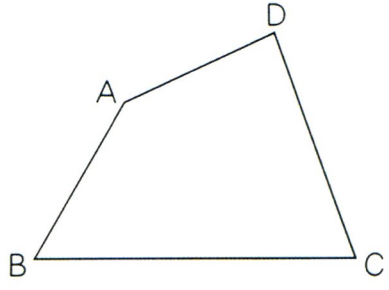


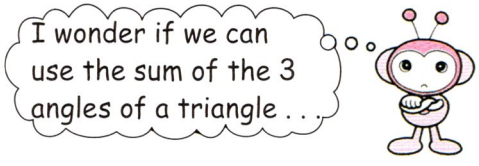
(2)



▶ The 4 angles of a quadrilateral

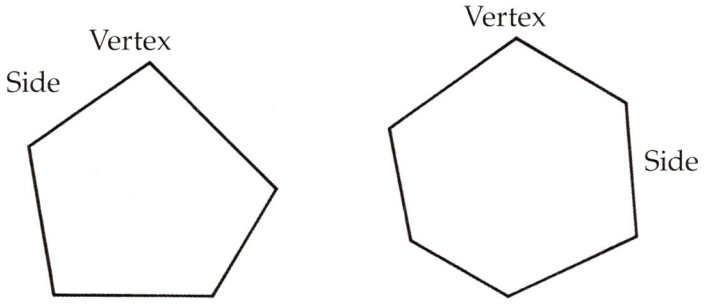
2 Let's investigate the sum of the 4 angles of a quadrilateral!





The sum of the 4 angles of a quadrilateral is 360° .

3 Let's find the number of sides, the number of vertices, and the sum of the angles of the following figures!

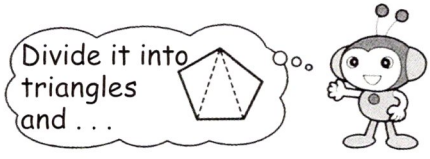


A shape surrounded by 5 straight lines is called a **pentagon** and a shape surrounded by 6 straight lines is called a **hexagon**.

Shapes that are surrounded by only straight lines such as triangles, quadrilaterals, pentagons, and hexagons are called **polygons**.

1 Find the number of sides and vertices of a pentagon. Then do the same for a hexagon.

2 Investigate the sum of the angles of a pentagon and a hexagon.

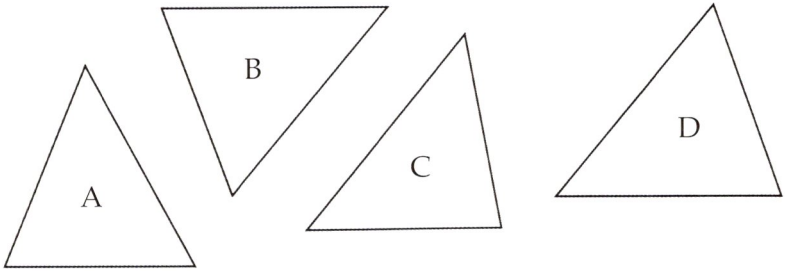


3 Organize the results you found above in a table.

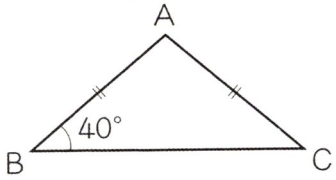
	Triangle	Quadrilateral	Pentagon	Hexagon
Number of Triangles	1			
Sum of Angles	180°			

Practice

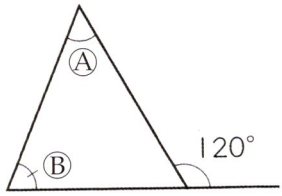
① Which shapes are congruent to each other? Use a compass and a protractor to find out.



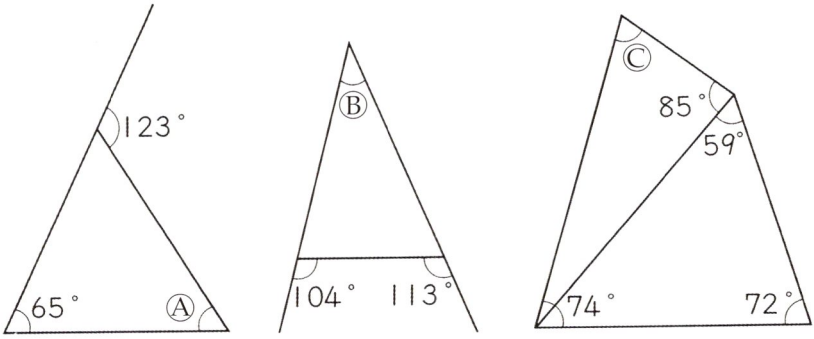
② Triangle ABC on the right is an isosceles triangle. What is the measure of angle A?



③ What is the sum of angles Ⓐ and Ⓑ?



④ Find the angles Ⓐ, Ⓑ, and Ⓒ by calculation.



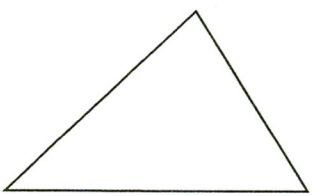
Check

1 Fill in the ☐ with the appropriate numbers and words.

(1) For congruent shapes, the lengths of ☐ sides are ☐. The size of the corresponding ☐ are also ☐.

(2) The sum of the 3 angles of a triangle is ☐°.

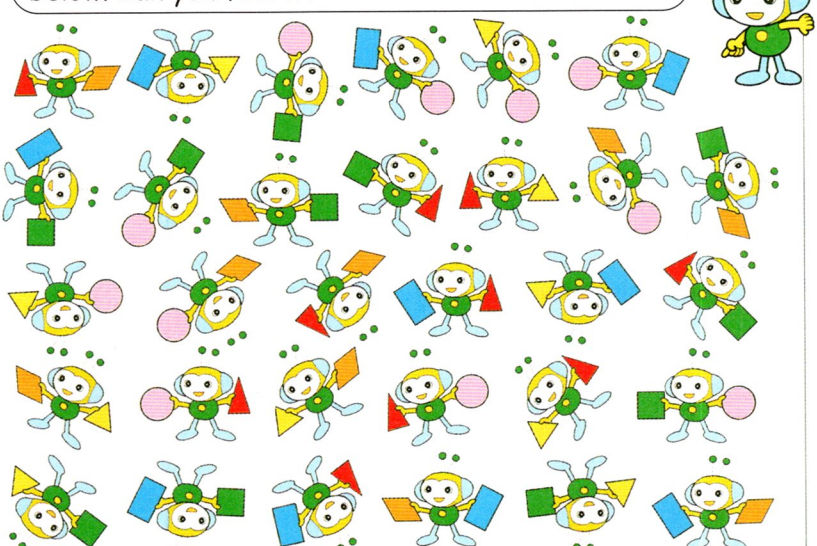
2 Please draw a triangle that is congruent to the triangle on the right by measuring the lengths of the sides and the angles.



Challenge

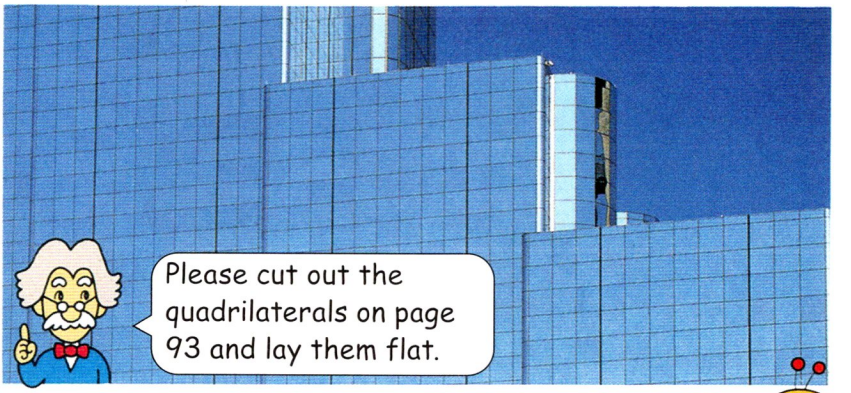
There is a pair of figures with the same shape below. Can you find it?



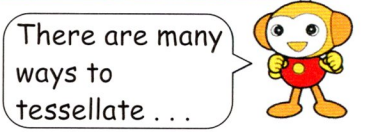


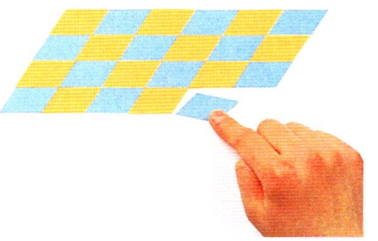
Math Plaza

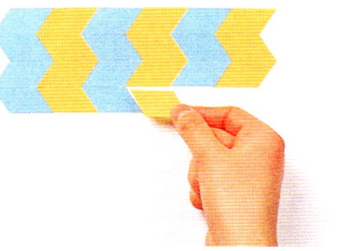
Tiling



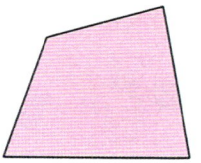
1 **Using the congruent parallelograms, let's make different tiling patterns with no overlap or gaps!**

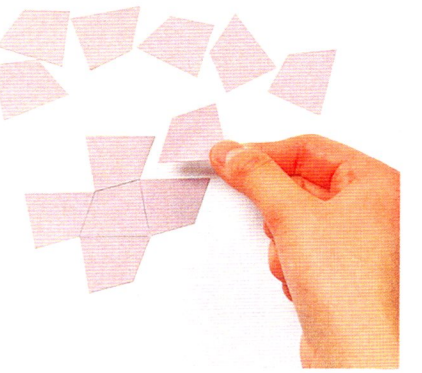




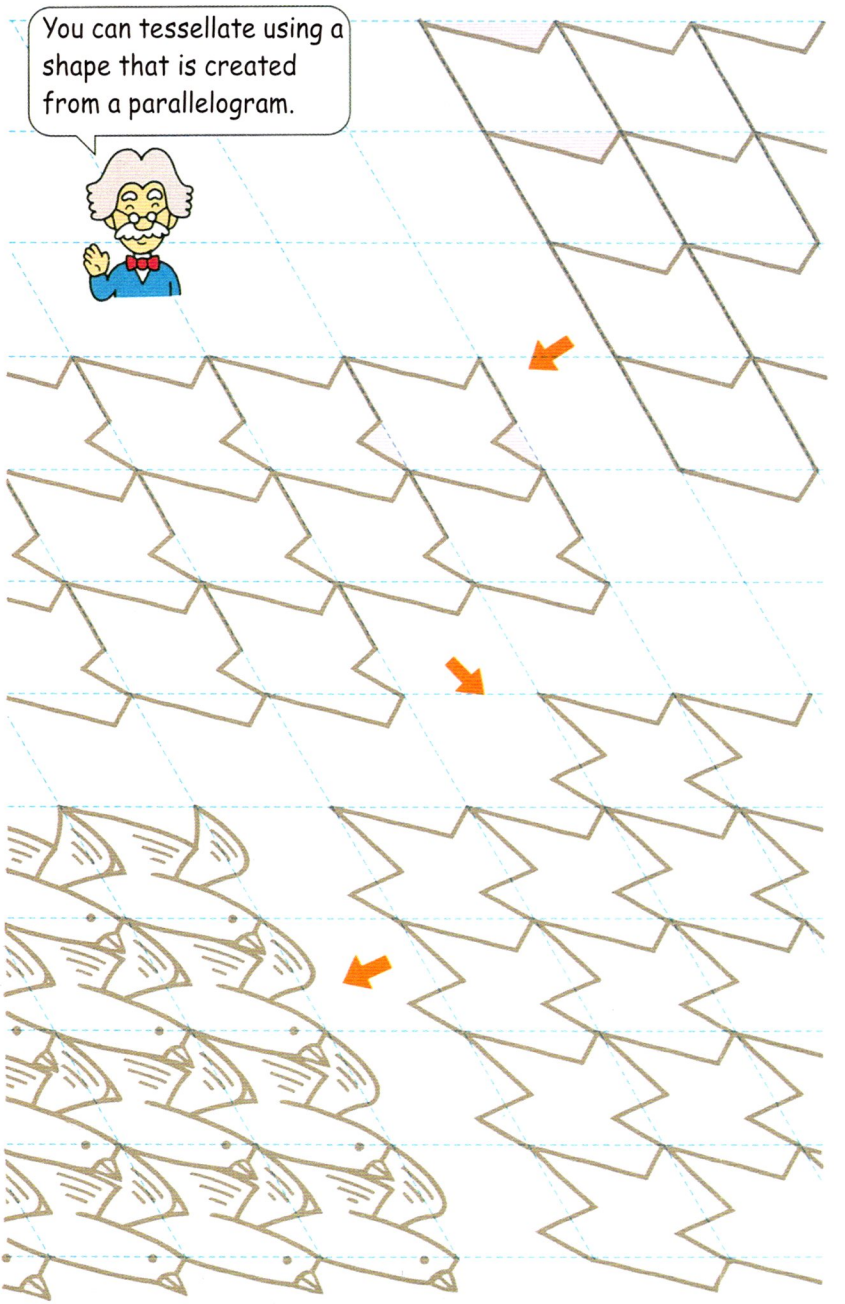


2 **Let's investigate whether we can make a tiling pattern with no overlap or gaps using the quadrilateral below!**



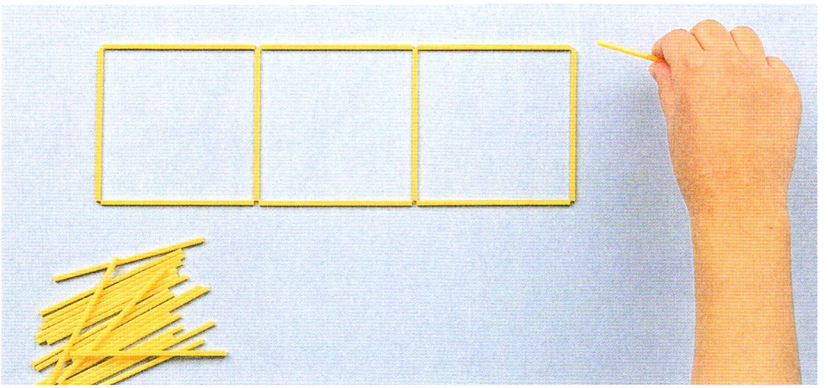


3 **Let's create your own fun shape that can be used to make tiling patterns!**

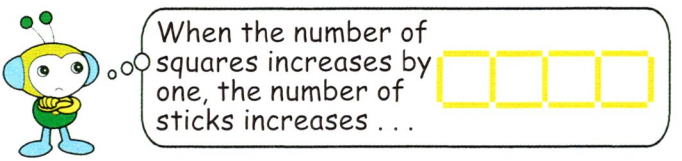


Problem Solving

Making Shapes with Sticks



1 **We are making squares using sticks that are the same length. If we build them as shown on the right, how many squares can we make with 31 sticks?**



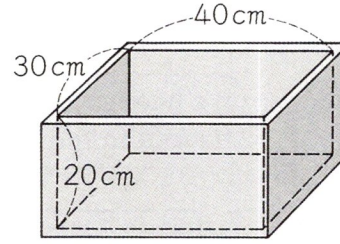
Number of squares	1	2	3	4	5	6	7
Number of sticks	4	7					

① We are making equilateral triangles using sticks that are the same length. If we build them as shown on the right, how many equilateral triangles can we make with 21 sticks?

How many sticks do we need to make 15 equilateral triangles?

Review · 3

① There is a container that has inside measures of 30cm length, 40cm width, and 20cm depth.
What is the capacity of the container in liters?



② Please do the following calculations.

(1) 7.4×5.3 (2) 9.8×4.5 (3) 6.5×0.8

(4) 1.82×5.4 (5) 3.25×2.4 (6) 8.35×2.02

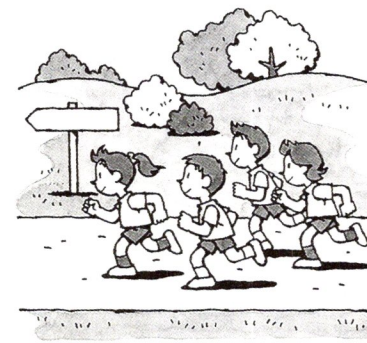
③ Please divide completely.

(1) $108.1 \div 4.7$ (2) $32.64 \div 9.6$ (3) $32.4 \div 0.9$

(4) $6.84 \div 5.7$ (5) $18.9 \div 0.45$ (6) $0.204 \div 8.5$

(7) $9.62 \div 2.6$ (8) $4.176 \div 7.2$ (9) $36.85 \div 1.34$

④ There are three different marathon courses, A, B, and C.
The distance of course B is 2.8km. The distance of course A is 1.2 times that of course B. The distance of course C is 0.8 times that of course B.



(1) Please find the distances of course A and course C.

(2) How many times is the distance of course A compared to the distance of course C?

6 Area of Quadrilaterals and Triangles



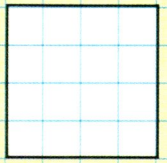
We have studied many different figures so far.

1 cm

1 cm



(Rectangle)



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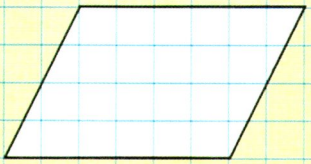

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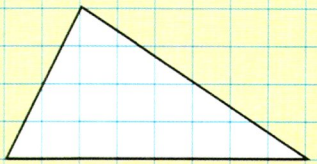

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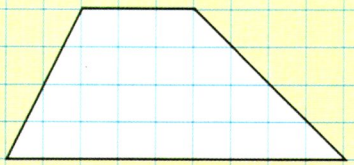

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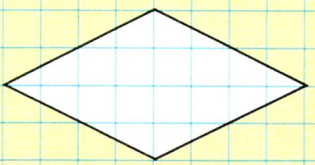

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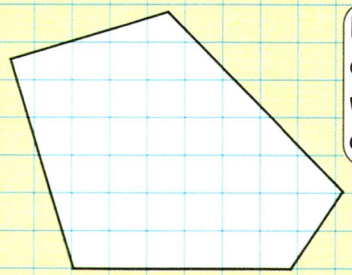
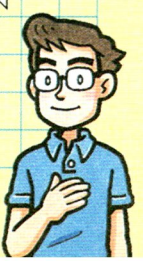


( )

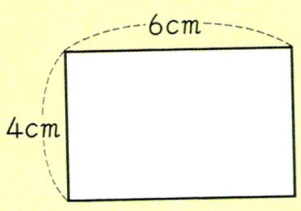
For which ones can we find the area?

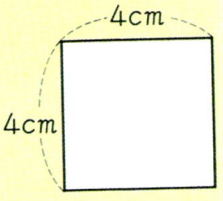






We have studied the area of rectangles and squares.





Math Sentence

Answer: $\square$ cm^2

Math Sentence

Answer: $\square$ cm^2

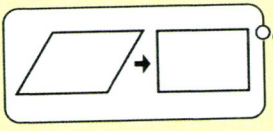


Can you find the area of other figures? What do you think?



Ritsuko

I think the shape of a parallelogram is closest to the shape of a rectangle.



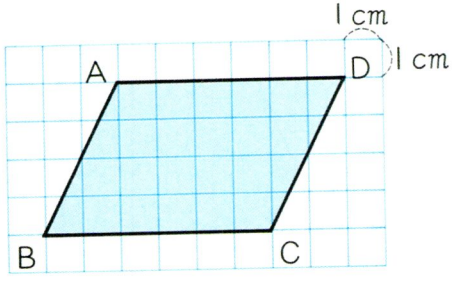


Yuji

Let's think about how to find the area of figures such as parallelograms and triangles!

1 Area of Parallelograms

1 Let's think about how to find the area of the following parallelogram!

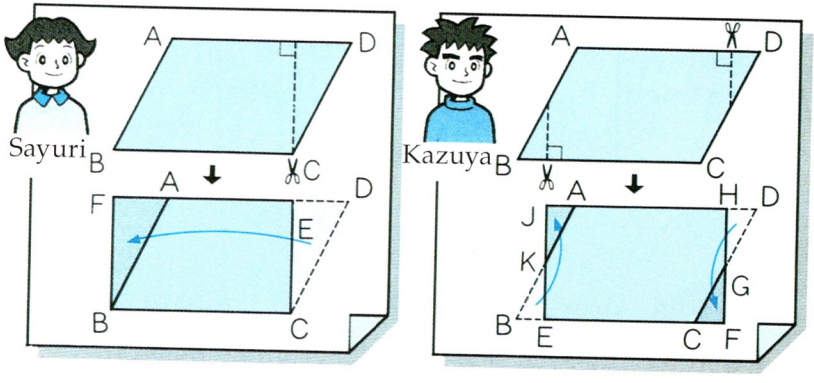


How can we change the shape into a rectangle?

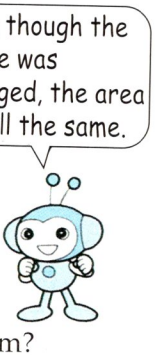


Sayuri and her friend thought about it in the following ways:

1 Let's describe how they found the area!



The area of a parallelogram can be found by changing its shape into a rectangle.

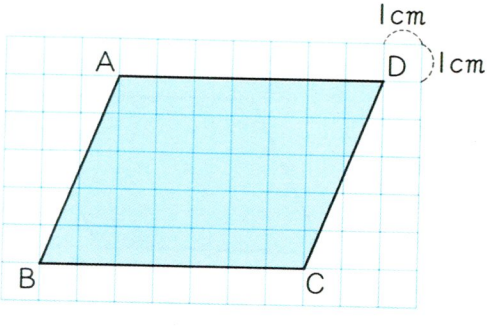


2 What is the length and width in cm of a rectangle with the same area as this parallelogram?

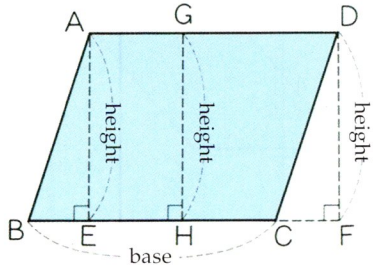
3 How many cm^2 is the area of this parallelogram?

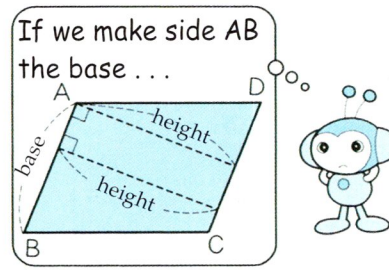
▶ The formula for finding the area of a parallelogram

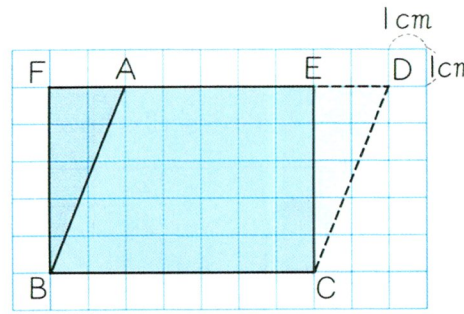
2 Let's use Sayuri's method to think about a formula for finding the area of a parallelogram!



If we make side BC the **base**, a straight line that is perpendicular to the base, such as line AE, is called the **height**.







1 What part of the length of the parallelogram ABCD is equal to the length and the width of rectangle FBCE?

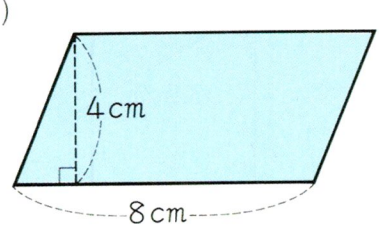
2 Let's find the area of parallelogram ABCD by calculation!

The area of a parallelogram can be found using the following formula.

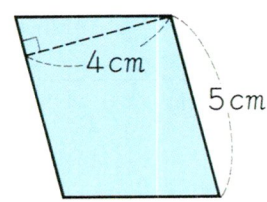
Area of Parallelogram = Base × Height

1 Let's find the area of the following parallelograms!

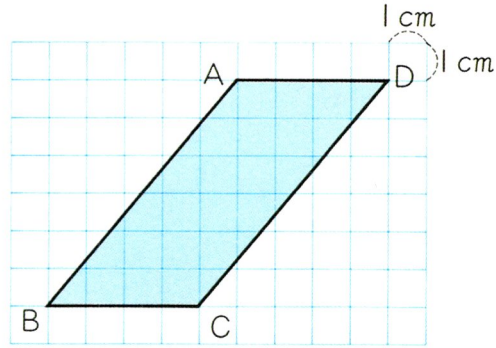
(1)

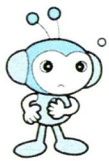


(2)



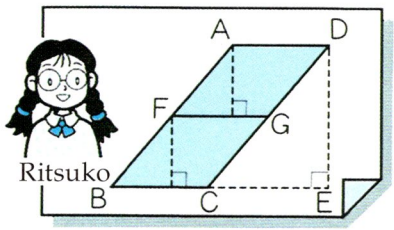
3 Let's think about how to find the area of the parallelogram on the right if we make side BC the base!

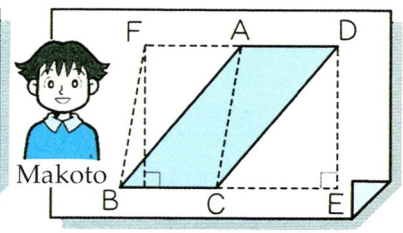




The height is located outside of the base, but I wonder if we can find the area in the same way.

1 Let's describe how they found the area!

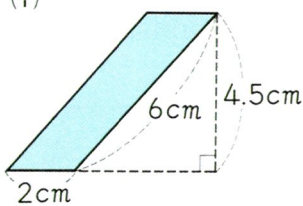




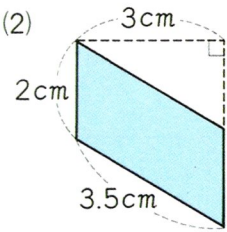
2 What is the area of parallelogram ABCD in cm^2?

1 Let's find the area of the following parallelograms!

(1)

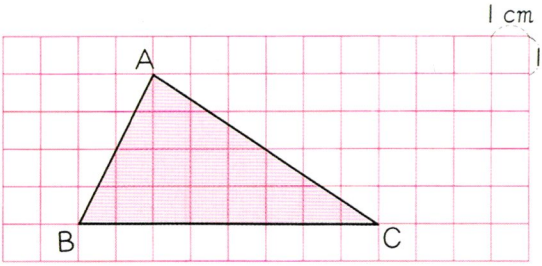


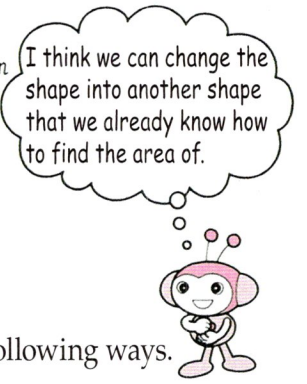
(2)



2 Area of Triangles

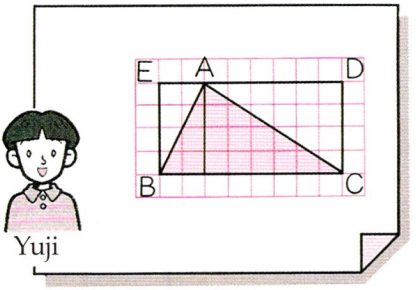
1 Let's think about how to find the area of the triangle below!

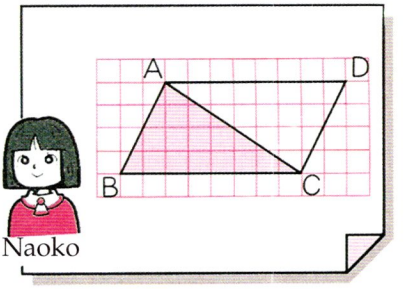


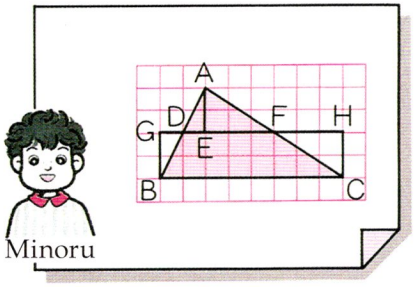


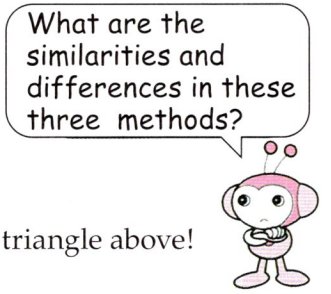
Yuji and his friends thought about it in the following ways.

1 Let's describe their methods!





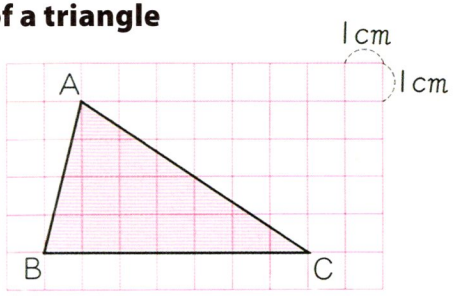




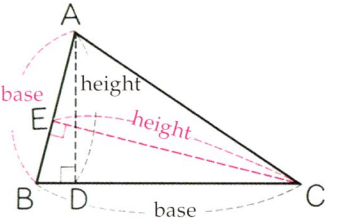
2 Let's calculate the area of the triangle above!

The formula for the area of a triangle

2 Let's think about a formula for the area of a triangle by using Naoko's idea!



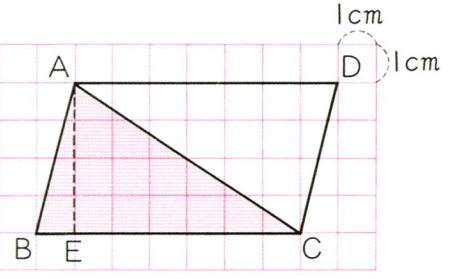
A straight line that is perpendiculor to the base is called the **height**.





If we make side BC the base, the height is line AD.
If we make side AB the base, then the height is . . . .

1 What part of triangle ABC is the same as the length of the base and the height of parallelogram ABCD?



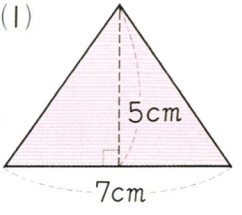
2 Let's calculate the area of triangle ABC!

The area of a triangle can be found by using the following formula.

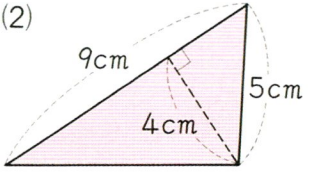
Area of Triangle = Base × Height ÷ 2

1 What are the areas of these triangles in cm^2?

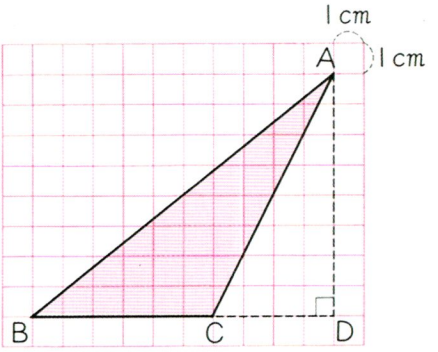
(1)

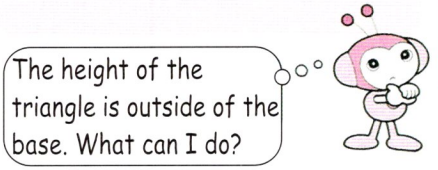


(2)

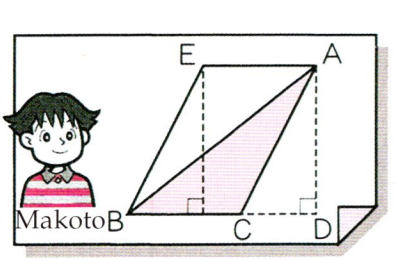


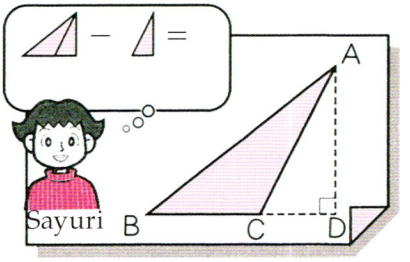
3 Let's think about how to find the area of the triangle on the right when you make side BC the base!





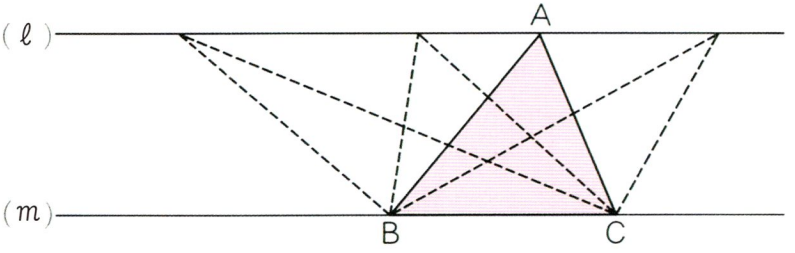
1 Please describe these two students' methods.





2 How many cm^2 is the area of triangle ABC?

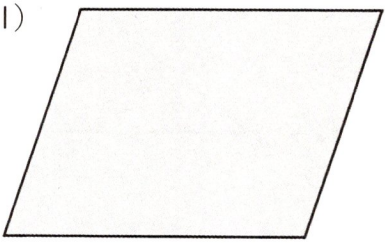
1 In the following diagram, two straight lines ℓ and m are parallel lines. The area of triangle ABC is always the same no matter where vertex A is located as long as it is on line ℓ. Please describe the reason why the area of these triangles is always the same.



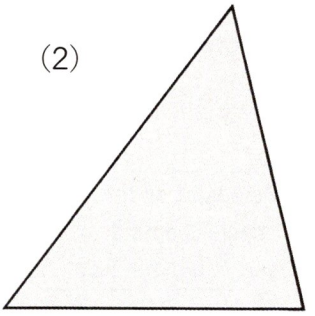
Practice 1

① Please measure the necessary parts of the following figures and find their areas.

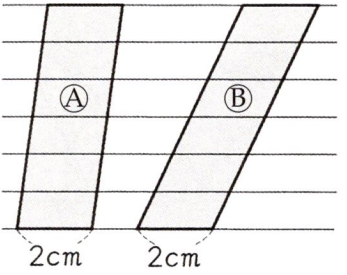
(1)



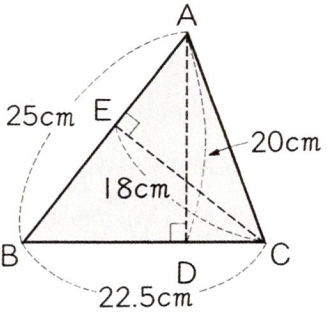
(2)



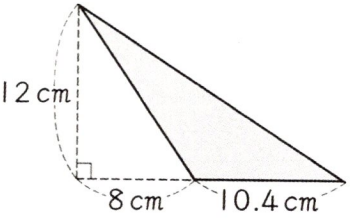
② The parallel lines are equally spaced and the distance between the lines is 1cm. Please compare the areas of these two parallelograms, Ⓐ and Ⓑ.



③ This diagram for triangle ABC contains 2 sets of base and height. Please describe each set of base and height.
Then please find the area of the triangle.

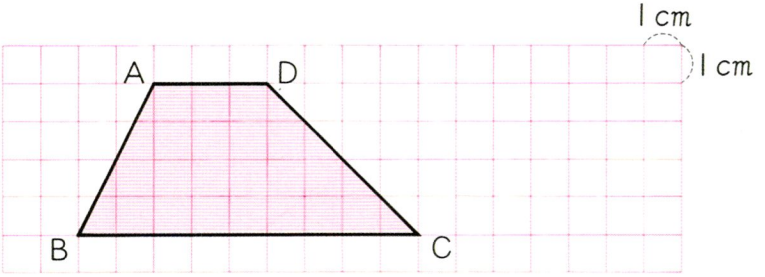


④ Please find the area of the triangle on the right.



3 Area of Various Quadrilaterals

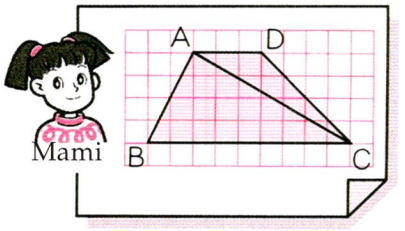
1 Let's think about the area of the following trapezoid!

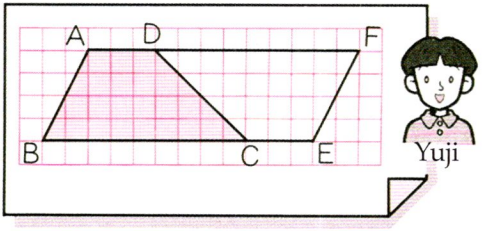


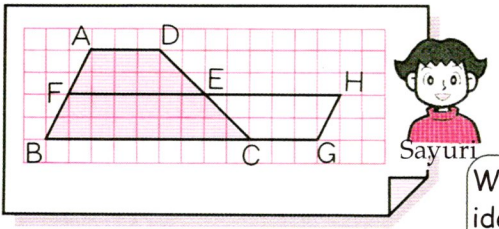
We can change the shape into other shapes that we already know how to find the areas of.

Mami and her friends thought about it in the following ways.

1 Let's describe these three friends' ideas!





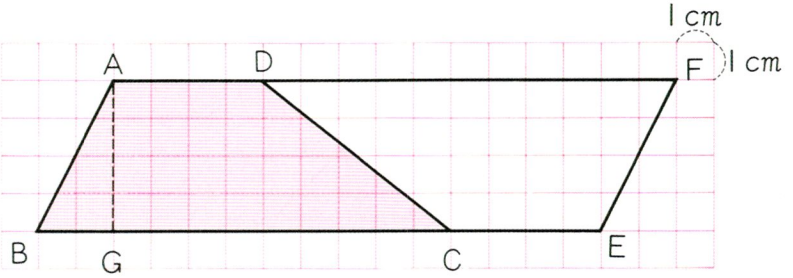


What part of their ideas are similar or different?

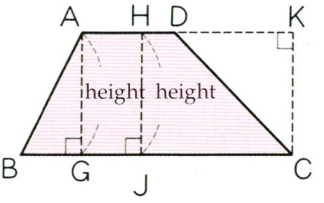
2 Let's find the area of the trapezoid above by calculation!

The formula for the area of a trapezoid

2 Let's think about a formula for the area of a trapezoid by using Yuji's idea!



In trapezoid ABCD on the right, sides AD and BC are parallel. AD is called the **top base** and the BC is called the **bottom base**. Straight lines that are perpendicular to the top and bottom bases, such as line AG, are called the height.



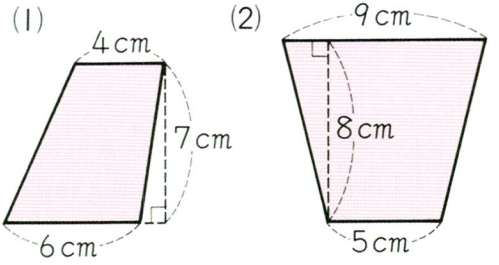
1 The base BE of parallelogram ABEF is equal to the sum of which sides of trapezoid ABCD?

2 Let's find the area of trapezoid ABCD above by calculation!

The area of a trapezoid can be found by using the following formula.

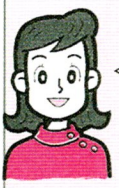
Area of Trapezoid = (**Top Base** + **Bottom Base**) × **Height** ÷ 2

1 How many cm^2 are the areas of these trapezoids on the right?

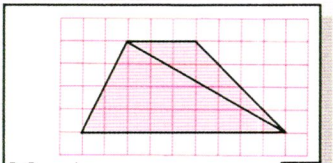


Story of Math

Methods for Finding the Area and Their Formula.

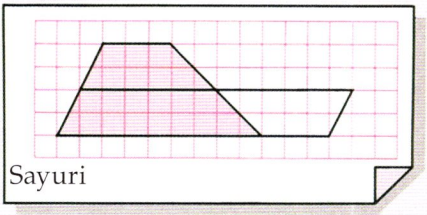


Can you develop formulas using Mami's and Sayuri's ideas?



Mami

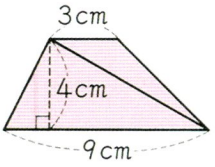
Let's develop the formula for the area of a trapezoid from Mami's and Sayuri's ideas!



Sayuri



Mami

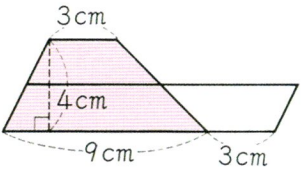


$3\times4\div2+9\times4\div2$

$3\times4\div2+9\times4\div2$

↓

$(3+9)\times4\div2$



$(9+3)\times(4\div2)$

↓

$(3+9)\times4\div2$



Sayuri

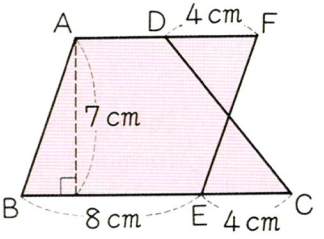
(Top Base+Bottom Base)×Height÷2

Can we construct the formula for the area of a triangle with Yuji's and Minoru's ideas on page 73?

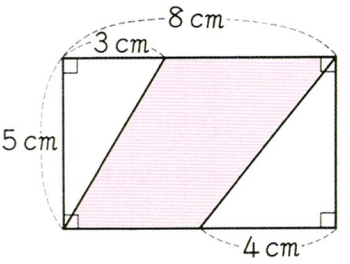


Practice 2

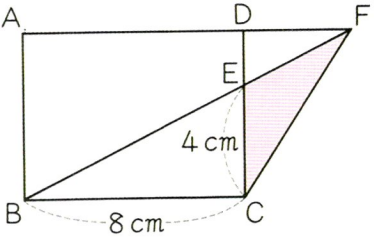
① Please compare the areas of parallelogram ABEF and trapezoid ABCD.



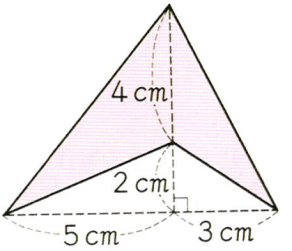
② Please find the area of the shaded part.



③ The area of rectangle ABCD is 48 cm^2. If line EC is 4cm, what is the area of triangle ECF in cm^2?



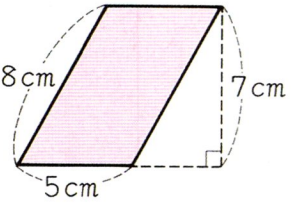
④ Please find the area of the shaded part in various ways.



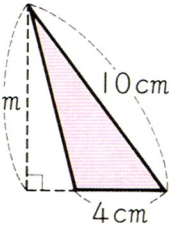
Check

1 Please find the area of the parallelogram, triangle, and trapezoid below.

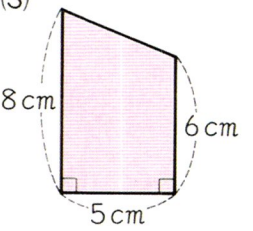
(1)



(2)



(3)

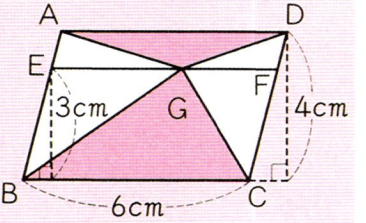


Challenge

Moving Area



In parallelogram ABCD, lines EF and AD are parallel. Please find the area of the shaded part.



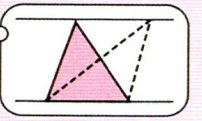


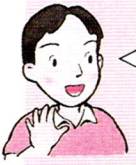
First, find the area of the triangle . . .

Hiroshi

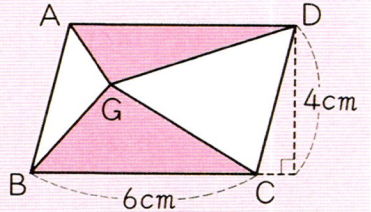


Emiko





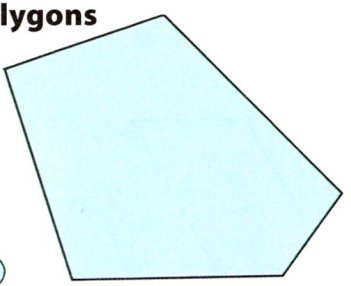
Then, write a point G anywhere in ABCD. Now, can you find the colored area?

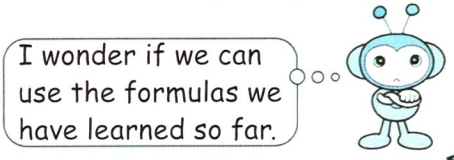


4 Ideas for Finding the Area

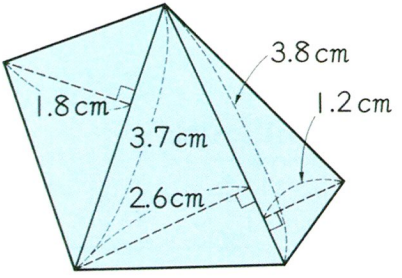
How to find the area of polygons

1 Let's think about ideas for finding the area of a pentagon!



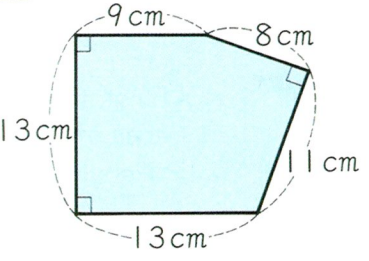


1 Let's make triangles using diagonal lines and find the area of a pentagon!

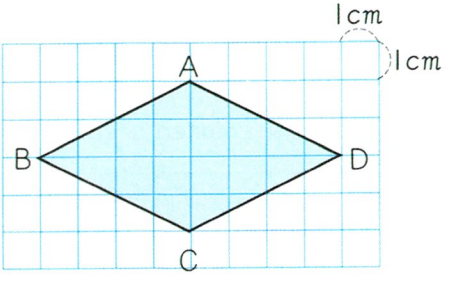




1 Let's find the area of the shaded part!

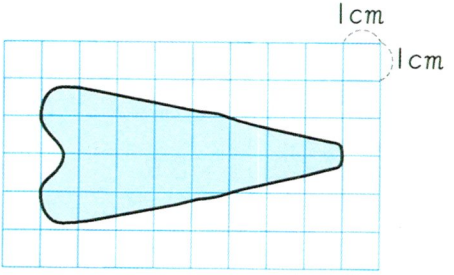


2 Let's think about how to find the area of the rhombus! What is the area?



Estimating area

2 There is a leaf shaped like the figure on the right. Let's think about how to estimate the area of this leaf!

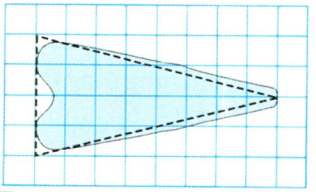


1 Please estimate the area of the leaf using Minoru's and Naoko's methods.



Minoru

Think about a shape that is close to the shape of the leaf.



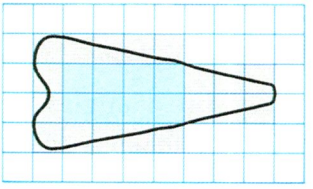
The shape looks like a ☐.

☐ × ☐ ÷ 2 = ☐ Answer: About ☐ cm^2



Naoko

I counted squares of the graph paper.



❶ The number of squares which are completely inside of the leaf ····· ☐ squares

❷ The number of squares which are not completely inside ····· ☐ squares

Think about squares that are not completely inside as half squares.

☐ + ☐ ÷ 2 = ☐ Answer: About ☐ cm^2

Review・4

① Please do the following calculations. For the division calculations divide completely.

(1) 6.7×5.8 (2) 0.62×3.5 (3) 2.8×0.76

(4) 0.55×8.9 (5) 13.7×0.48 (6) 7.16×2.14

(7) $25.2 \div 0.6$ (8) $663 \div 8.5$ (9) $655.5 \div 1.9$

(10) $16.2 \div 0.25$ (11) $3.22 \div 4.6$ (12) $59.52 \div 1.28$

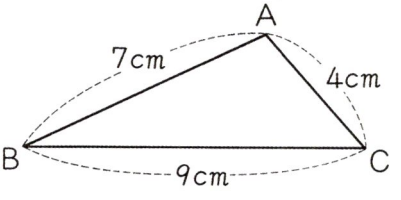
② Please find the volume of the following.

(1) A cube with $9cm$ edges

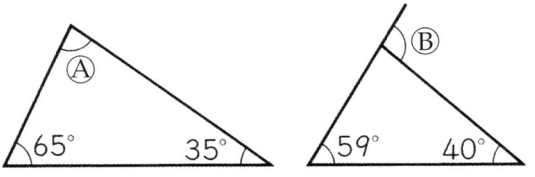
(2) A rectangular prism with $8cm$ length, $5cm$ width, and $7cm$ height

(3) A rectangular prism with $3.5m$ length, $2.8m$ width, and $4m$ height

③ Please draw a congruent triangle to triangle ABC on the right.



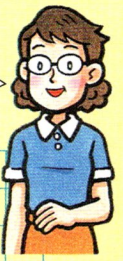
④ Please find the measure of angles Ⓐ and Ⓑ by calculation.



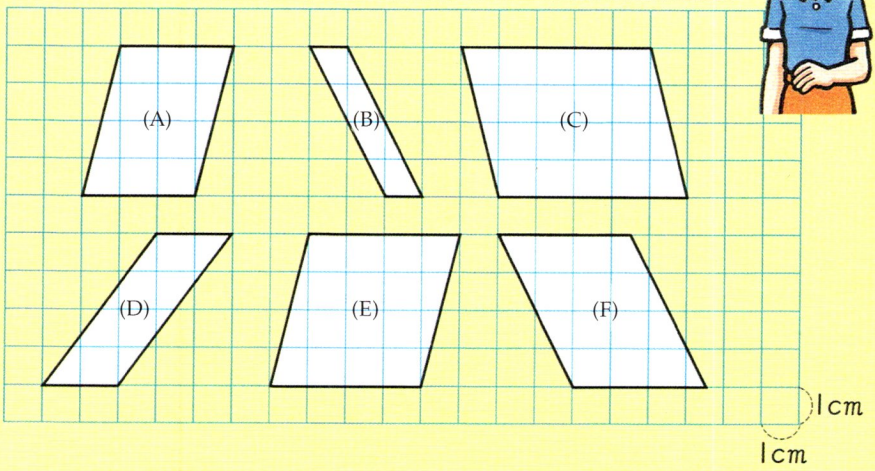
⑤ The distance from Town A to Town B is $58.2km$. The distance from Town A to Town C is $203.7km$. How many times is the distance from Town A to Town C as the distance from Town A to Town B?



7 Variables and Mathematical Equations



There are many parallelograms which have a height of 4cm. Find math sentences to find the area of these shapes.





Makoto

(A) $3 \times 4 = 12$ (cm^2)

(B) $1 \times 4 = 4$ (cm^2)

(C) $5 \times 4 = 20$ (cm^2)

(D) $2 \times 4 = 8$ (cm^2)

I wonder if we can show it in one math sentence . . .



Minoru

Let's think about how to write a mathematical equation!

▶ Mathematical equations using variables

1 Let's discuss how to write a mathematical equation that shows the area of many different kinds of parallelograms that have heights of 4cm!

1 Please write a math sentence for the area using the length of the base as □cm and the area as ○cm^2.

$\square \times 4 = \bigcirc$

When you write a mathematical equation, you may use x and y instead of symbols like □ and ○, and write it in the following way.

$\square \times 4 = \bigcirc \quad \rightarrow \quad x \times 4 = y$

2 Using the mathematical equation $x \times 4 = y$, please find y when x is 10, 15, 20, and 4.5.

3 In the mathematical equation $x \times 4 = y$, for what value of x do you have $x \times 4 = 52$?



The area of a parallelogram with a height of 4 cm can be written in one mathematical equation, $x \times 4 = y$.

(1) Besides using the mathematical equation $x \times 4 = y$ to find the area of a parallelogram with a height of 4cm, what other relationships can you show with this equation?

The same volume of juice . . .



Yuji

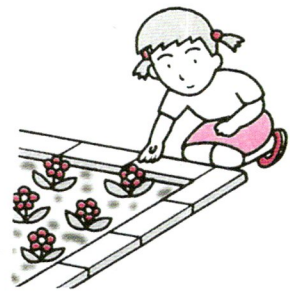
(2) $x d\ell$ of juice is shared equally by 4 people. If $y d\ell$ is the amount each person gets, find a mathematical equation that shows the relationship between x and y.

Let's use x and y for many different problem solving situations in the future!



Problems involving finding the value of x

2 The area of the 5th graders' flower garden is $4.4m^2$. The area is 0.8 times the area of the 6th graders' flower garden.
Let's find the area of the 6th graders' flower garden!



1 Make the area of the 6th graders' flower garden xm^2 and write a mathematical equation with multiplication.

2 Please find the value of x.

$$x \times 0.8 = 4.4$$

$$x = 4.4 \div 0.8$$

$$x = \square$$

Answer: $\square m^2$

3 Put the answer above into the mathematical equation $x \times 0.8 = 4.4$ and check the answer.

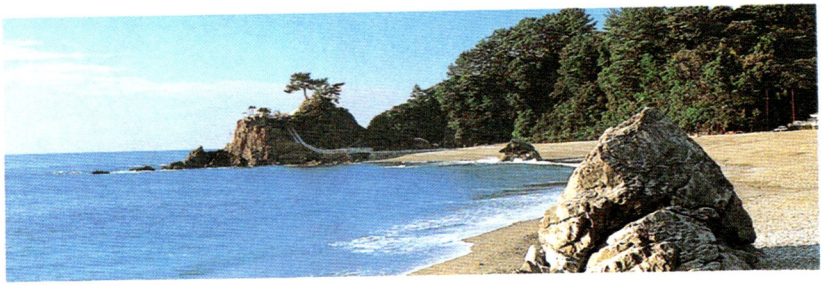
1 Please find the values of x in the following mathematical equations.

(1) $76 + x = 96$ (2) $25 \times x = 200$ (3) $x \div 3 = 7$

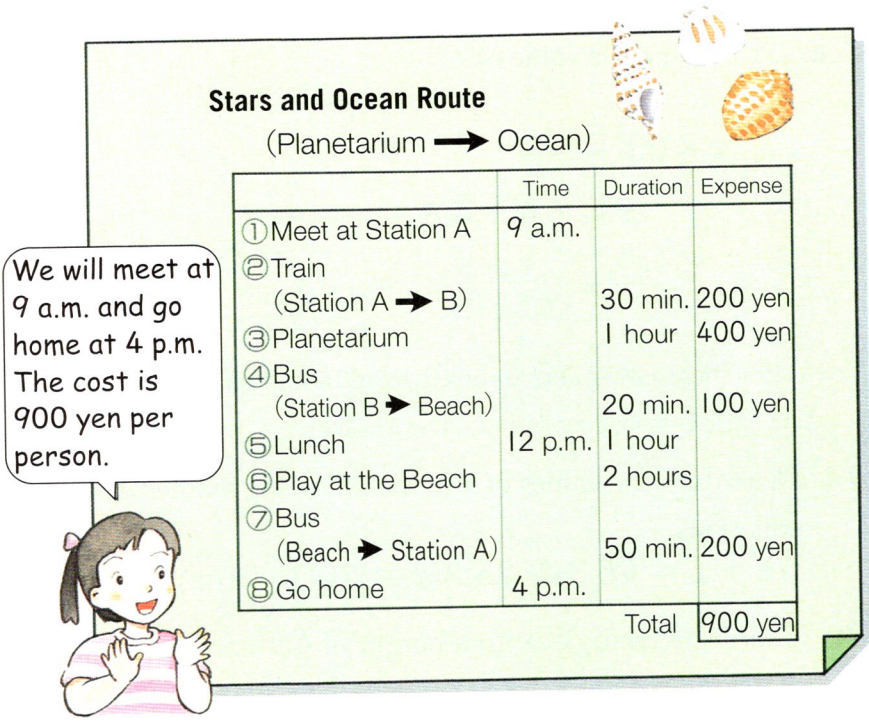
2 There is a triangle with a height of $4cm$ and an area of $16cm^2$.
Please set the length of the base as xcm, and construct a mathematical equation. Then find the length of the base.

Life and Math

Let's Make a Plan!



1 **Emiko and her friends are planning a trip for the next holiday.**



Stars and Ocean Route

(Planetarium → Ocean)

	Time	Duration	Expense
①Meet at Station A	9 a.m.		
②Train (Station A → B)		30 min.	200 yen
③Planetarium		1 hour	400 yen
④Bus (Station B → Beach)		20 min.	100 yen
⑤Lunch	12 p.m.	1 hour	
⑥Play at the Beach		2 hours	
⑦Bus (Beach → Station A)		50 min.	200 yen
⑧Go home	4 p.m.		
		Total	900 yen

Plan different routes for the trip and calculate the time and cost.


The fare that is given here is the children's price.
Station A
Bus 50 min. 200 yen
Aquarium 500 yen
Beach Front
Train 30 min. 200 yen
Bus 20 min. 100 yen
Planetarium 1 hour 400 yen
Bus 40 min. 180 yen
Bus 20 min. 120 yen
Station B
Forest Park Entrance
Train 15 min. 100 yen
Bus 30 min. 150 yen
Forest Park
Bus 30 min. 150 yen
Walking 10 min.
Station C
Vineyard Entrance
Vineyard 500 yen

Calculation Practice

① (1) 6.59×9.3 (2) 0.82×0.74 (3) 16×5.02

(4) 7.5×5.44 (5) 13.4×4.2 (6) 5.36×37.5

(7) 1.65×91 (8) 3.88×0.27 (9) 2.9×0.28

(10) 0.056×7.5 (11) 24×41.7 (12) 0.18×0.37

(13) 3.15×0.32 (14) 8.6×4.07 (15) 29.2×89

(16) 8.5×80.3 (17) 48×0.95 (18) 8.14×27.3

(19) 0.065×420 (20) 0.75×0.096 (21) 58×4.85

(22) 4.7×0.64 (23) 850×2.36 (24) 0.036×1.5

② (1) $9.62 \div 3.7$ (2) $2.709 \div 4.3$ (3) $24 \div 3.75$

(4) $6.9 \div 0.75$ (5) $46.62 \div 0.63$ (6) $9.006 \div 2.37$

(7) $72.8 \div 2.8$ (8) $27 \div 0.25$ (9) $4.028 \div 5.3$

(10) $2.4 \div 0.96$ (11) $6.96 \div 0.87$ (12) $8.652 \div 4.2$

(13) $21.28 \div 0.07$ (14) $3 \div 62.5$ (15) $73.1 \div 0.86$

(16) $7.47 \div 8.3$ (17) $5.1 \div 0.24$ (18) $44.18 \div 9.4$

①	0.812	1.008	0.6068	1.0476	27.3	80.32	40.8	
	0.42	150.15	56.28	61.287	201	222.222	0.0666	
	2006	0.072	682.55	3.008	45.6			
	0.054	2598.8	35.002	1000.8	281.3			
②	85	9.2	3.8	0.63	304	4.7	74	0.76
	2.6	0.9	26	6.4	108	2.5	8	0.048
	2.06	21.25						

Please color the answers.



③ (1) $1.6 + 0.62 \times 3.5$ (2) $8.9 - 10.35 \div 2.3$

(3) $(5.8 + 1.36) \times 2.14$ (4) $(5.57 - 1.28) \div 15.6$

(5) $10.56 \div (1.82 + 3.46)$ (6) $6.41 \times (1.08 - 0.95)$

(7) $(11.01 - 2.46) \times 0.98$ (8) $(1.908 + 0.885) \div 0.57$

(9) $52.7 \div (40.4 - 7.6 \times 4.5)$

(10) $301.4 - 0.56 \times 22.5 + 11.43$

④ Please round off the quotients to the nearest $\frac{1}{10}$.

(1) $9.45 \div 5.7$ (2) $6.71 \div 0.83$ (3) $2.6 \div 3.5$

(4) $46 \div 3.72$ (5) $2.08 \div 4.2$ (6) $6.72 \div 0.76$

(7) $1.17 \div 2.8$ (8) $7.28 \div 0.37$ (9) $27 \div 4.19$

⑤ Please find the quotients to one decimal place and show the remainders.

(1) $5.2 \div 0.84$ (2) $4.14 \div 2.2$ (3) $29.57 \div 0.73$

(4) $2.33 \div 2.7$ (5) $78 \div 8.4$ (6) $3.16 \div 6.38$

(7) $48.7 \div 3.6$ (8) $9.8 \div 0.43$ (9) $32 \div 4.75$

③	300.23	2	3.77	8.379	0.275	8.5			
	4.4	15.3224	0.8333	4.9					
④	0.7	19.7	0.5	8.8	6.4	12.4	1.7	8.1	0.4
⑤	0.8	R 0.17	1.8	R 0.18	22.7	R 0.039			
	40.5	R 0.005	6.7	R 0.175	9.2	R 0.72			
	0.4	R 0.608	6.1	R 0.076	13.5	R 0.1			

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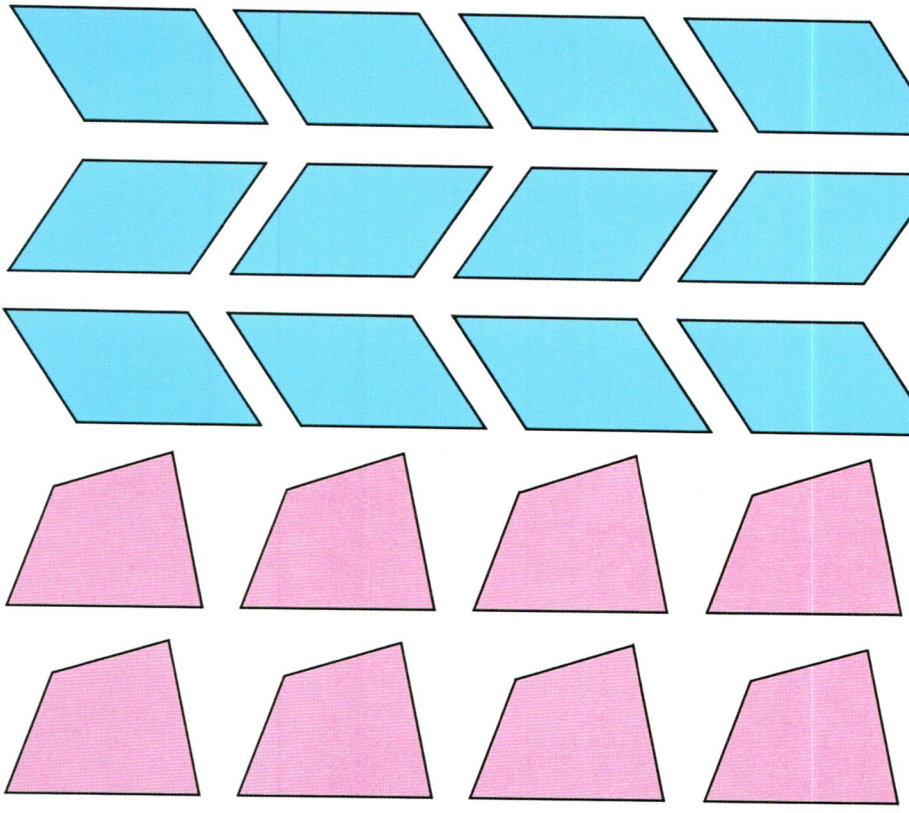

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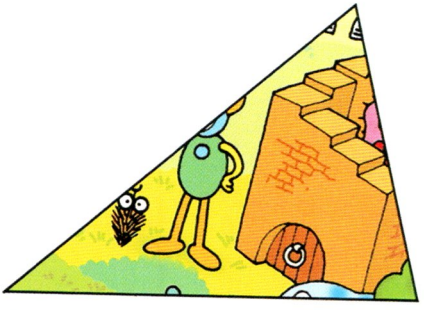

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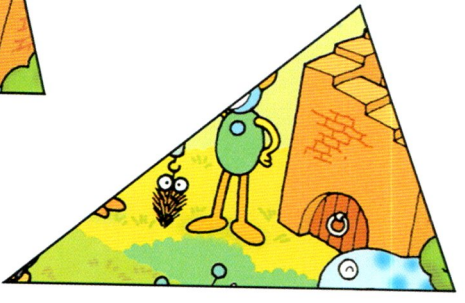

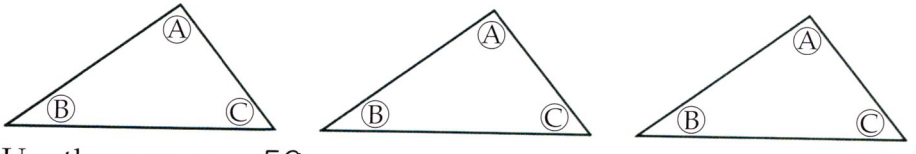

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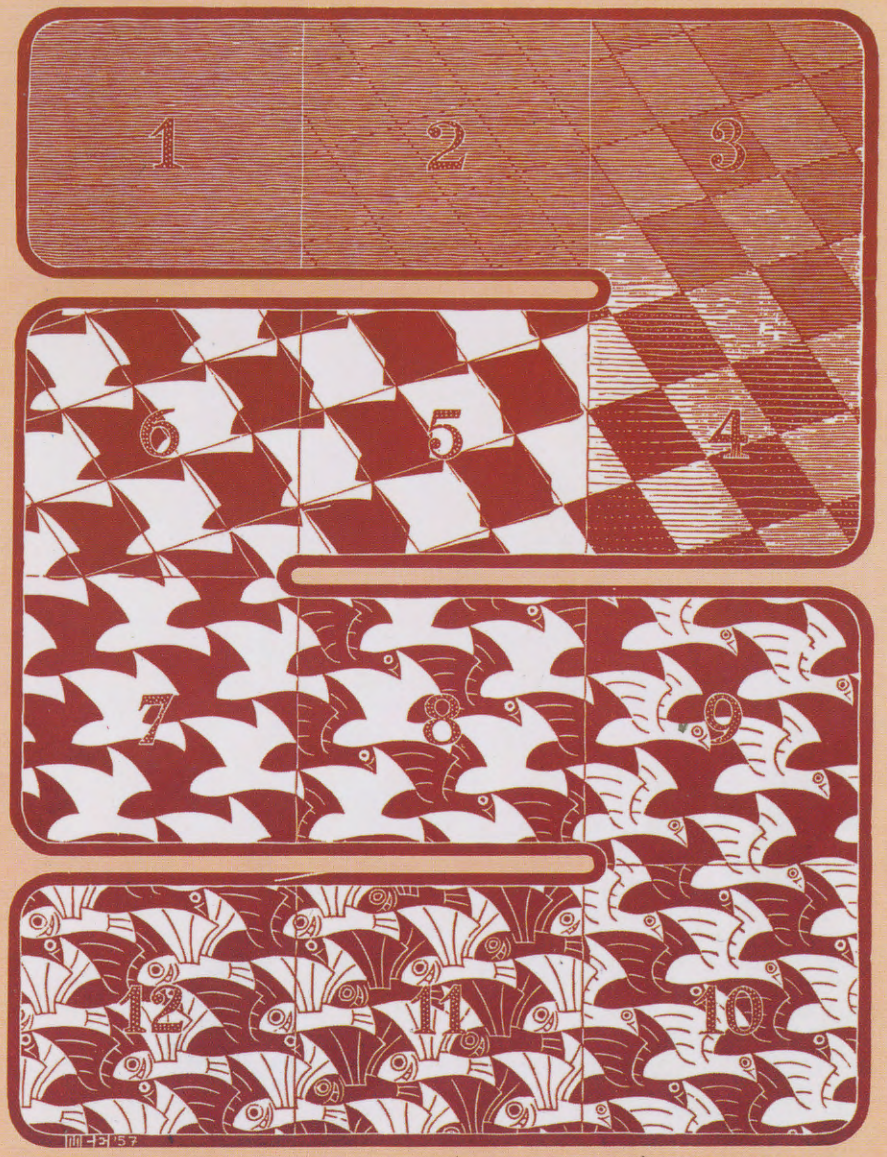

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TOKYO SHOSEKI

Mathematics 5B

for Elementary School



TOKYO SHOSEKI



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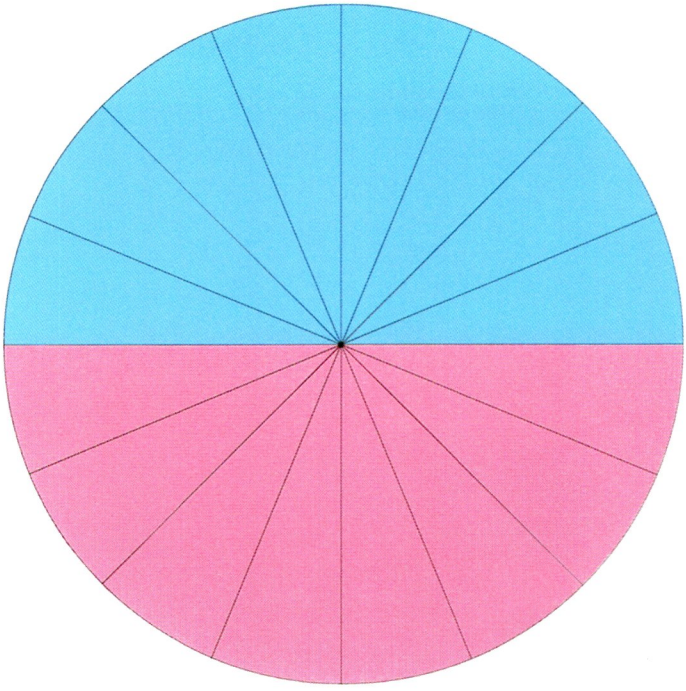

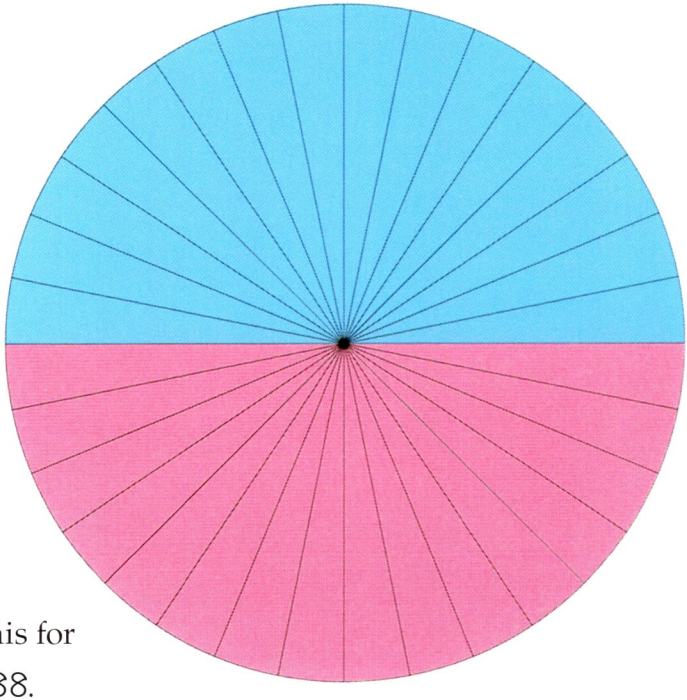

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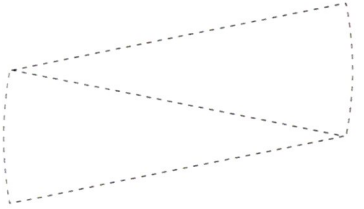



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Calculation Practice

① (1) $\frac{3}{5}+\frac{1}{3}$ (2) $\frac{1}{2}+\frac{3}{7}$ (3) $\frac{2}{7}+\frac{2}{5}$ (4) $\frac{1}{6}+\frac{1}{12}$

(5) $\frac{2}{5}+\frac{4}{15}$ (6) $\frac{1}{4}+\frac{3}{10}$ (7) $\frac{3}{4}+\frac{5}{6}$ (8) $\frac{2}{3}+\frac{4}{5}$

(9) $\frac{4}{7}+\frac{1}{2}$ (10) $\frac{7}{9}+\frac{5}{6}$ (11) $2\frac{2}{3}+1\frac{1}{6}$ (12) $3\frac{2}{5}+2\frac{1}{7}$

② (1) $\frac{3}{5}-\frac{1}{4}$ (2) $\frac{5}{7}-\frac{2}{5}$ (3) $\frac{3}{4}-\frac{1}{6}$ (4) $\frac{2}{3}-\frac{1}{6}$

(5) $\frac{7}{12}-\frac{1}{3}$ (6) $\frac{3}{4}-\frac{5}{16}$ (7) $1\frac{1}{4}-\frac{2}{3}$ (8) $1\frac{2}{5}-\frac{5}{6}$

(9) $2\frac{1}{3}-\frac{3}{4}$ (10) $2\frac{1}{7}-\frac{4}{5}$ (11) $2\frac{4}{5}-1\frac{3}{4}$ (12) $6\frac{1}{4}-2\frac{3}{8}$

③ (1) $\frac{5}{6}+\frac{5}{8}-\frac{11}{24}$ (2) $1\frac{1}{2}+2\frac{7}{10}-\frac{4}{5}$

(3) $\frac{6}{7}-\frac{3}{14}+\frac{1}{2}$ (4) $1\frac{5}{8}-\frac{1}{3}+\frac{5}{6}$

(5) $\frac{1}{3}+\left(\frac{7}{9}-\frac{5}{12}\right)$ (6) $2\frac{2}{3}-\left(\frac{4}{5}+\frac{1}{2}\right)$

①	$\frac{1}{4}$	$\frac{2}{3}$	$1\frac{7}{12}$	$5\frac{19}{35}$	$\frac{24}{35}$	$1\frac{11}{18}$
	$1\frac{1}{14}$	$\frac{14}{15}$	$3\frac{5}{6}$	$\frac{11}{20}$	$1\frac{7}{15}$	$\frac{13}{14}$
②	$\frac{11}{35}$	$3\frac{7}{8}$	$1\frac{12}{35}$	$\frac{7}{20}$	$1\frac{7}{12}$	$\frac{7}{16}$
	$\frac{7}{12}$	$1\frac{1}{20}$	$\frac{17}{30}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{7}{12}$
③	$1\frac{1}{7}$	1	$\frac{25}{36}$	$2\frac{1}{8}$	$1\frac{11}{30}$	$3\frac{2}{5}$

Please color the answers.



▶ Quantitative relationships

① Please find the value of x in the following equations.

(1) $x \div 0.4 = 3.2$ (2) $x \times 6 = 42$

② Please express the following percentages in decimals and fractions.

(1) 3 % (2) 80 % (3) 45.5 % (4) 140 % (5) 0.5 %

③ What % of the quantities in ( ) are the following quantities?

(1) 68*yen* (80*yen*) (2) 3.2 m (8 m) (3) 370 g (250 g)

④ A rectangular prism has a volume of 1008 cm^3. Its length and width are 7 cm and 8 cm, respectively.
Using x cm as its height, please write an equation to calculate its volume.
Then, find the height.

⑤ A whole class with 40 students had dental checkups. There were 15 students who did not have cavities. What is the ratio of students who have cavities to the number of students in the whole class?

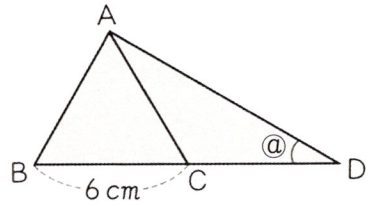
⑥ As shown in the table on the right, we make two kinds of rice and beans, A and B, by mixing rice with beans. Which one has a higher ratio of beans?

Weights of Rice and Beans

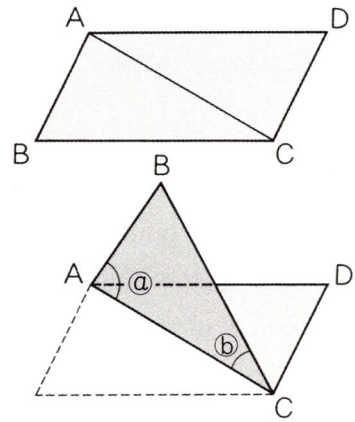
	Rice (g)	Beans (g)
A	450	108
B	580	145

▶ Geometric figures

① Triangle ABC on the right is an equilateral triangle with 6cm sides.
If side CD is the same length as side BC, what is the measure of angle ⓐ?



② A parallelogram was folded along AC as shown on the right. If angle ⓐ is 85° and angle ⓑ is 30°, what is the measure of angle A of the parallelogram?

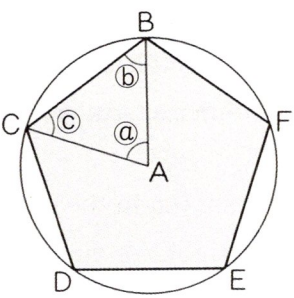


③ The pentagon BCDEF on the right is a regular pentagon.

(1) What is the measure of angle ⓐ?

(2) What are the measures of angles ⓑ and ⓒ?

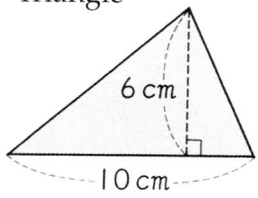
(3) What is the measure of an interior angle of a regular pentagon?



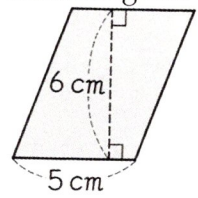
▶ Quantities and measurements

① What are the areas of the figures below?

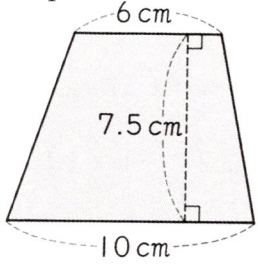
(1) Triangle



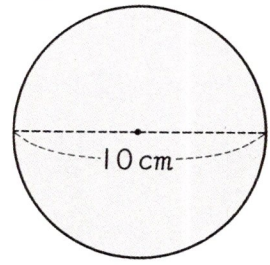
(2) Parallelogram



(3) Trapezoid

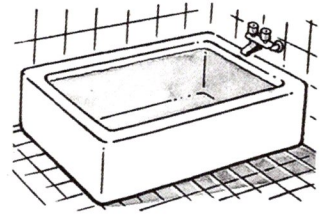


(4) Circle



② How many m is the diameter of a circle with a circumference of $21.98m$? How many m^2 is the area of the circle?

③ A bathtub is shaped like a rectangular prism with an inside measure of $1.5m$ length, $1.2m$ width, and $0.7m$ depth. It took 28 minutes to fill the bathtub. How many ℓ of water was poured in each minute?



④ How many km per hour is a train moving if it takes 17 hours to go $1229km$ from station A to station B? Round off the answer to the second highest digit.

⑧ Are the following numbers even or odd?

(1) Multiples of 2

(2) Numbers that are one more than multiples of 2

⑨ Please calculate the following.

(1) $\frac{3}{5}+\frac{2}{3}$ (2) $\frac{5}{6}+\frac{2}{21}$ (3) $1\frac{4}{9}+\frac{1}{6}$ (4) $\frac{5}{7}+3\frac{1}{3}$

(5) $\frac{7}{9}-\frac{1}{12}$ (6) $1\frac{1}{6}-\frac{3}{10}$ (7) $3\frac{5}{6}-1\frac{2}{3}+\frac{5}{9}$

⑩ Please change the fractions into decimals and the decimals into fractions.

(1) $\frac{2}{5}$ (2) $\frac{11}{8}$ (3) $1\frac{13}{50}$ (4) 0.3 (5) 1.32 (6) 3.65

⑪ What numbers are indicated by the arrows? Please express each one in decimals and fractions.



Challenge

Mysterious Characteristics



There are two numbers, 12 and 15.
(1) Find the product.
(2) Find the Least Common Multiple and the Greatest Common Factor and then find their products.
(3) Compare the two products.

❶ $12 \times 15 =$ ☐

The least common multiple — The greatest common factor

❷ ☐ × ☐ = ☐

How about other numbers?



5th Grade **Review**

▶ Numbers and calculations

① Please fill in the ☐ with the appropriate numbers.

(1) $50.84 = 10 \times \square + 1 \times \square + 0.1 \times \square + 0.01 \times \square$

(2) $6.382 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

② Please fill in the ☐ with the appropriate numbers.

(1) ☐ times 0.648 is 64.8

(2) ☐ of 809 is 8.09

③ Please calculate the following after estimating.
Round your answers to the second highest place.

(1) 45.6×3.8 (2) 6.02×0.52 (3) 8.8×2.24

(4) $5.27 \div 1.34$ (5) $4.504 \div 2.14$ (6) $0.77 \div 1.86$

④ 1*kg* of salt costs 75*yen*. How much is the price of 0.8*kg* salt?

⑤ If you make 2.5*kg* bags of sugar from a total of 83.5*kg* sugar, how many bags can you make? How much sugar will be left?

⑥ Please find all the common factors of the numbers in each ( ).

(1) (12, 15) (2) (10, 18) (3) (27, 36)

⑦ Please find the two smallest common multiples of the numbers in each ( ).

(1) (4, 7) (2) (8, 18) (3) (16, 48)

Areas of Countries and Their Populations (1995)

	Areas ($10000km^2$)	Populations (10000 people)	Population Density
Japan	38	12520	
China	960	122146	
India	329	93574	
Mongolia	157	241	
Singapore	0.06	299	

Areas of the Continents (1995)

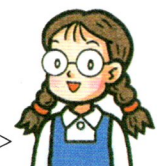
	Areas ($10000km^2$)
North and South America	4205
Asia	3176
Africa	3030
Others	3153
Total	13564

Areas of Forests (1995)

	Areas ($10000km^2$)
North and South America	1407
Asia	565
Africa	520
Others	962
Total	3454

(4) What % of the world's forests are located in Africa?

① Using this information about the earth, let's create some problems of your own. Then solve those problems.



It is assumed that the earth was born about 4.6 billion years ago. Lately, there have been many problems, such as global warming, declining forest areas, and acid rain.

Ritsuko

Life and Math

Our Earth

Akira studied many things about the earth.



Areas of the Oceans (1995)

	Areas ($10000km^2$)
The Pacific	16525
The Atlantic	8244
The Indian	7344
Others	3993
Total	36106



1 Please answer the following questions. Round off the answers to the second highest place.

(1) The radius of the earth is about $6378km$. About how many km is the length of the equator?

(2) The earth spins once every twenty-four hours. How many km per hour does the earth spin at the equator line?

(3) Draw a circle graph and show the areas of the oceans.

Check

1 Fill in the ( ) with the proper words and numbers.

(1) A polygon whose six sides and six interior angles are all equal is called ( ).

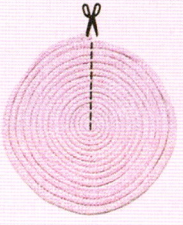
(2) The number that shows the ratio of the circumference of a circle to its diameter is called ( ). It is about ( ).

(3) The area of a circle = ( ) × ( ) × 3.14

2 Please find the length of the circumference of a circle with a diameter of 16cm. Then find its area.

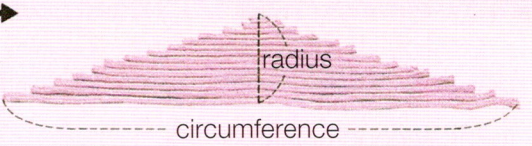
Challenge

Let's Make a Formula for the Area of a Circle Using a Rope!



Make a circle by spiraling a rope. Then, cut along a radius and open it up.





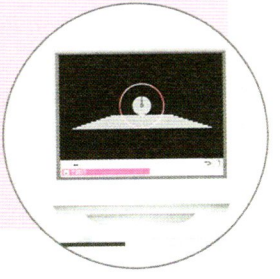
The area of the triangle = the circumference × radius ÷ 2

↓

diameter × pi



Is it the same as: the area of the circle = radius × radius × pi?



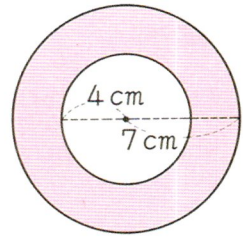
Practice 2

① Please find the circumference and the area of the following circles.

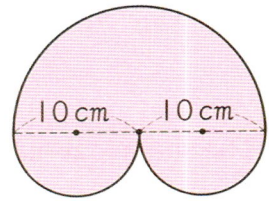
(1) A circle with a radius of 3.5cm.

(2) A circle with a diameter of 9m.

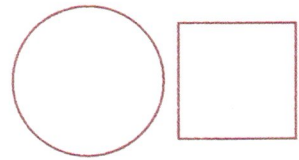
② In the drawing on the right, how much longer is the circumference of the outside circle than that of the inside circle in cm?
How many cm^2 is the area of the shaded part?



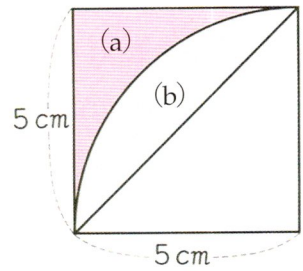
③ In the drawing on the right, please find the perimeter and area of the figure.



④ We made a circle and a square from two 12.56m wires. Which area is bigger and by how much?



⑤ In the drawing on the right, how can we find area (a)? How about area (b)? Find these areas.



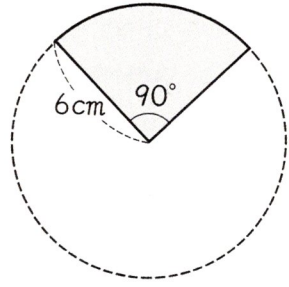
1 Please find the areas of the following circles.

(1) A circle with a radius of 6cm

(2) A circle with a diameter of 20cm

▶ Sector

3 Let's find the area of the shape on the right!

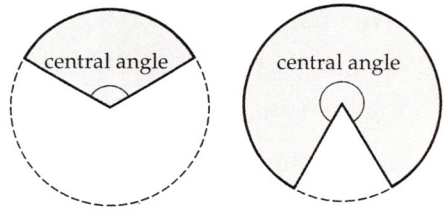


1 How many of the shape do we need to make the circle?

2 How many cm^2 is the area of the shape?

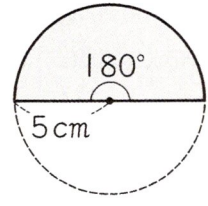
The shape cut out from a circle along two radii is called a **sector.**

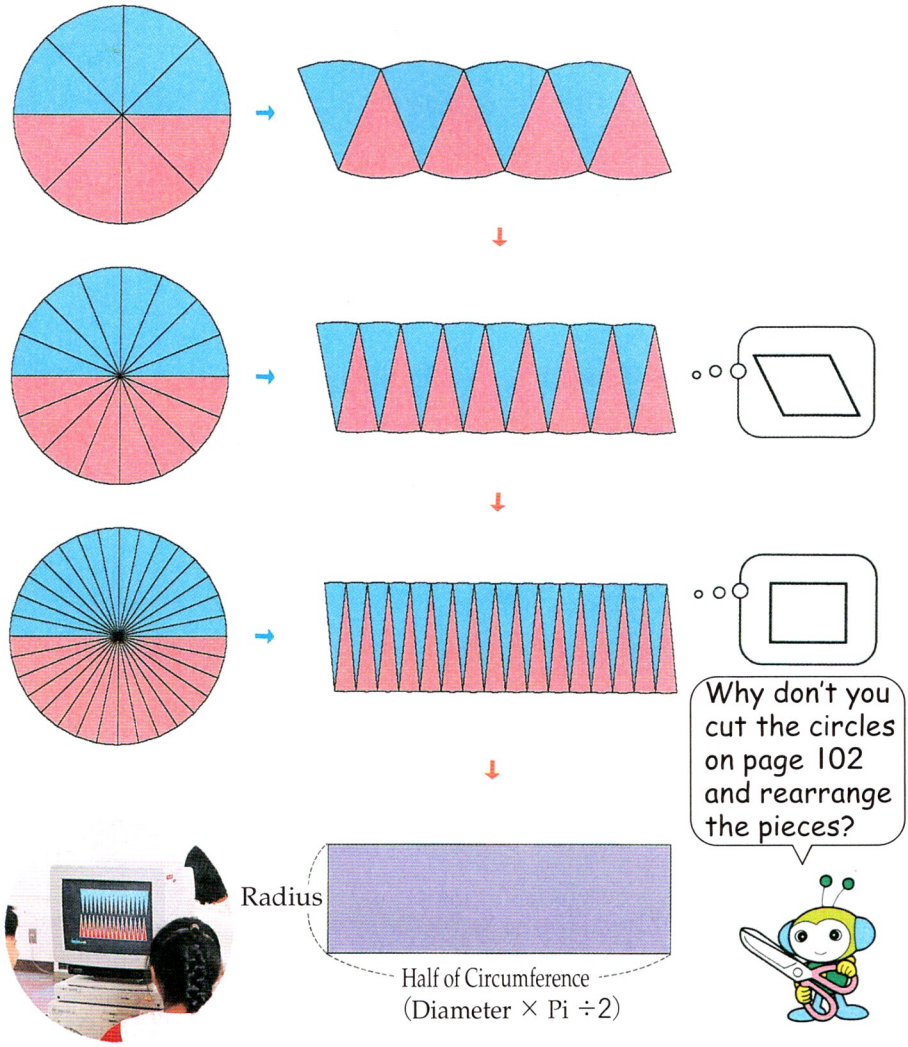
In sectors, the angle between the two radii is called the **central angle.**



1 Please find the length around the semicircle with a 5cm radius. Then find the area.

Semicircle

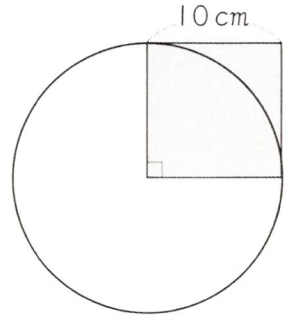




Half of a circumference equals the diameter × pi ÷ 2. Because this is the same as radius × pi, we can find the area of a circle with the following formula.

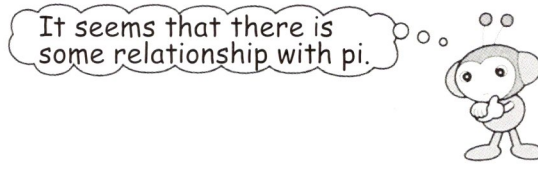
The Area of a Circle = Radius × Radius × Pi

3 About how many times the area of a 10cm square is the area of a circle with a radius of 10cm?

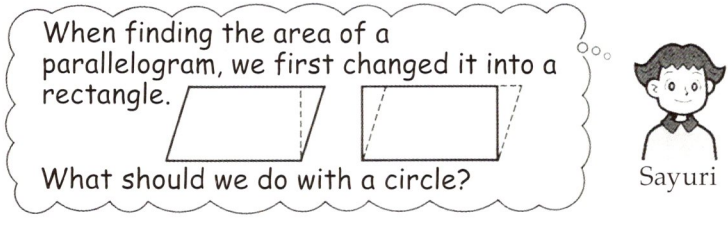


4 Please draw circles with different radii on a sheet of grid paper. Let's find about how many times as big the area of each circle is as the area of a square whose sides are equal to the radius of the circle!

The area of a circle is about 3.1 times that of a square whose sides are the same as the circle's radius.

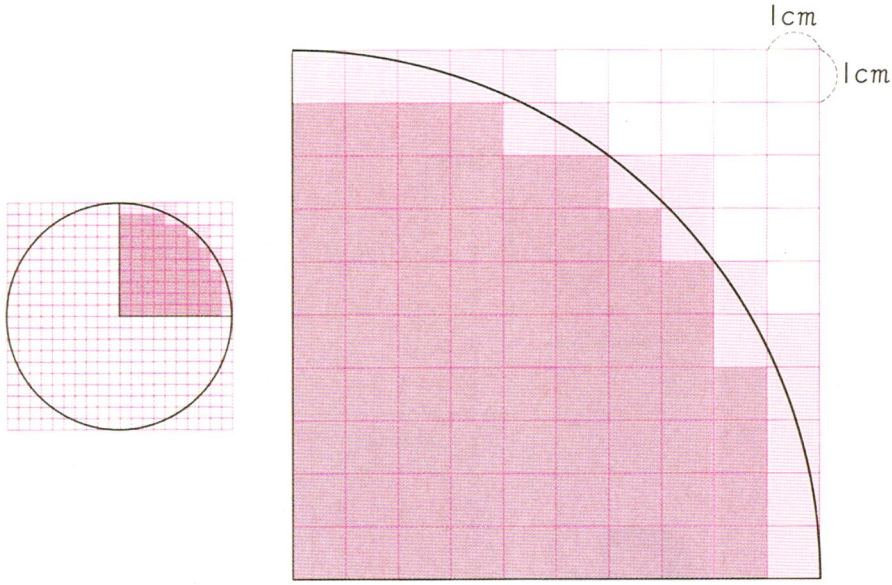


2 Let's think about a formula for finding the area of a circle!



1 When a circle is divided into smaller and smaller pieces and rearranged as shown on the next page, what kind of shape is it turning into?

2 Please draw a circle with a radius of 10cm on 1cm grid paper. Let's find the approximate area of the circle by counting the number of $1cm^2$ squares! Count all partial squares as one half.





Minoru

The number of ▢ is ▢ ..... ▢ cm^2

The number of ▢ is ▢

Counting these squares as halves ..... ▢ cm^2

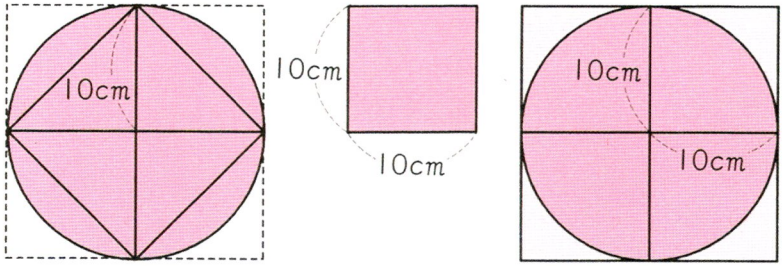
The area of the original circle is ▢ × 4 = ▢

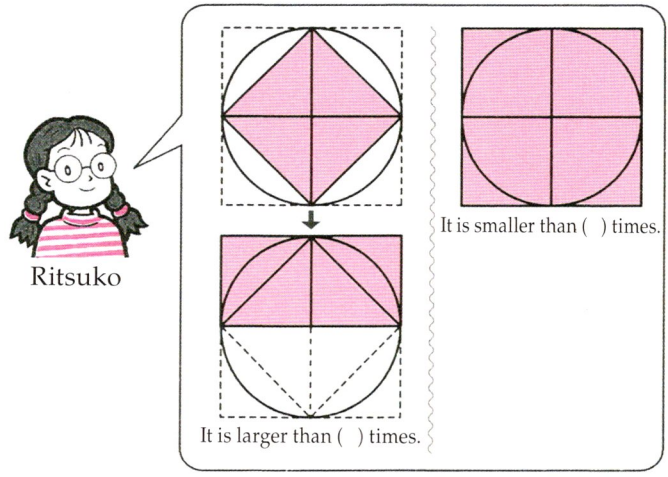
Answer: about ▢ cm^2

3 Area of Circles

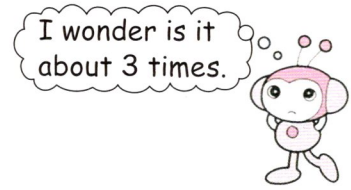
1 Let's find the area of a circle with a radius of 10cm!

1 Referring to the drawings below, estimate how many times as big the area of the circle is as the area of a 10cm square.





The area of a circle is more than 2 times the area of a square whose sides are the same as the circle's radius but smaller than 4 times the area of the square.



Practice I

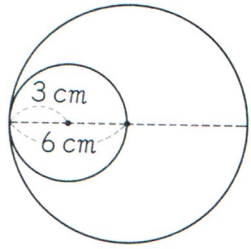
① Please draw a regular pentagon and a regular hexagon by using a circle that has a radius of 4cm.

② Please find the circumference of the following circles.

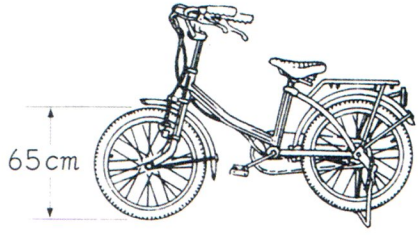
(1) A circle with a diameter of 7cm

(2) A circle with a radius of 6cm

③ A circle has a radius of 3cm. If you make the radius twice as long, how many times the circumference of the original circle will the circumference of the new circle be?



④ A bicycle has a wheel with a diameter of about 65cm. The wheel spins 84 times per minute. What is the speed of this bicycle per minute? Round off the answer to the second highest place.

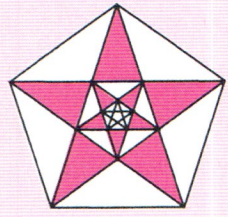


Challenge

Designing Patterns



Draw beautiful patterns using the regular polygons we drew for question ① above.



Approximate lengths

4 A tree has a circumference of 75cm. Approximately how many cm is the tree's diameter?



To find an approximate length, we sometimes use 3 for pi.



1 Please write an equation when the diameter is $x$$cm$. Then find x.

$x \times 3 = 75$

$x = 75 \div 3$

$x =$ ☐

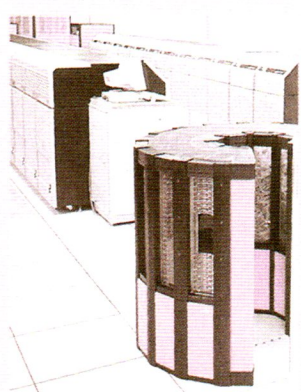
Answer: about ☐ cm

1 About how many m is the diameter of a circular pond that has a circumference of about 51m?

Story of Math

The Story of Pi

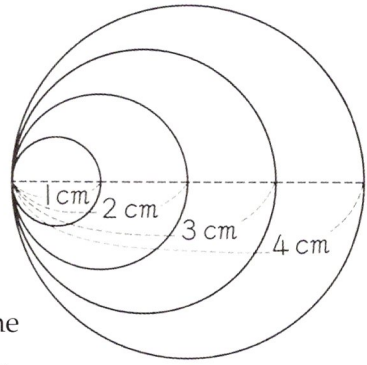
We usually use 3.14 as the value of pi. But it actually goes on and on like 3.141592653..... Until about 4000 years ago, people used the value $3\frac{1}{8}$. People have tried to find a more accurate value of pi. Today we know pi to 50 billion decimal places.



Changes in diameters and circumferences

3

Let's investigate how the circumferences of circles change as their diameters change!



1 If the diameter is x cm and the circumference is y cm, write a equation to find the circumference.

2 As x becomes 1, 2, 3, ···, 6, what will y become? Fill out the table below.

Diameter x (cm)	1	2	3	4	5	6
Circumference y (cm)	3.14					

Because 3.14 is fixed in the equation $y = x \times 3.14$, we can find the circumference when we know the diameter.

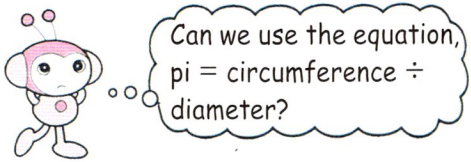
3 Please find y when x equals 12.

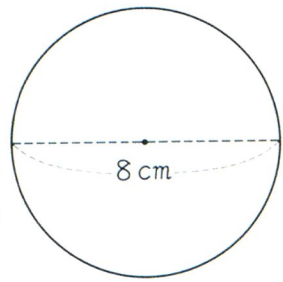
4 As x increases by 1, by how much does y increase?

5 As x becomes 2, 3, 4 times as big as 1, how many times as much as 3.14 does y become?

2 How many cm is the circumference of a circle that has a diameter of 8cm?

Let's think about a way to find circumference!





Circumference can be calculated by the formula below.

Circumference = Diameter × Pi

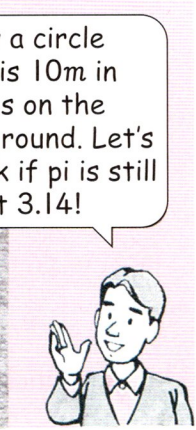
1 Please write a math sentence to calculate the circumference using the radius.

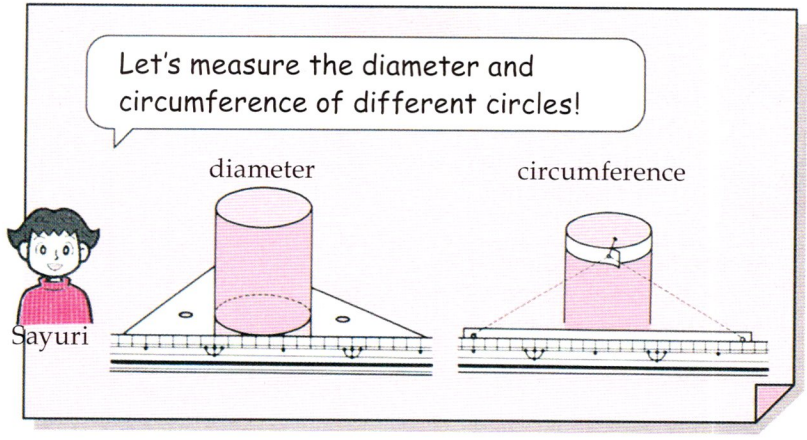
1 Please find the circumference of a circle that is 9cm in diameter.

Challenge

Let's Investigate the Circumference of Larger Circles!







2 Let's investigate how many times the diameter the circumference is by measuring various circular objects. Round your answers to the nearest hundredth!

	Circumference (cm)	Diameter (cm)	Circumference ÷ Diameter (times)
Can	22	7	
Bottle	36.1	11.5	
Pan	78.6	25	

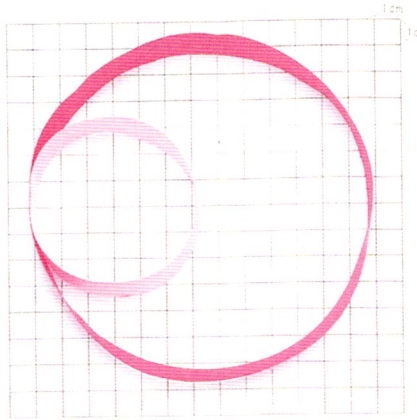
The circumference of any circle is about 3.14 times as long as its diameter. This number is called **pi.**

Pi = Circumference ÷ Diameter

2 Circumference



I made two circles with 20cm and 40cm ribbons. About how long are the diameters?

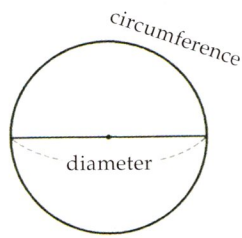


As the length around the circle becomes twice as long, does the diameter also double? I wonder.



Mami

1 The length around a circle is called its **circumference.**
Let's look into the relationship between circumference and diameter!

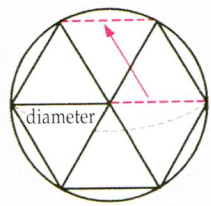


1 How many times the diameter of a circle is its circumference?



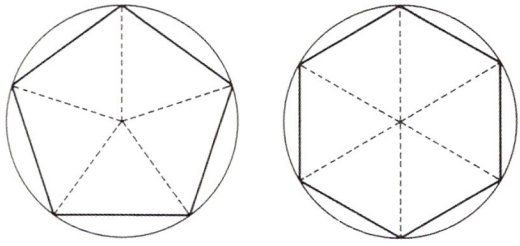
Minoru

Because the length around a regular hexagon is ( ) times its diameter, the circumference is a little longer than ( ) times its diameter.



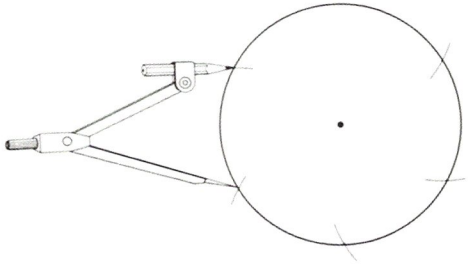
Circumference is a little longer than 3 times the diameter.

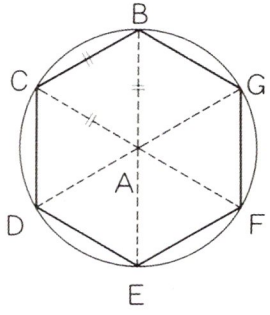
1 Let's draw a regular pentagon and a regular hexagon!



▶ Ideas for drawing a regular hexagon

3 As shown below, you can draw a regular hexagon by dividing the circumference of the circle by the length of the radius.
Let's think of the reason why!



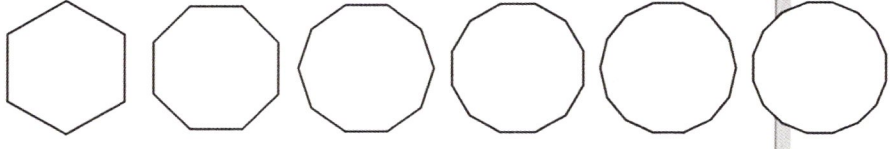


Story of Math

Regular Polygons with Many Sides

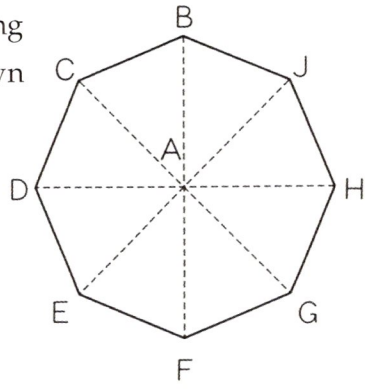


Let's increase the numbers of sides of regular polygons. They become increasingly closer to a certain shape.



▶ How to draw regular polygons

2 Let's think of a way to draw a regular octagon!



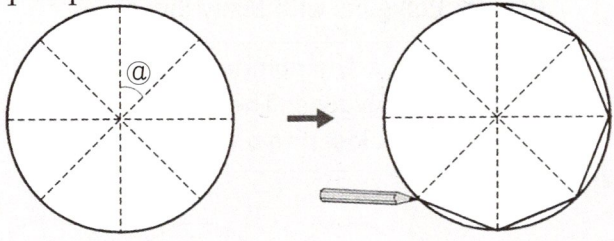
1 Please compare the following in the regular octagon shown on the right.

(1) The distance from A to each vertex.

(2) The eight angles formed around A.

2 Please draw a circle with A as a center and AB as a radius. Make sure that all the vertices touch the circle.

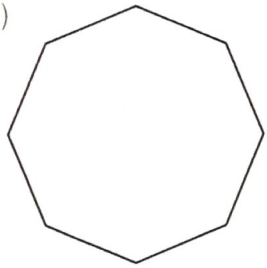
As shown below, you can draw a regular octagon by dividing the angles around the center into eight equal parts.



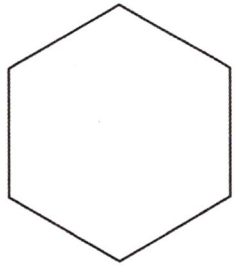
3 How many degrees is angle @ in the drawing above?

4 Please draw a regular octagon by following the drawing method explained above.

(1)



(2)



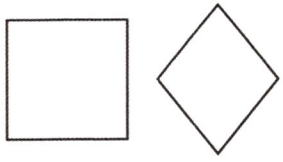
1 Compare the measures of the sides and angles of the two polygons above.

As for the octagon (1), the lengths of the sides are all equal and the measures of the angles are also all equal. This kind of octagon is called a **regular octagon.**

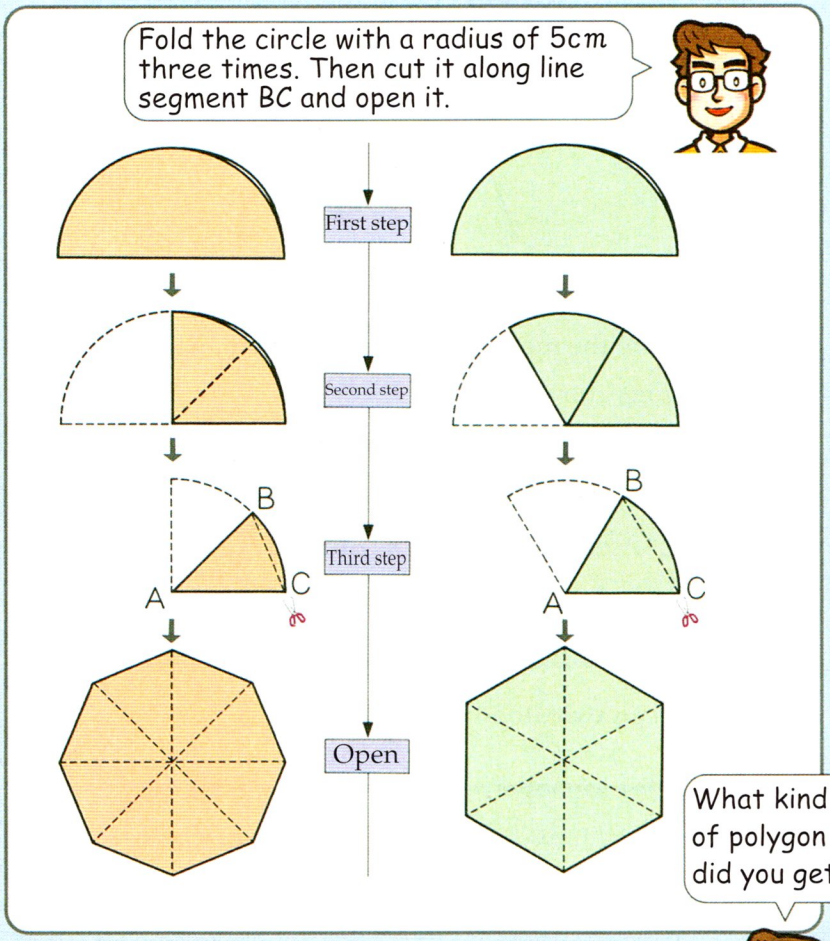
2 What can we call polygon (2)?

Polygons that have sides that are the same length and angles of equal measure are called **regular polygons**.

3 Can we call a square a regular polygon? How about a rhombus?



4 What regular polygon has the fewest sides?



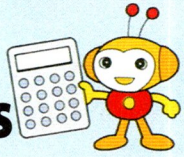
Let's investigate the characteristics of polygons!

1 Regular Polygons

1 Let's investigate the characteristics of the polygons we made above!



13 Circles and Regular Polygons





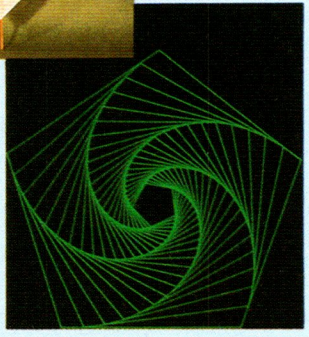
There are many different shapes around us.

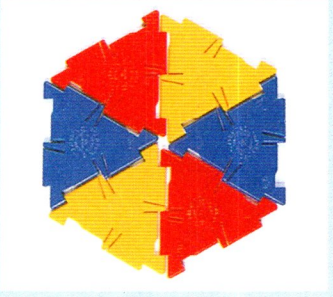








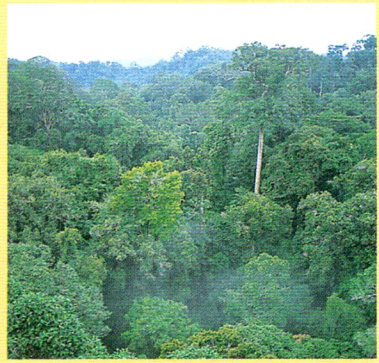




Story of Math

Disappearing Tropical Forest

On the earth, there are 17 *oku* 3000 *man* *ha*(1730000000 *ha*) of tropical forest. It is more than 40 times of the area of Japan.
The tropical forest has been shrinking because of burned fields and lumberjacking. Every year, almost 3 *wari* of the area of Japan is disappearing.







Area of Tropical Forest (unit: 1 *man* *ha* (10000 *ha*))

	1980	1995	Relative values of Reduction
Africa	57000	50000	
Asia	35000	32000	
USA	99000	91000	
Total	191000	173000	



Check

1 Please answer the questions about the following sentence.

> There are 40 students in Emiko's class. There are 22 girls and they are 55% of the whole class.

(1) What is the compared quantity? What is the base quantity?

(2) What do you call the relative value shown as 55%?

(3) Show 55% as a decimal.

2 Please find the relative value of 2.7km based on 4.5km.

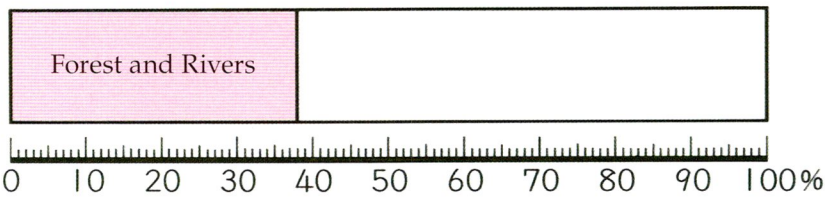
Challenge

Let's Find Relative Values!

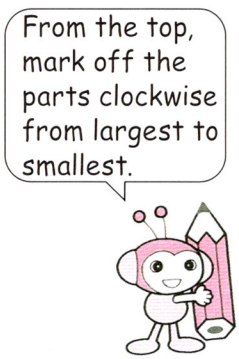


1 Let's show how the land is used using a percentage bar graph!

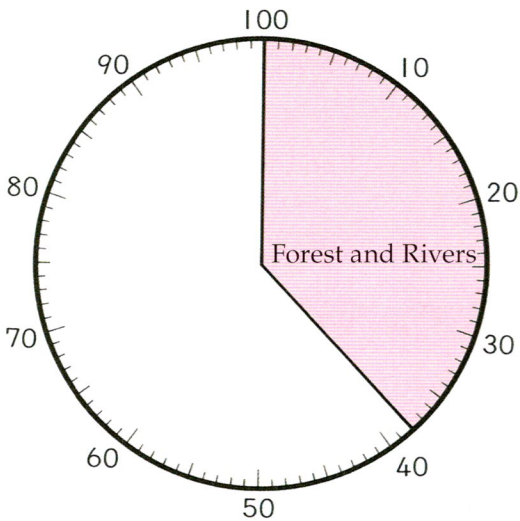
Relative Values of Land Use in Nishikawa City



2 Let's show how the land is used using a circle graph!



Relative Values of Land Use in Nishikawa City



1 The table on the right shows the number of different kinds of books in the library.
Let's represent this data in a percentage bar graph and a circle graph!

Number of Library Books

Kind	Number of books
Novels	1589
Science books	932
Reference books	507
Dictionaries	280
Other	612
Total	3920



How to draw percentage bar graphs and circle graphs

3 Yoshiko researched the land use in Nishikawa City. Let's make a percentage bar graph and a circle graph!

Land Use in Nishikawa City

Use	Area (km^2)	Percent (%)
Forest, Rivers	10.2	38
Housing	6.3	
Industry	4.7	
Agriculture	4.4	
Other	1.6	
Total	27.2	

How to Draw Percentage Bar Graphs and Circle Graphs

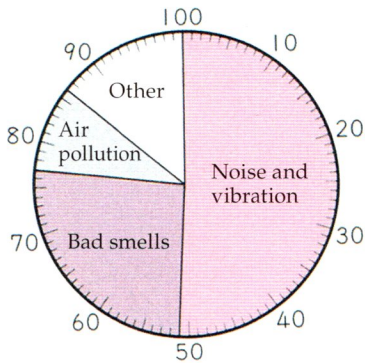
1. Find the percentage of each type of land use. If the total does not add up to 100%, adjust the largest by making it bigger or smaller so that the total is 100%.
2. Use graph paper with a scale partitioned into 100 and determine each part accordingly.

2

The graph below shows the relative values of the number of complaints about different types of pollutions in Nishikawa City.
Let's investigate the relative values of complaints based on the total number of complaints!



Relative Values of Complaints About Various Pollution



A graph like the one above is called a **circle graph**. The circle shows the total amount and it is divided into different parts based on their relative values.

1 What % of the complaints were about noise and vibration?
2 What % of the complaints were about bad smells? How about air pollution?

Percentage bar graphs and circle graphs are useful to represent the relative values of the parts to the whole, or the relative values among different parts.

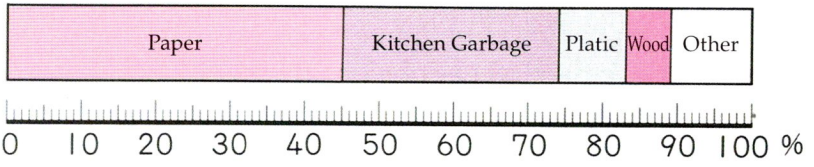
3 Graphs of Relative Values

Percentage bar graphs and circle graphs

1 The graph below shows the relative values of different kinds of waste from Nishikawa City. Let's find the relative value of each kind based on the total amount of waste!



Relative Values of Kinds of Waste in Nishikawa City



A graph like the one above is called a **percentage bar graph**. It uses a long rectangle to represent the whole and is divided into different sections based on their relative values.

1 What % of the waste was paper? How about plastic waste?

2 The amount of kitchen garbage is about how many times as much as the plastic waste?

Practice

① Please write the following relative values as percentages.

(1) 0.96 (2) 0.814 (3) 1.3 (4) 3 (5) $\frac{1}{10}$

② Please write the following percentages and *Buai* in decimal numbers.

(1) 67 % (2) 40 % (3) 3.5 % (4) 8 *wari* (5) 5 *bu*

③ Please answer the following questions.

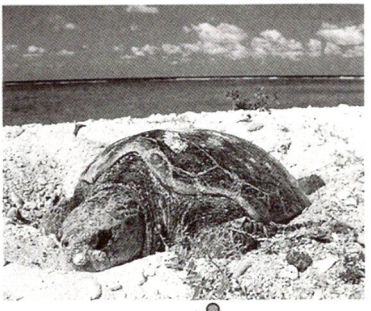
(1) What is the relative value of 8g based on 12.5g?

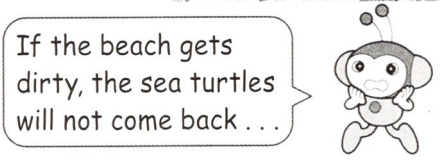
(2) 9ℓ is what percent of 12ℓ?

(3) $8.4m^2$ is what percent of $3.36m^2$?

(4) 3 *wari* 6 *bu* of what amount is 180 yen?

④ In Minamikawa City, they are protecting the eggs of loggerheads (sea turtles). This year, 646 eggs hatched out of 680 eggs. What % of the eggs hatched?





⑤ We planted carnations in a greenhouse. The area of space where we planted carnations is $132m^2$ which is 48% of the total area of the greenhouse.
How many m^2 is the total area of the greenhouse?

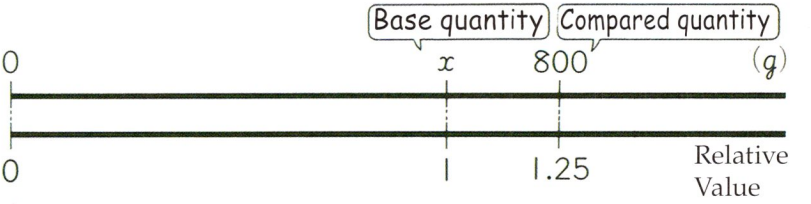
2

In Nishikawa City, each person produced 800g of waste each day last year. This is 125% of the amount produced 10 years ago.



What was the amount of waste produced 10 years ago?

Let's think about how to find the base quantity!



1 Make the amount 10 years ago xg and write a multiplication sentence. Then find x.

$$x \times 1.25 = 800$$

$$x = 800 \div 1.25$$

$$x = \square$$

Answer: $\square$ g

To find the base quantity, it is helpful to write a math sentence to find the compared quantity using x as the base quantity.

1

About 22% of the total weight of beef is protein. To eat 88g of protein, how many g of beef do you need to eat?

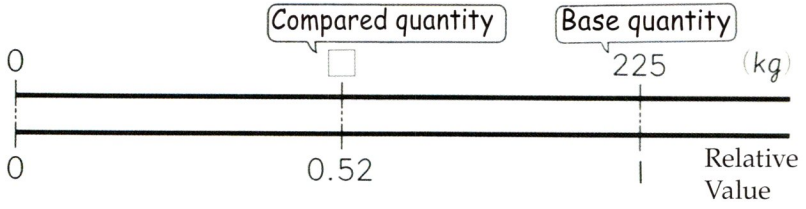
2 Problems involving Percentage

1 In Nishikawa City, each person uses 225kg of paper every year on average. 52% of the paper is recycled.
How many kilograms of paper will be recycled out of 225kg?



Let's find a way to find the compared quantity, given the base quantity and relative value!

1. Please express 52% using a decimal number.
2. Write a math sentence to find the compared quantity.



52% of 225kg is 0.52 times as much as 225kg.

$225 \times 0.52 = \square$ (kg)

The compared quantity can be found using the following math sentence:
Compared Quantity = Base Quantity × Relative Value

1 On a train with a capacity of 160 people, the relative value of passengers is 130%.
How many people are on the train?

3

In Nishikawa City, they are planning to plant 3000 trees and they have planted 2250 trees so far.
Compared to the goal, what is the relative value of the number of trees they have already planted?



$2250 \div 3000 =$ ☐

In Japanese, the relative value of 0.1 is called "1-***wari*** (1割)", 0.01 is called "1-***bu*** (1分)", and 0.001 is called "1-***rin*** (1厘)."
Relative value expressed this way is called "***Buai*** (歩合)."

1 Show the number of trees they have already planted in *Buai*.

(1) Please express the relative value expressed as 0.125 in *Buai*.

(2) Please express the following *Buai* in decimals or whole numbers.

(1) 7 *wari* 6 *bu* (2) 3 *bu* 8 *rin* (3) 5 *wari* 2 *rin* (4) 10 *wari*

(3) Please express the following relative values as percentages and *Buai*.

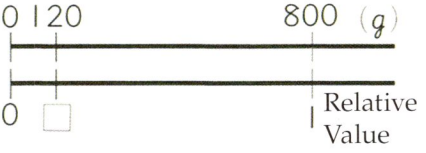
	1	0.1	0.01	0.001
Percentage	100 %			
Buai	10 *wari*			

2 In Nishikawa City, each person produced 800g of waste a day last year. This year they are campaigning for each person to reduce their waste by 120g a day.

If you set last year's amount as the base quantity, what is the relative value of the amount of reduction?



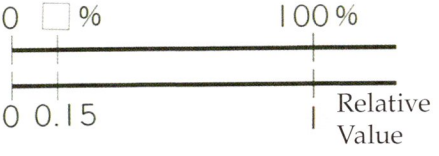
$120 \div 800 = \square$



The relative value of 0.01 is called 1 **percent** and written as 1%. Relative value expressed as percent is called **percentage**.



1 Please express the amount of reduction as a percentage.



Percentage is based on the base quantity of 100. The relative value of 1 is 100% when expressed as a percentage.

1 Please rewrite the relative values that are shown as decimal numbers as percentages.

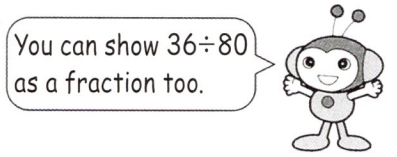
(1) 0.07 (2) 0.54 (3) 0.3 (4) 1.48 (5) 0.604

2 Please rewrite the relative values that are shown as percentages as decimal numbers.

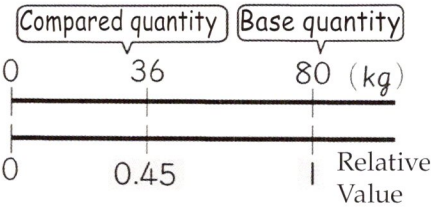
(1) 37 % (2) 8 % (3) 90 % (4) 120 % (5) 0.6 %

2 At East Elementary School, the amount of paper waste is how many times as much as the total amount of waste?

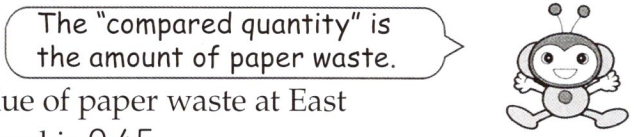
$36 \div 80 =$ ☐ (times)



If you consider the total amount of waste as 1, the paper waste will be 0.45.



When you set one number as the base quantity and compare another quantity with it, the value that is expressed is called "relative value."

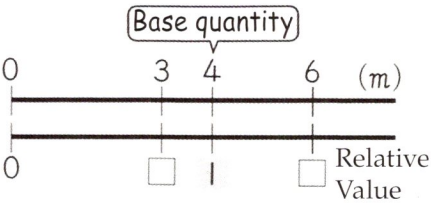


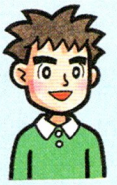
The relative value of paper waste at East Elementary School is 0.45.

3 Please find the relative value of paper waste at West and North elementary schools.

You can find the relative value using the following math sentence:
Relative Value = Compared Quantity ÷ Base Quantity

1 Please find the relative values for $3m$ and $6m$ when you set $4m$ as the base.



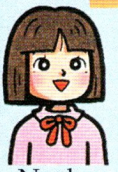


Kazuya

We asked a city official to find out how much waste is produced by 3 elementary schools in Nishikawa City each day.

Amount of Waste of Each School and Amount of Paper in the Waste (Data from Jan. 12)

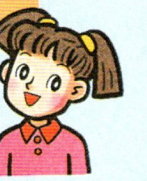
	Total amount of waste	Amount of paper
East Elementary	80 kg	36 kg
West Elementary	70 kg	35 kg
North Elementary	60 kg	33 kg



Naoko

East Elementary School produces the most paper waste.

East Elementary School produces the most waste, too.



Mami

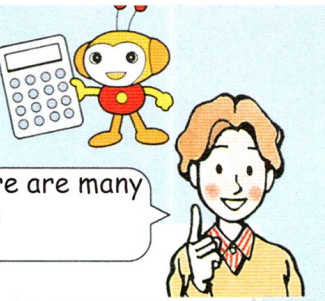
Let's think about how we can represent the relationship of the parts and the whole!

1 Relative Value and Percentage

1 Among East, West and North elementary schools, which school has more paper waste in comparison to the total waste?
Let's think about how to compare them!

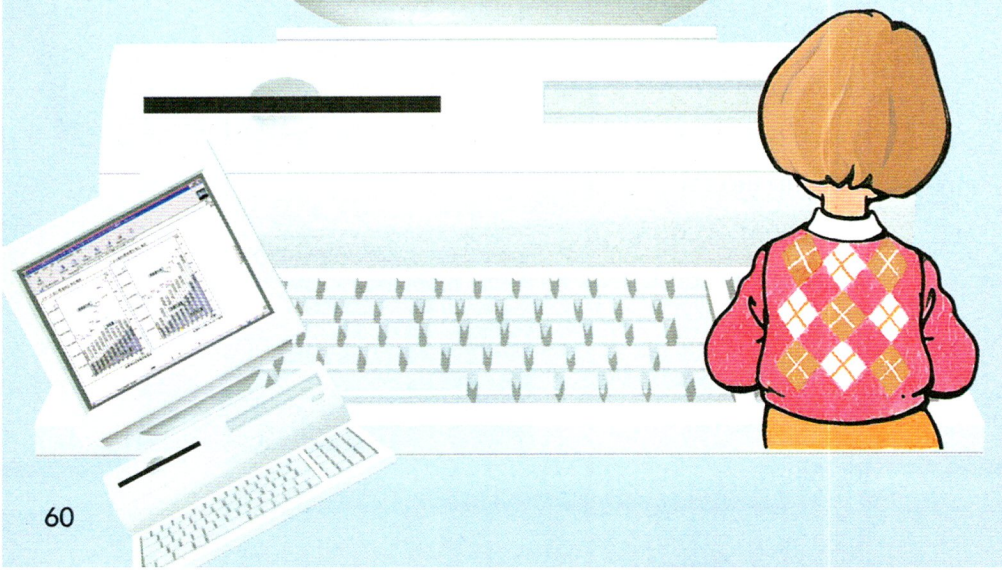
1 At which school was the amount of paper waste more than $\frac{1}{2}$ of the total amount of waste?

12 Percentage and Graphs



On the Internet, there are many websites about waste management issues.





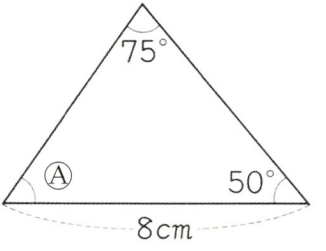
④ Please find all the common factors of the numbers in ( ). Also find the Greatest Common Factor and the Least Common Multiple.

(1) (9, 12)　　(2) (16, 24)　　(3) (8, 12, 24)

⑤ There is a triangle like the one on the right.

(1) What is the measure of angle Ⓐ?

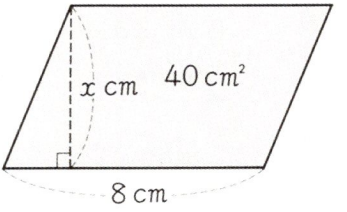
(2) Please draw a triangle congruent to the figure.



⑥ There is a parallelogram with an $8cm$ base and an area of $40cm^2$.

If the height is $x cm$, write a math sentence that represents this relationship.

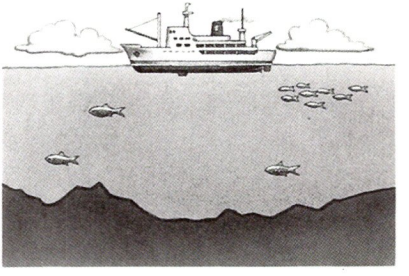
Also, find the height.



⑦ The sound created at the bottom of a ship traveled through water, hit the bottom of the ocean, and bounced back up to the ship after 1.2 seconds.

The speed of sound in sea water is about $1.5km$ per second.

What is the depth (m) of the ocean at this location?



Review・7

① Please calculate the following.

(1) 4.8×0.09

(2) 1.08×3.2

(3) $3.12 \div 2.6$

(4) $0.77 \div 0.14$

(5) $26 \times 0.75 - 13.25$

(6) $4.6 \div 2.3 \div 0.5$

(7) $0.67 + 0.33 \times 7.5$

(8) $(0.292 + 4.91) \div 0.34$

(9) $27.51 \div (6.38 - 2.45)$

(10) $7.34 \times (4.17 + 1.98)$

② Please calculate the following.

(1) $\frac{1}{4} + \frac{1}{3}$

(2) $\frac{3}{5} + \frac{1}{7}$

(3) $\frac{5}{8} + \frac{1}{4}$

(4) $\frac{4}{7} + \frac{2}{21}$

(5) $\frac{5}{9} + 3\frac{2}{3}$

(6) $2\frac{5}{6} + \frac{5}{14}$

(7) $\frac{6}{7} - \frac{3}{4}$

(8) $\frac{5}{6} - \frac{3}{10}$

(9) $\frac{11}{12} - \frac{1}{4}$

(10) $\frac{8}{9} - \frac{4}{5}$

(11) $1\frac{1}{6} - \frac{3}{14}$

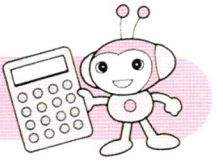
(12) $1\frac{1}{2} - \frac{11}{12}$

③ The Hokutosei Express runs 1207.9km between Ueno and Sapporo in about 16 hours. What is the approximate speed of Hokutosei Express in km per hour? Round the answer to the nearest tenth.



Math Plaza

Let's Make 100!



Here are the numbers 1 to 9 from least to greatest.

1 2 3 4 5 6 7 8 9



I put + and − signs between the numbers and made a math sentence like this. What is the answer?

1 2 3 + 4 − 5 + 6 7 − 8 9

123 67 89

$$123 + 4 - 5 + 67 - 89 = \square$$

1 Place +, −, ×, or ÷ between these 9 numerals and make a math sentence with an answer of 100.

1 2 3 + 4 5 − 6 7 8 9

123 45 67

123+45−67 is 101... so . . .



1 2 3 4 5 6 7 8 9



1+2+3+4+5+6+7 is 28... so . . .

Why don't you make many more different math sentences?



Check

1 Please fill in the ☐ with the appropriate numbers.

(1) When you divide 3ℓ of juice into 7 equal parts, the math sentence is ☐ ÷ ☐ and the answer is $\frac{\square}{\square}$ ℓ.

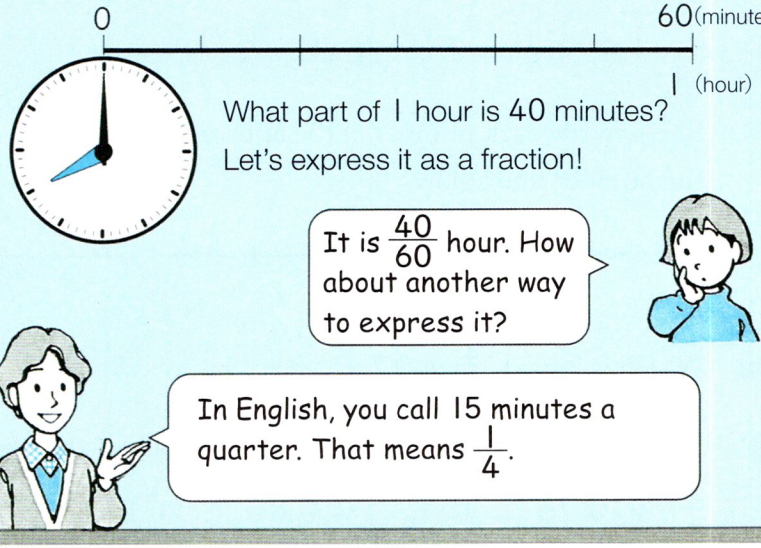
(2) To convert $\frac{7}{4}$ to a decimal number, you calculate ☐ ÷ ☐.
When you convert $\frac{7}{4}$ to a decimal number, it is ☐.

(3) To convert 0.09 to a fraction, make ☐ the denominator, and write $\frac{9}{\square}$.

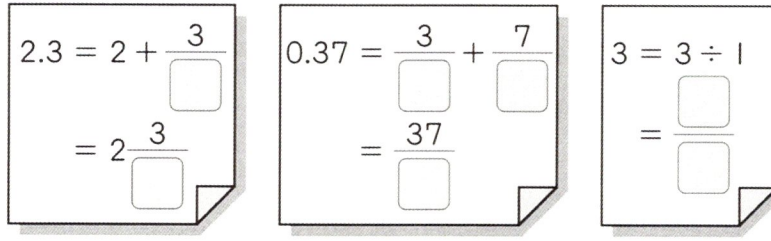
(4) To convert 7 to a fraction, make 1 its denominator, and write $\frac{\square}{1}$.

Story of Math

Fractions and Time



(3) Please convert 2.3, 0.37, and 3 to fractions.



You can convert decimal numbers to fractions with denominators of 10, 100, etc. Also whole numbers can be converted into fractions with denominators of 1.

1 Please convert the fractions to decimal numbers, and the decimal numbers to fractions.

(1) $\frac{3}{8}$ (2) $\frac{12}{5}$ (3) $\frac{5}{4}$ (4) 0.7 (5) 0.08 (6) 3.01

2 Show the following numbers on a number line.

$\left(2,\ 1.4,\ \frac{4}{5},\ 2\frac{1}{2},\ 0.7,\ \frac{6}{2} \right)$

1 Please write each number at the appropriate mark on the number line below.



2 Which is bigger, $\frac{4}{5}$ or 0.7?

1 Please fill in the □ with the correct inequality symbols.

(1) $\frac{3}{4}$ □ 0.76 (2) $\frac{7}{8}$ □ 0.85 (3) 0.4 □ $\frac{3}{7}$

2 Fractions, Decimal Numbers, and Whole Numbers

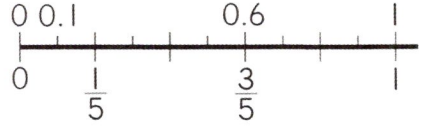
1 Please answer the following questions (1) ~ (3), and investigate the relationship between fractions, decimal numbers and whole numbers.

(1) When you divide a $3m$ tape into 5 equal pieces, how many m is a piece?

Find the answer as both a decimal number and a fraction.

$3 \div 5 = \square$ (m) $\qquad$ $3 \div 5 = \dfrac{\square}{\square}$ (m)

1 Using the figure on the right, check whether 0.6 and $\frac{3}{5}$ are equal.



(2) Please rewrite $\frac{3}{4}$ and $\frac{2}{9}$ as decimal numbers.

$\frac{3}{4} = \square \div \square$

$= \square$

$\frac{2}{9} = 2 \div 9$

$= 0.222\cdots\cdots$

To convert a fraction into a decimal number, you need to divide the numerator by the denominator.

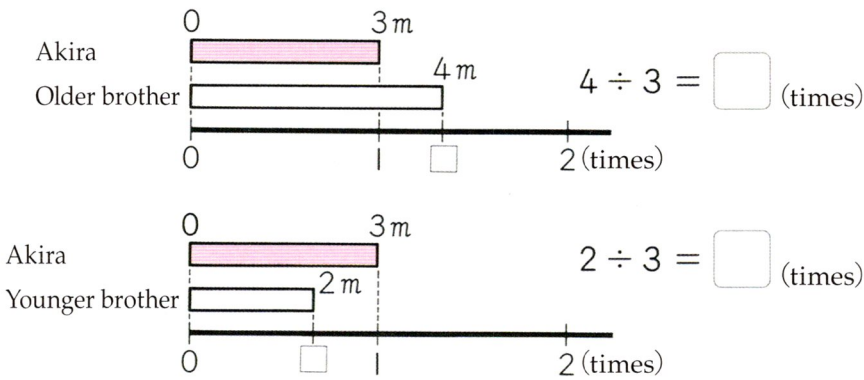
There are fractions which you can convert to decimal numbers exactly and fractions which you cannot.

Times as much

2 Akira and his brothers did the long jump. Akira jumped 3m, his older brother jumped 4m, and his younger brother jumped 2m.
If you compare the other jumps to Akira's record, how many times as far are each of his brother's jumps as Akira's jump?



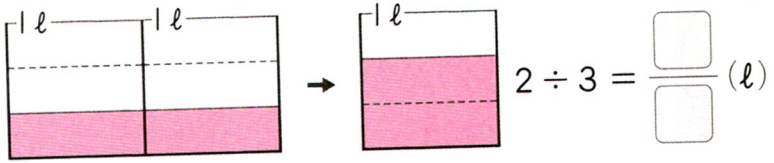
1 What kind of calculation do we need to do?



When you talk about how many times as many/much, we can use fractions like $1\frac{1}{3}$ times or $\frac{2}{3}$ times.
$\frac{2}{3}$ times as much represents the relationship that if 3m were 1, 2m would be $\frac{2}{3}$.

1 How many times as much are 20ℓ and 8ℓ as 12ℓ?

2 Look at the figure below and express the amount you get when you divide 2ℓ into 3 equal amounts using a fraction.



The quotient of 2÷3 can be shown accurately by using a fraction.

3 When you share 4ℓ of juice among 3 people, how many ℓ of juice will each person get?

$$4 \div 3 = \frac{\square}{\square} = \square \ (\ell)$$

The quotient of the division of two whole numbers can be expressed as a fraction. $a \div b = \frac{a}{b}$

1 Please show the quotients using fractions.

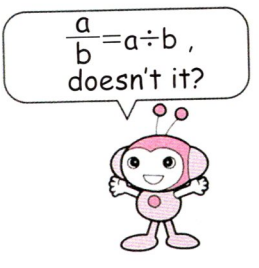
(1) $1 \div 6$ (2) $4 \div 5$ (3) $8 \div 12$ (4) $18 \div 14$

(5) $7 \div 3$ (6) $28 \div 6$ (7) $9 \div 2$ (8) $17 \div 9$

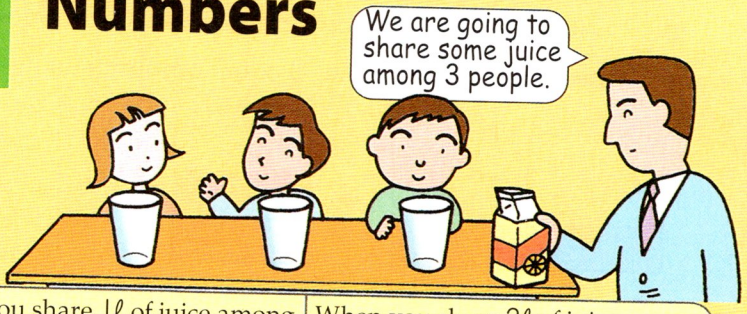
2 Please fill in the □ with the appropriate numbers.

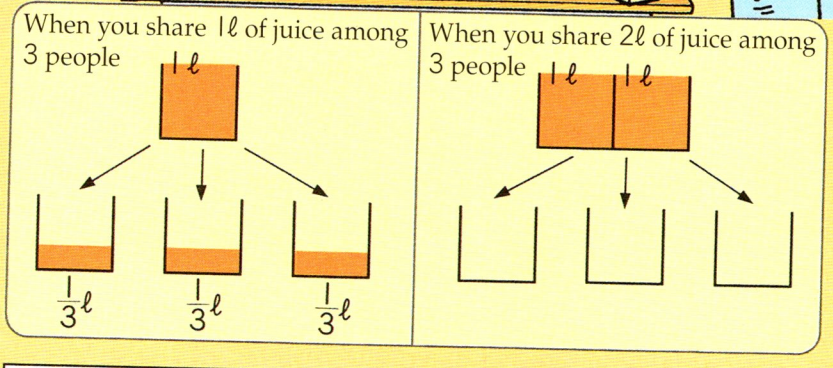
(1) $\frac{5}{9} = 5 \div \square$ (2) $\frac{1}{4} = \square \div 4$

(3) $\frac{7}{2} = \square \div 2$



11 Fractions and Decimal Numbers



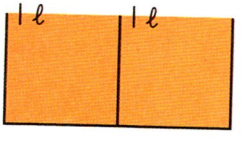


Let's think about the relationship between division and fractions!

1 Fractions and Division

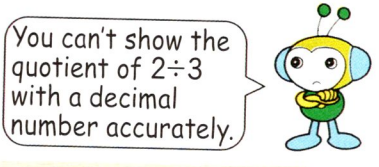
1 If you share 2ℓ of juice among 3 people, how many ℓ of juice will each person get?

$2 \div 3$



T Please express the quotient of $2 \div 3$ as a decimal number.

$2 \div 3 = 0.666$ ....., and the process does not stop.

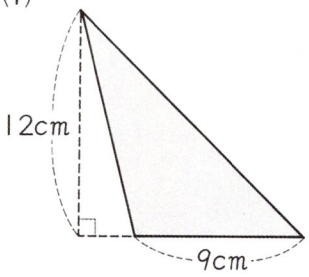


? Let's think about another way to show the answer for division!

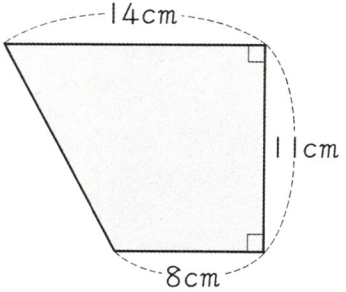
Review·6

① Please find the area of the following triangle and trapezoid.

(1)



(2)



② The number of books borrowed from the school library last week is shown below.
What is the average number of books borrowed in a day?



Number of Books Borrowed

Day of the Week	Mon	Tue	Wed	Thu	Fri	Sat
Number of Books	58	42	49	45	52	66

③ The table on the right shows the areas of school playgrounds and the number of students at 2 different schools.
Please compare the crowdedness of each school's playground.

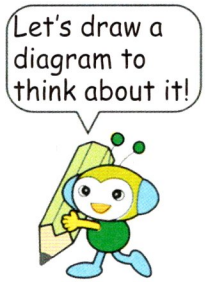
Area of Playground and Number of Students

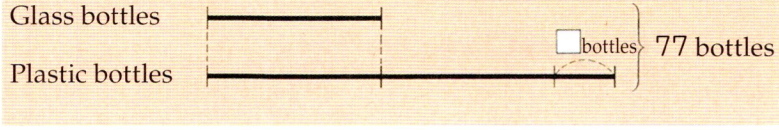
	Area (m^2)	Number of Students (people)
East Elementary School	8850	354
West Elementary School	10340	517





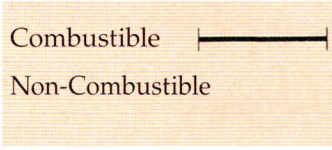
2 **Kiyoshi's group collected bottles. There was a total of 77 bottles. The number of plastic bottles was 8 more than twice the number of glass bottles. How many bottles of each type were there?**





① **The total non-combustible waste weighed 3.5 times as much as the combustible waste and the difference was 115kg. What was the weight of each kind of waste?**





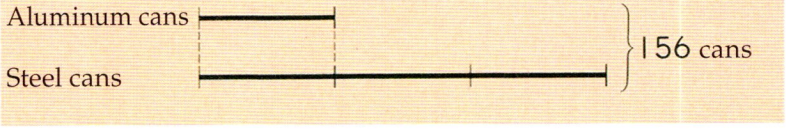
Problem Solving

Keeping a Park Clean



1 **Emiko's group collected cans. The total number of aluminum and steel cans was 156. There were 3 times as many steel cans as aluminum cans.**
How many aluminum cans and steel cans were there?





Practice

① Please fill in the ☐ with the appropriate numbers.

(1) $\frac{2}{5} = \frac{\square}{15} = \frac{\square}{45}$ (2) $\frac{10}{12} = \frac{5}{\square} = \frac{\square}{18}$

② Please simplify the following fractions.

(1) $\frac{5}{10}$ (2) $\frac{15}{21}$ (3) $\frac{42}{48}$ (4) $\frac{11}{77}$ (5) $1\frac{16}{24}$ (6) $1\frac{75}{100}$

③ Please compare the fractions by finding a common denominator.

(1) $\left(\frac{4}{5}, \frac{3}{4}\right)$ (2) $\left(2\frac{5}{6}, 2\frac{11}{15}\right)$ (3) $\left(\frac{5}{12}, \frac{7}{10}, \frac{2}{5}\right)$

④ Please calculate.

(1) $\frac{1}{3} + \frac{2}{5}$ (2) $\frac{1}{10} + \frac{1}{15}$ (3) $3\frac{1}{10} + \frac{1}{8}$ (4) $1\frac{2}{9} + 1\frac{1}{3}$

(5) $\frac{2}{3} - \frac{3}{7}$ (6) $\frac{5}{6} - \frac{4}{5}$ (7) $2\frac{3}{4} - \frac{5}{7}$ (8) $2\frac{1}{18} - 1\frac{1}{2}$

Check

1 Please find all the fractions with denominators from 1 to 20 that are equivalent to $\frac{1}{2}$.

2 Please fill in the ☐ with the appropriate numbers by thinking about how to add and subtract fractions with different denominators.

(1) $\frac{1}{6} + \frac{3}{8} = \frac{\square}{\square} + \frac{\square}{24} = \frac{\square}{24}$

(2) $\frac{6}{7} - \frac{2}{5} = \frac{\square}{\square} - \frac{14}{\square} = \frac{\square}{\square}$

3 Let's think about how to calculate $1\frac{1}{3}+\frac{4}{5}$!

1 Explain these two students' methods.

$$1\frac{1}{3}+\frac{4}{5}=\frac{4}{3}+\frac{4}{5}=\frac{20}{15}+\frac{12}{15}=\frac{32}{15}=2\frac{2}{15}$$

Makoto

$$1\frac{1}{3}+\frac{4}{5}=1\frac{5}{15}+\frac{12}{15}=1\frac{17}{15}=2\frac{2}{15}$$

Ritsuko

If you use a common denominator, you can add mixed numbers just like proper fractions.

4 Let's think about how to calculate $2\frac{2}{5}-\frac{1}{2}$!

$$2\frac{2}{5}-\frac{1}{2}=2\frac{4}{10}-\frac{5}{10}=1\frac{14}{10}-\frac{5}{10}=1\frac{9}{10}$$

1 Please explain the calculation method on the left.

1 (1) $\frac{2}{3}+1\frac{1}{2}$ (2) $1\frac{1}{4}+1\frac{3}{10}$ (3) $1\frac{1}{3}-\frac{5}{6}$ (4) $2\frac{3}{7}-1\frac{3}{4}$

2 (1) $\frac{2}{3}+1\frac{5}{6}+\frac{4}{9}$ (2) $4\frac{1}{4}-1\frac{1}{2}-\frac{1}{6}$

You can add and subtract fractions with different denominators by finding a common denominator.

1 (1) $\frac{1}{2}+\frac{2}{5}$ (2) $\frac{1}{4}+\frac{1}{6}$ (3) $\frac{5}{6}-\frac{2}{3}$ (4) $\frac{5}{9}-\frac{1}{6}$

2 Let's think about how to calculate $\frac{2}{3}+\frac{5}{6}$ and $\frac{5}{6}-\frac{1}{3}$!

1 Please explain the following addition and subtraction methods.

$$\frac{2}{3}+\frac{5}{6}=\frac{4}{6}+\frac{5}{6}$$
$$=\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{6}}}$$
$$=1\frac{1}{2}$$

$$\frac{5}{6}-\frac{1}{3}=\frac{5}{6}-\frac{2}{6}$$
$$=\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}}$$
$$=\frac{1}{2}$$

If you can simplify the answer, simplify it as much as possible.

1 (1) $\frac{1}{2}+\frac{1}{6}$ (2) $\frac{1}{12}+\frac{2}{3}$ (3) $\frac{5}{6}-\frac{7}{10}$ (4) $\frac{9}{10}-\frac{2}{5}$

2 (1) $\frac{1}{2}+\frac{3}{4}-\frac{4}{5}$ (2) $\frac{7}{8}-\frac{4}{5}+\frac{1}{2}$

2 Addition and Subtraction of Fractions

1 Let's think about the following problems!

(1) There is $\frac{1}{3}$ ℓ of milk in a bottle and $\frac{1}{2}$ ℓ in a milk carton. How many ℓ of milk do we have altogether?

Math Sentence



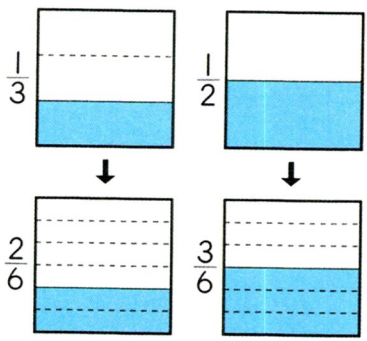


Let's think about how to calculate!

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}$$

$$=\square$$

Answer: ☐ ℓ



(2) We have $\frac{2}{3}$ ℓ of milk. We used $\frac{1}{2}$ ℓ to make a cake. How many ℓ of milk are left?

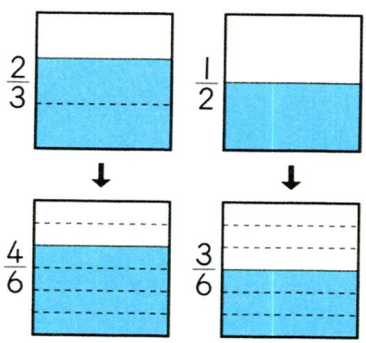
Math Sentence

Let's think about how to calculate!

$$\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}$$

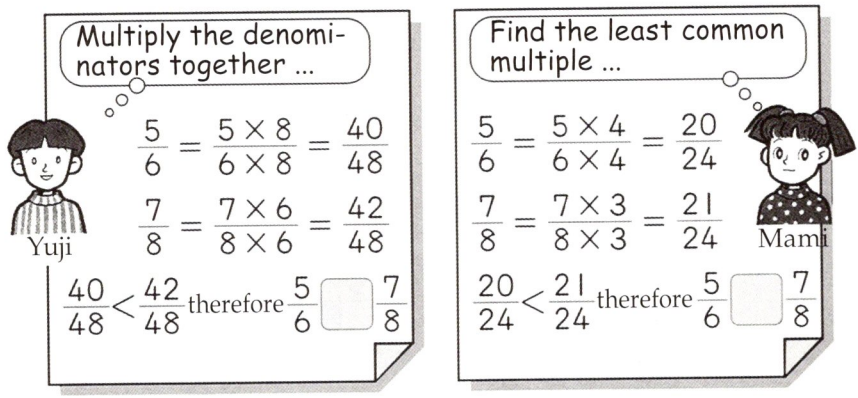
$$=\square$$

Answer: ☐ ℓ



4 Which fraction is greater, $\frac{5}{6}$ or $\frac{7}{8}$?

Let's think about how to find a common denominator!



Usually you use the least common multiple of the denominators as the common denominator.

1 Please find the common denominator and rewrite the fractions in ( ).

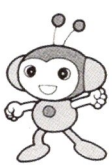
(1) $\left(\frac{4}{9}, \frac{5}{6}\right)$ (2) $\left(\frac{7}{12}, \frac{5}{8}\right)$ (3) $\left(\frac{1}{2}, \frac{2}{3}, \frac{2}{5}\right)$

Challenge

Comparing the Size of Fractions

$\frac{\square}{\square} > \frac{\square}{\square}$

Insert the cards below into the □ and make the math sentence corret.

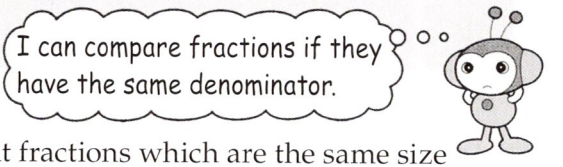


1 2 3 4 5 6 7 8 9

▶ Finding a common denominator

3 Which fraction is greater, $\frac{3}{5}$ or $\frac{2}{3}$?

Let's think about how to compare fractions with different denominators!



1 Create equivalent fractions which are the same size as $\frac{3}{5}$ and $\frac{2}{3}$. Then find fractions that have the same denominator.

$\frac{3}{5} = \frac{6}{10} = \boxed{\frac{9}{15}} = \frac{12}{20}$

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \boxed{\frac{10}{15}}$

Because $\frac{9}{15} < \frac{10}{15}$, thus $\frac{3}{5} \square \frac{2}{3}$


Minoru

The process of changing 2 or more fractions so that they have the same denominator is called **finding a common denominator**.
By finding a common denominator, you can compare the value of different fractions.

A common denominator is a common multiple of the denominators of the original fractions.

1 Please fill in the $\square$ with the appropriate symbols (=, <, or >).

(1) $\frac{1}{2} \square \frac{1}{3}$ (2) $\frac{2}{3} \square \frac{3}{4}$ (3) $\frac{2}{5} \square \frac{3}{7}$ (4) $1\frac{1}{3} \square 1\frac{2}{5}$

▶ Simplifying fractions

2 Let's think about how to make a fraction with the smallest numerator and denominator that is equivalent to $\frac{18}{24}$!

1 Please explain the following two students' methods.



Sayuri

$$\frac{\overset{3}{\cancel{18}}}{\underset{12}{\cancel{24}}} = \frac{3}{4}$$



Kazuya

$$\frac{\overset{3}{\cancel{18}}}{\underset{4}{24}} = \frac{3}{4}$$

When you divide the numerator and the denominator by their common factor, you can make a fraction with smaller numbers in the numerator and denominator. This process is called **simplifying a fraction**.

When you simplify a fraction, usually you try to make the numerator and denominator as small as possible.

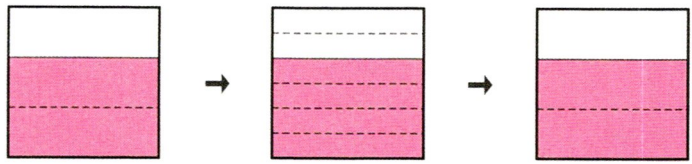
$$\frac{18}{24} = \frac{3}{4}$$



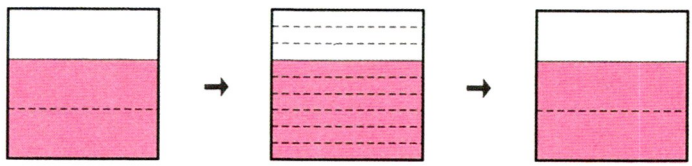
A smaller denominator makes it easier to see the size of a fraction. $\frac{64}{96} = \frac{2}{3}$

1 Please simplify the following fractions.

(1) $\frac{4}{10}$ (2) $\frac{12}{15}$ (3) $\frac{14}{21}$ (4) $1\frac{9}{18}$ (5) $2\frac{20}{24}$



$$\frac{2}{3} = \frac{2 \times \square}{3 \times \square} = \frac{4}{6} \qquad \frac{4}{6} = \frac{4 \div \square}{6 \div \square} = \frac{2}{3}$$

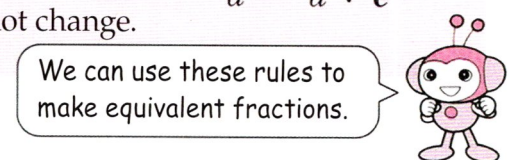


$$\frac{2}{3} = \frac{2 \times \square}{3 \times \square} = \frac{6}{9} \qquad \frac{6}{9} = \frac{6 \div \square}{9 \div \square} = \frac{2}{3}$$

When you multiply the numerator and the denominator of a fraction by the same number, or divide them by the same number, the size of the fraction will not change.

$$\frac{b}{a} = \frac{b \times c}{a \times c}$$

$$\frac{b}{a} = \frac{b \div c}{a \div c}$$



1. Please make 3 equivalent fractions that are equal to $\frac{3}{5}$.

2. Please fill in the $\square$ with the appropriate numbers.

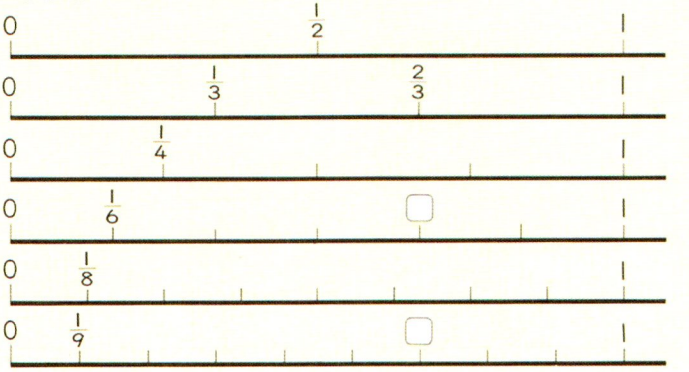
(1) $\frac{4}{5} = \frac{8}{\square} = \frac{\square}{20}$ (2) $\frac{6}{8} = \frac{\square}{4} = \frac{15}{\square}$



Ritsuko

Fractions may be equal to each other even though their denominators are different.

$$\frac{2}{3} = \frac{\square}{6} = \frac{\square}{9}$$





I wonder if we can compare the size of fractions that have different denominators.

$$\frac{2}{3} \square \frac{3}{4}$$

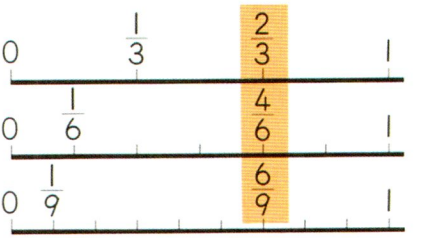
Minoru

Let's think about how to compare and calculate fractions with different denominators!

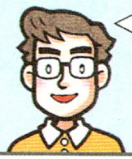
1 Simplifying Fractions and Finding Common Denominators

▶ Equivalent fractions

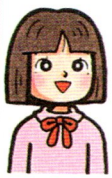
1 $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ are equivalent fractions. Let's think about how to create equivalent fractions.



10 Addition and Subtraction of Fractions

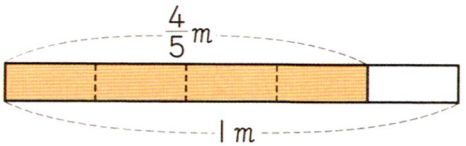


Let's review what we learned about fractions up to now!



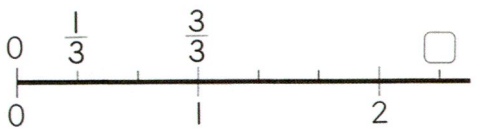
Naoko

$\frac{4}{5}$ m is ☐ of 5 equal parts of 1 m.



Seven $\frac{1}{3}$'s make $\frac{\square}{3}$.

You can write ☐ to show it as a mixed number.





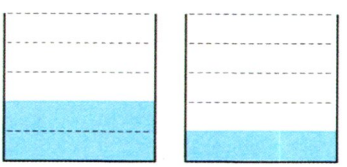
Kazuya

We can add and subtract fractions.



Yuji

$\frac{2}{5} + \frac{1}{5} = \square$



Review · 5

① Please calculate the following.

(1) 7.89×5.8 (2) 0.13×4.37 (3) 6.68×0.45

(4) 5.52×7.25 (5) 62.7×0.54 (6) 0.075×0.9

② Please divide the following completely.

(1) $99.94 \div 3.8$ (2) $0.42 \div 5.6$ (3) $34.2 \div 0.75$

(4) $11.34 \div 0.63$ (5) $9.89 \div 4.3$ (6) $16 \div 2.56$

③ Please find the 3 smallest multiples of 7 and 12, respectively. Also, find all factors of 21 and 45, respectively.

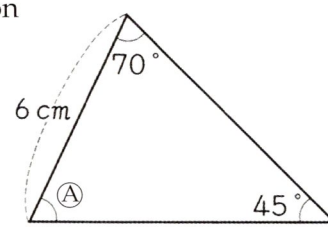
④ Please find the Least Common Multiple as well as the Greatest Common Factor for the following pairs of numbers.

(1) (9, 15) (2) (8, 12) (3) (24, 32)

⑤ There is a triangle like the one on the right.

(1) What is the measure of angle Ⓐ ?

(2) Please draw a triangle that is congruent to this triangle.



⑥ Please calculate the following.

(1) $\frac{2}{7}+\frac{3}{7}$ (2) $\frac{3}{5}+\frac{4}{5}$ (3) $1\frac{1}{8}+\frac{5}{8}$ (4) $2\frac{7}{9}+1\frac{4}{9}$

(5) $\frac{5}{6}-\frac{2}{6}$ (6) $1\frac{1}{3}-\frac{2}{3}$ (7) $2\frac{4}{7}-1\frac{1}{7}$ (8) $3\frac{1}{4}-1\frac{2}{4}$

Check

1 Mami measured the weights of 4 oranges.
What is the average weight of the oranges?
Please fill in the ▭ with the appropriate numbers.

80g	90g	82g	92g

(▭ + ▭ + ▭ + ▭) ÷ ▭ = ▭ Answer: ▭ g

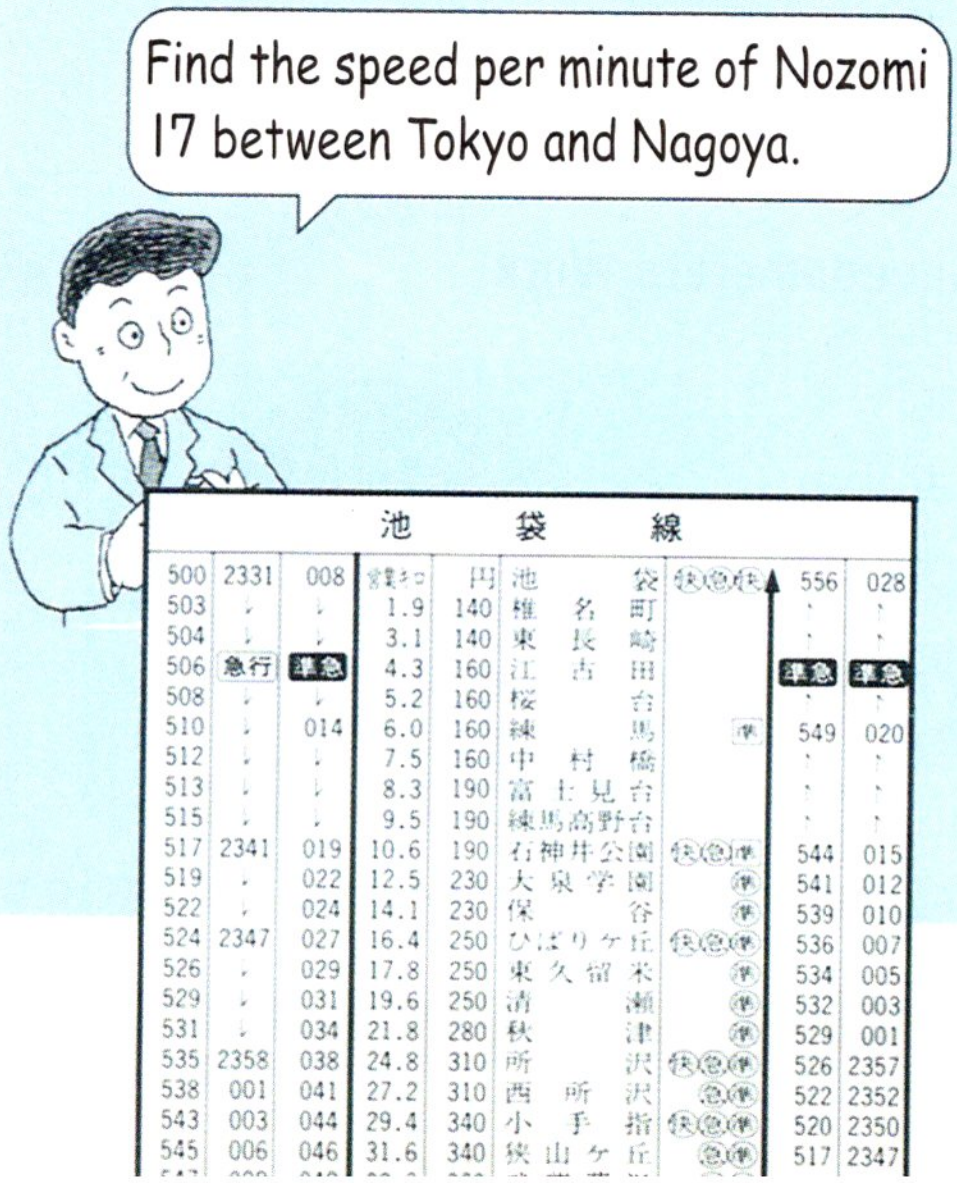
2 Please fill in the ▭ with the appropriate numbers.

(1) A car travels 70*km* in 2 hours. We can find the speed of the car per hour by ▭ ÷ ▭ = ▭ . The speed of the car is ▭ *km* per hour.

(2) When a bicycle travels 150*m* per minute for 5 minutes, it travels ▭ *km*.

Challenge

Let's Find the Speed!



500	2331	008	営業キロ	円	池袋	快急快	556	028
503	↓	↓	1.9	140	椎名町		·	·
504	↓	↓	3.1	140	東長崎		·	·
506	急行	準急	4.3	160	江古田		準急	準急
508	↓	↓	5.2	160	桜台		·	·
510	↓	014	6.0	160	練馬	準	549	020
512	↓	↓	7.5	160	中村橋		·	·
513	↓	↓	8.3	190	富士見台		·	·
515	↓	↓	9.5	190	練馬高野台		·	·
517	2341	019	10.6	190	石神井公園	快急準	544	015
519	↓	022	12.5	230	大泉学園	準	541	012
522	↓	024	14.1	230	保谷	準	539	010
524	2347	027	16.4	250	ひばりヶ丘	快急準	536	007
526	↓	029	17.8	250	東久留米	準	534	005
529	↓	031	19.6	250	清瀬	準	532	003
531	↓	034	21.8	280	秋津	準	529	001
535	2358	038	24.8	310	所沢	快急準	526	2357
538	001	041	27.2	310	西所沢	急準	522	2352
543	003	044	29.4	340	小手指	快急準	520	2350
545	006	046	31.6	340	狭山ヶ丘	急準	517	2347

Nozomi 17

営業キロ	列車名		こだま 463	ひかり 231	こだま 465	ひかり 165	のぞみ 17
	季節・臨時					◆	
	発車番線		17	16	19	15	14
0.0	Tokyo	発	1331	1338	1342	1345	1356
28.8	新横浜	〃	1348	1355	1401	↓	↓
83.9	小田原	〃	1413	↓	1427	↓	↓
104.6	熱海	〃	1423	↓	1437	↓	↓
120.7	三島	〃	1437	↓	1452	↓	↓
146.2	新富士	〃	1448	↓	1505	↓	↓
180.2	Shizuoka	着	1501	↓	1518	1445	↓
		発	1504	↓	1521	1451	↓
229.3	掛川	〃	1523	↓	1540	↓	↓
257.1	浜松	〃	1537	↓	1554	1517	↓
293.6	豊橋	〃	1554	↓	1617	↓	↓
336.3	三河安城	〃	1611	↓	1633	↓	↓
366.0	Nagoya	着	1624	1528	1646	1551	1532
		発	···	1529	···	1552	1534
396.3	岐阜羽島	〃	···	1547	···	↓	↓
445.9	Maibara	着	···	1601	···	↓	↓
		発	···	1602	···	↓	↓
513.6	Kyoto	着	···	1622	···	1629	1610
		発	···	1624	···	1631	1612
552.6	新大阪	着	···	1639	···	1647	1626
552.6	新大阪	発	···	···	···	1649	1628
589.5	新神戸	〃	···	···	···	1703	↓
612.3	西明石	〃	···	···	···	↓	↓
644.3	Himeji	着	···	···	···	1720	↓
		発	···	···	···	1721	↓
665.0	相生	〃	···	···	···	↓	↓
732.9	Okayama	着	···	···	···	1744	1711

Practice 2

① Hiroshi walks 110 steps in one minute on average. When his stride measures 60cm, what is his walking speed in m per minute?

② 5 seconds after we saw lightning we heard thunder. How far was the lightning from where we were? The speed of sound in the air is about 0.34km per second. Assume we saw the lightning at the same time as it flashed.

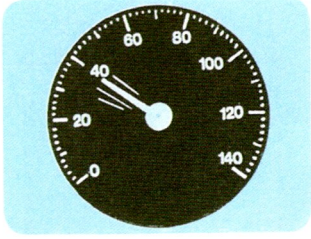


③ A bicycle traveled 18km in 1 hour and 30 minutes. Find the speed per minute of this bicycle. Also find the speed per hour.

Story of Math

How Speedometers Work

The speedometer of a car shows an instant speed at the moment by measuring how many times a wheel rotates in a second and computing the speed per hour.



Production speed

5 Copy machine A can copy 4500 pages of documents in 1 hour. Copy machine B can copy 300 pages of documents in 5 minutes. Which machine can make copies faster?



Let's think about production speed!

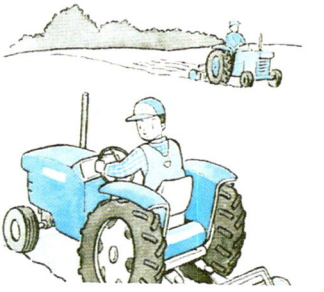
1 Let's compare the speed by comparing the number of copies per minute!

A ··· Math Sentence ( ) (sheets of paper)

B ··· Math Sentence ( ) (sheets of paper)

Production speed is represented as the amount of work per unit of time.

1 Two tractors cultivate the field. Tractor A cultivates 150a (ares) in 1 hour, and Tractor B does 120a in 45 minutes.
Which tractor cultivates the field faster?





1 Consider the traveling time as x hours and write a multiplication sentence. Then find the value of x.

$138 \times x = 552$

$x = 552 \div 138$

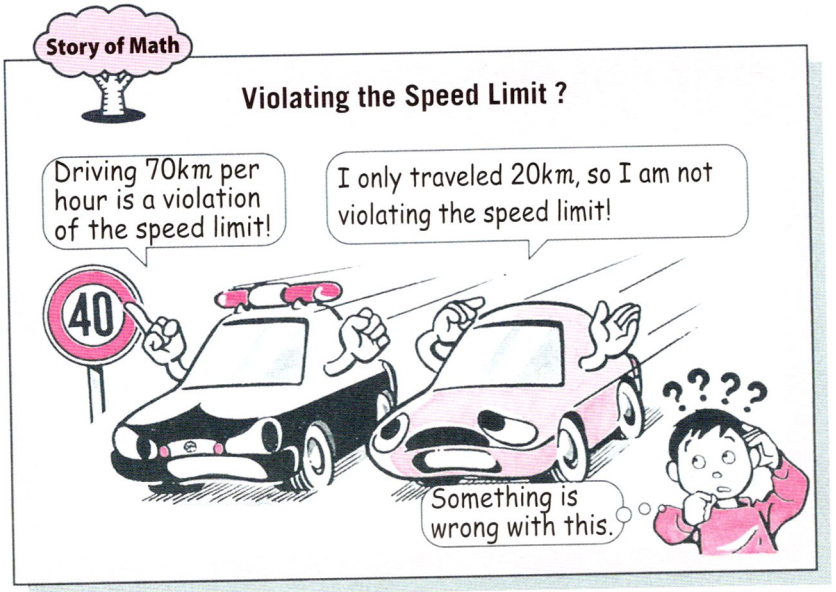
$x =$ ☐

Time can be found using the formulas for speed or distance, can't it?

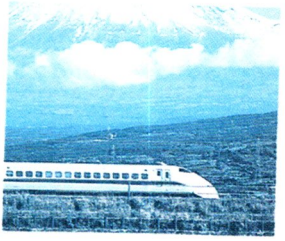


Answer : ☐ hours

1 How many minutes does it take a mini motorcycle that travels $500m$ per minute to travel $4.5km$?



How to find distance



3 Find the distance that the bullet train "Nozomi" travels in 3 hours if it is going at 232*km* per hour.

Let's think about how to find distance when you are given speed and time!



232 × 3 = ☐ (*km*)

Distance can be found by the following formula.

Distance = Speed × Time

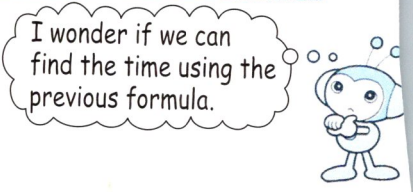
1 How many *km* will a person travel in 40 minutes if he/she walks 65*m* per minute?

How to find time



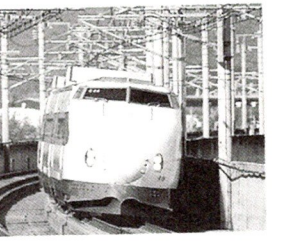
4 Find how long it will take for the bullet train "Kodama" to travel 552*km* when the train travels at 138*km* per hour.

Let's think about how to find time when you know the speed and the distance!



How to express speed

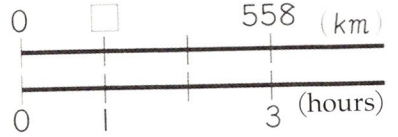
2 The bullet train "Hikari" travels 558km in 3 hours, and "Yamabiko" travels 392km in 2 hours.
Which travels faster?



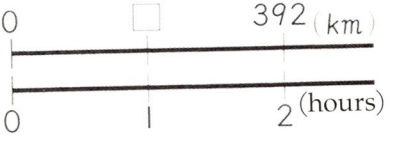
Let's think about how we can express speed!

1 Let's compare the distance that the trains travel in one hour!

Hikari



Yamabiko



$558 \div 3 =$ ☐ (km) $392 \div 2 =$ ☐ (km)

Speed is expressed as the distance traveled in a unit of time.

Speed = Distance ÷ Time

Speed is expressed differently depending on the unit of time used.

Speed per hour ..... speed expressed as the distance traveled in 1 hour

Speed per minute ..... speed expressed as the distance traveled in 1 minute

Speed per second ..... speed expressed as the distance traveled in 1 second

① Please find the speed per hour of a car that travels 216km in 5 hours.
Also, find the speed per minute and per second.

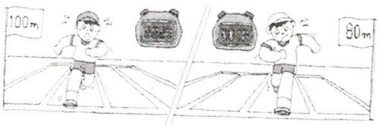
3 Speed

How to compare speed

1 The table on the right shows the distance and time that Susumu and Kiyoshi ran.
Who ran faster, Susumu or Kiyoshi?

Distance and Time

	Distance (m)	Time (sec.)
Susumu	100	20
Kiyoshi	80	18



Let's think about how we can compare speed!



Naoko

How much did they run in a second?

How long did it take to run 1m?



Yuji

1 Think about which person run faster using the two methods presented above.

Speed can be compared by using per unit quantity such as the distance run per second or the time it takes to run each meter.

1 Which is faster, a bicycle that goes 3000m in 15 minutes, or a bicycle that goes 2000m in 8 minutes?

Practice 1

① Emiko recorded how many minutes she read last week. How many minutes a day did she read on average?

Amount of Time Read

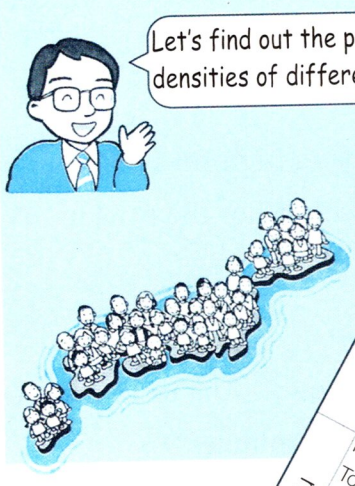
Day of the Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Time	45min	0	50min	30min	45min	1 h	2 hs

② Five people are in a car. If the average weight of a person is 62kg, and the weight of the car is 1200kg, how many kilograms is the weight of the people and the car altogether?

③ A car can go 7.5km on 1ℓ of gasoline and another car can go 12km on 1ℓ of gasoline.
If the two cars travel the same 270km route, what will be the difference in the amount of gasoline used by the cars?

Challenge

Let's Find Population Densities!

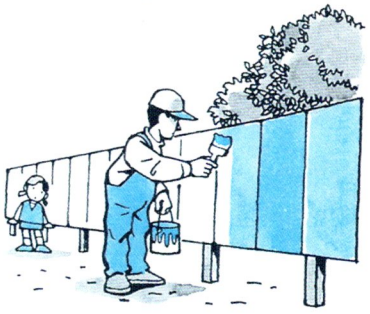


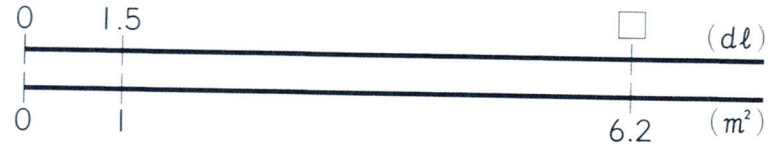
Area & Polulation (1996)

Region	Prefecture	Area (km^2)	Population
Hokkaido	Hokkaido	78,412.59	5,692,321
Tohoku	Aomori	9,605.57	1,481,663
	Iwate	15,274.40	1,419,505
	Miyagi	7,284.30	2,328,739
	Akita	11,612.66	1,213,667
	Yamagata	9,323.27	1,256,958
	Fukushima	13,781.41	2,133,592
Kanto	Ibaraki	6,093.40	2,955,530
	Tochigi	6,408.28	1,984,390
	Gunma	6,363.18	2,003,540
	Saitama	3,797.34	6,759,311
	Chiba	5,155.75	5,797,782
	Tokyo	2,183.26	11,773,605
	Kanagawa	2,412.11	8,245,9
Chubu	Niigata	12,581.62	2
	Toyama	4,246.09	
	Ishikawa	4,184	
	Fukui		
	Yamana		

Multiplication and division of per unit quantities

4 We would like to paint a wall which is $1m$ in height and $6.2m$ in width. We need $1.5d\ell$ of paint to paint $1m^2$.
How many $d\ell$ of paint do we need to paint the wall ?

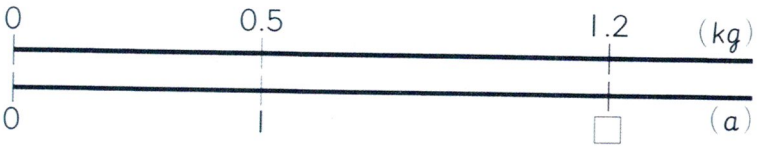




Math Sentence ☐

Answer: ☐ dℓ

5 We need $0.5kg$ of fertilizer for a $1a$ (*are*) field.
If we have $1.2kg$ of fertilizer, how many ares of field can we cover?



Math Sentence ☐

Answer: ☐ a

1 We have $8.1d\ell$ of paint. If we use $1.5d\ell$ of paint to paint $1m^2$ of wall, how many m^2 of the wall can we paint?

Population density

3 We researched the areas and populations of Toyama city and Ohita city.
Let's compare the crowdedness of Toyama city and of Ohita city!

Area and Populaion of Toyama city and Ohita city (1995)

	Area (km^2)	Population (people)
Toyama city	209	325303
Ohita city	361	426981

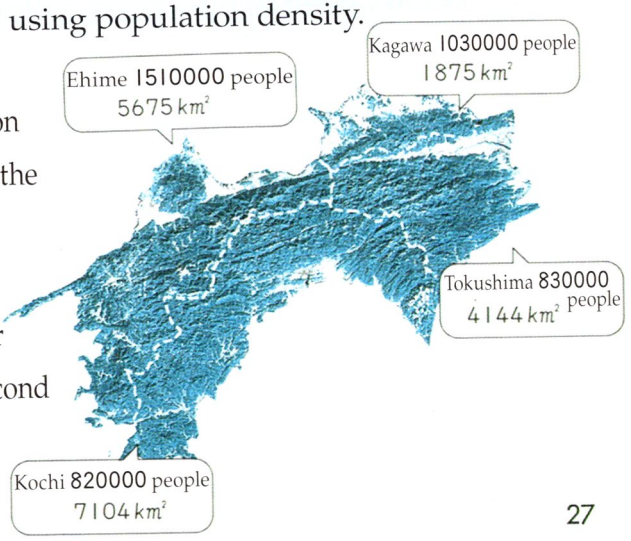
1 How many people are there per $1km^2$ in Toyama city and Ohita city ?
Please round your answers to the second highest place.

Toyama city··· $325303 \div 209 =$ ☐ Answer: ☐ people

Ohita city ··· $426981 \div 361 =$ ☐ Answer: ☐ people

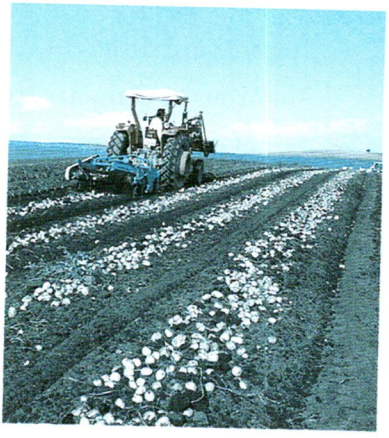
The number of people who live in an area of $1km^2$ is called "population density."
The crowdedness of a country, prefecture, county, or city is expressed using population density.

1 Find the population density of each of the 4 prefectures in Shikoku.
Please round your answers to the second highest place.

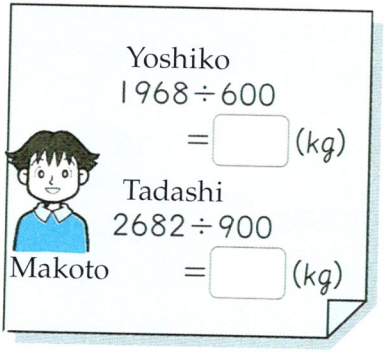


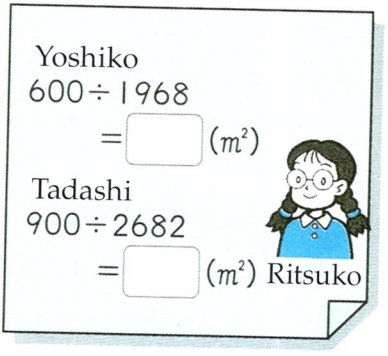
Various per unit quantities

2 At Yoshiko's farm, which is 600m^2, 1968kg of potatoes were produced.
At Tadashi's farm, which is 900m^2, 2682kg of potatoes were produced.
Which farm was better at producing potatoes?

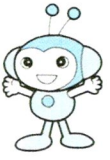


1 Please explain these 2 students' methods.



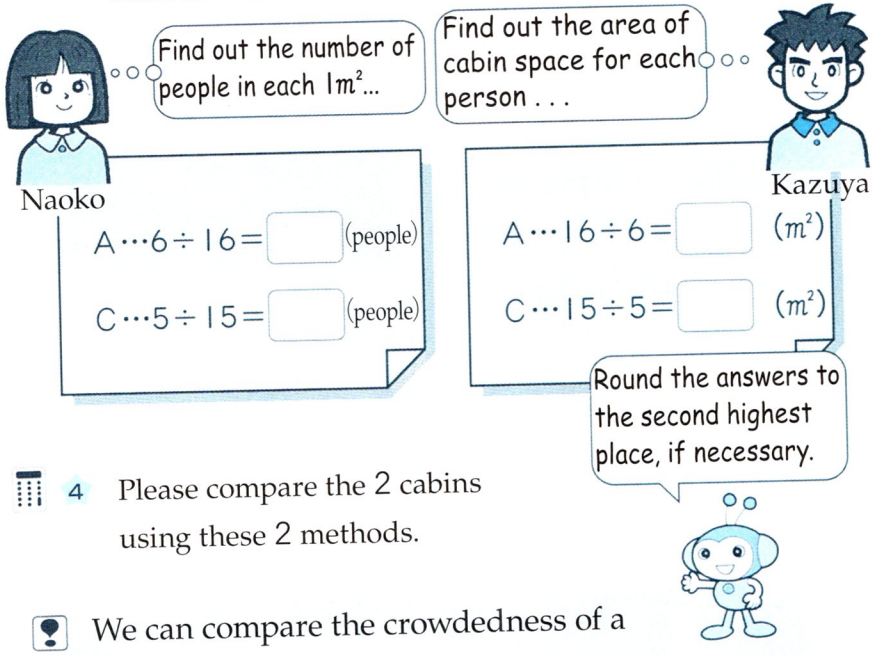


2 Which farm did better at producing potatoes?



1 A car can go 135km on 15ℓ of gasoline. Another car can go 262km on 25ℓ of gasoline.
Which car is more fuel efficient?

3 For A and C, the areas as well as the numbers of people are different.
In order to compare the crowdedness of those cabins, what can we do?



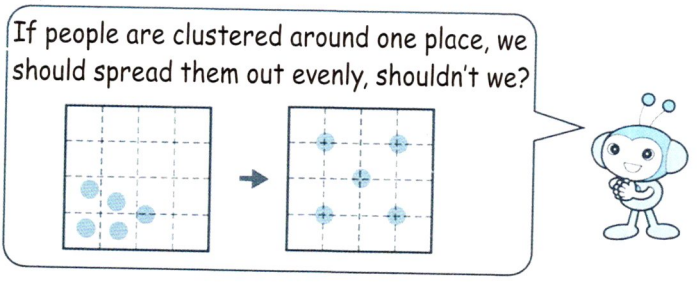
4 Please compare the 2 cabins using these 2 methods.

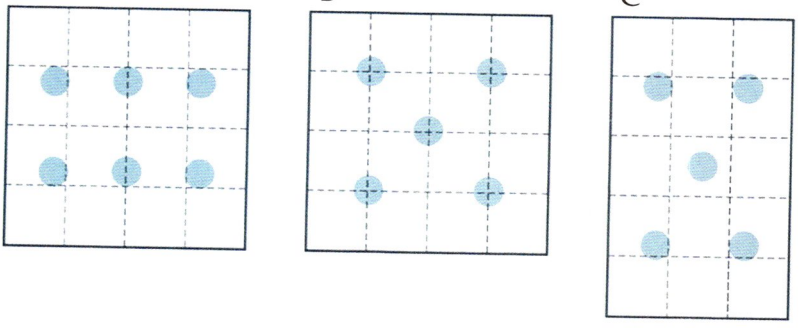
We can compare the crowdedness of a space by computing the number of people per $1m^2$ or the area per person.
We refer to these quantities as "per unit quantity."

1 Please compare the crowdedness of the gardens of Classes A and B.

Area of Gardens and Number of Plants

	Area (m^2)	Number of Plants
Class A	15	120
Class B	12	100





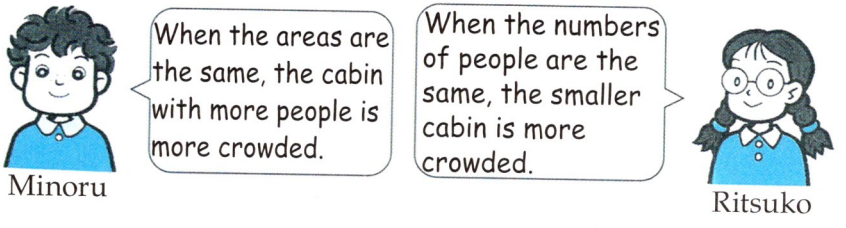
Area of Cabin and the Number of People

	Area (m^2)	Number of People
A	16	6
B	16	5
C	15	5



1 Which cabin is more crowded, A or B?

2 Which cabin is more crowded, B or C?



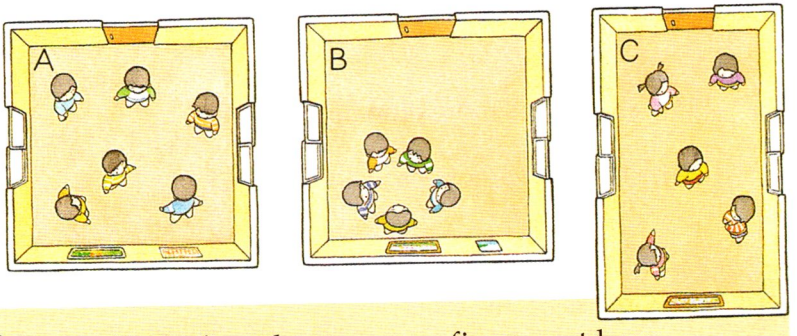
2 Per Unit Quantity



▶ Crowdedness

1 Kiyoshi and his friends will sleep in cabins A, B and C at camp.

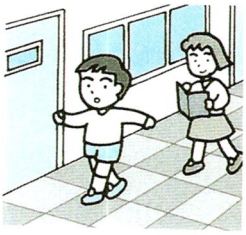
Which cabin is the most crowded?



? Let's think about how we can figure out how crowded something is!

Using average

3 Akira decided to measure the length of the hallway in his school. Let's think about how we can use our steps to measure it!

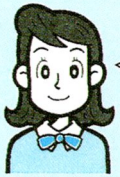


1 The total length of Akira's 20 steps was 12.8m.
How long is the average length of Akira's step?

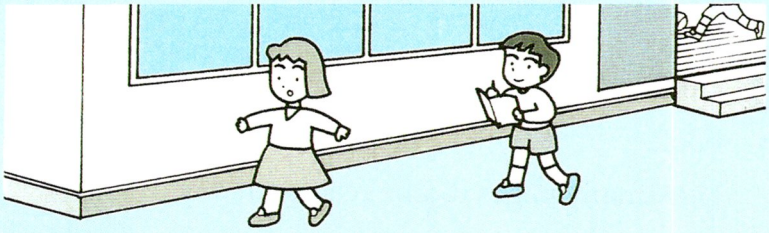
2 When Akira walked from one side of the hallway to the other, he took 125 steps.
About how long is the length of the hallway?

Challenge

Let's Find Out a Distance Using our Steps!



Measure the length of your own step and let's use it to measure various things in the school!



▶ Average number of people

2 The table below shows the number of students who borrowed books from the school library last week.
How may students borrowed books each day on average?



Number of Students Who Borrowed Books in a Week

Day of the Week	Mon	Tue	Wed	Thu	Fri	Sat
Number of People	6	5	4	0	3	9

I wonder if it is possible to find the average of a number of people...



Math Sentence



We should include the day that nobody borrowed a book, shouldn't we?

Yuji

Answer: ☐ people

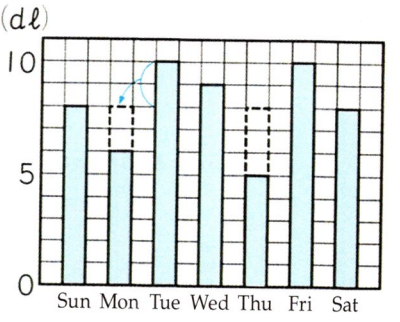
Sometimes we use decimal numbers to express an average quantity, even for quantities that cannot be expressed with decimal numbers, like number of people.

1 Takeshi took 8 days to finish a 220-page book.
How many pages did he read each day, on average?



The process to make varied quantities into equal sized quantities is called "evening out."

Milk Consumption in a Week



1 Think about how you can even out the quantities using the bar graph on the right. How many dℓ is the evened out quantity?

Let's find out how to even out the different quantities by calculation!

2 What kind of calculations do we need to do?
We can find out an evened out quantitiy by using the following calculation.

$(8+6+10+9+5+10+8) \div 7 =$ ☐

Answer: ☐ dℓ

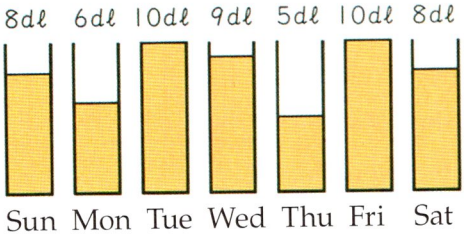
The quantity that is made by evening out different quantities is called the **average**.
The average can be calculated by using the following formula:

Average = the sum of the quantities ÷ the number of quantities

1 Please find the average of the following quantities.

18g 21g 20g 16g 22g 17g

The amount of milk Sayuri's family drinks in a day varies.



If we even out the amounts throughout the week, how much milk do we drink in a day?



Sayuri

Let's investigate how to find and use evened out amounts!

1 Average

How to find average

1 The chart below shows the amount of milk Sayuri's family drank in a day.
If they had drunk the same amount of milk everyday, how much milk would they have drunk in a day?

Milk Consumption in a Week

Day of the Week	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Quantity of Milk (dℓ)	8	6	10	9	5	10	8

9 Per Unit Quantity

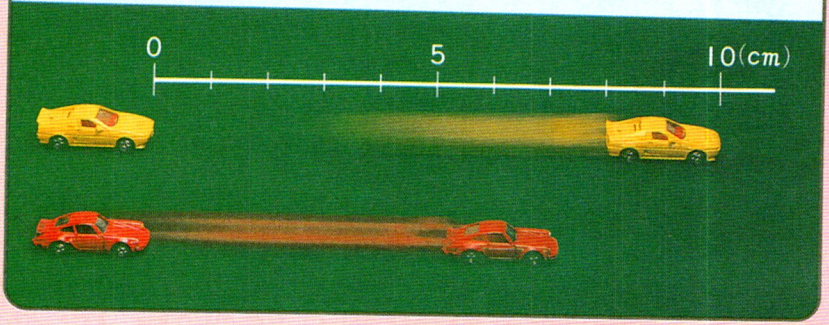




10 Yamanote Line

for Shibuya & Shinagawa

These cars ran the distance shown in the picture below in one second.



Story of Math

The Days of the Week on a Calendar

Today is Wednesday, Oct. 4th. Which day of the week is Oct. 26th?







Tadashi

It will take time to list all the dates in order...

I will write down the first two weeks and then think about it.



Yoshiko

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14



Kiyoshi

The dates in the second week are all 7 more than the dates in the first week.



Hiroko

Because the dates increase by 7 each week, I checked the remainder when the dates are divided by 7.

Sunday ..... remainder 1

Monday ..... remainder 2

⋮

The 26th is ( ).

Check

1 Please think about the whole numbers from 1 to 39.

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39

(1) Circle the even numbers.

(2) Find the multiples of 6 and the multiples of 8.

(3) Find the Least Common Multiple of 6 and 8.

(4) Find the factors of 32 and the factors of 24.

(5) Find the Greatest Common Factor of 32 and 24.

Challenge

Transforming Numbers



Try to write a number as the product of small factors other than 1.

$4 = 2 \times 2$
$6 = 2 \times 3$
$9 = 3 \times 3$
....



18 $18 = \square \times \square \times \square$
$\square \times 9$
$\square \times \square$

30 $30 = \square \times \square \times \square$
2×15
3×5



Try other numbers as well.

$64 = \square \times \square \times \square \times \square \times \square \times \square$

$84 = \square \times \square \times \square \times \square$

Practice

① Of all the 3-digit whole numbers you can make using the digits 2, 3, and 4 once and only once, please find the largest even number and the smallest odd number.

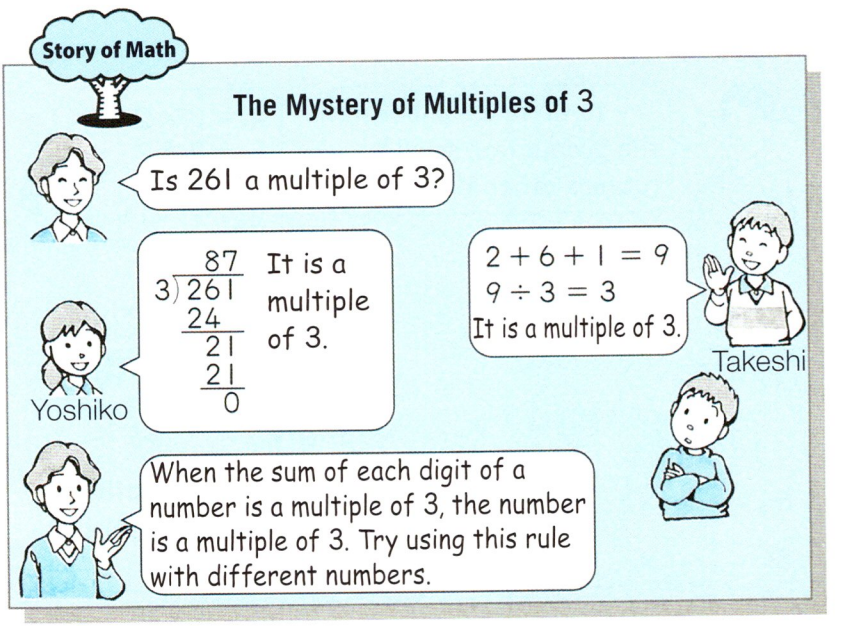
② Please find the three smallest multiples of 8 and the three smallest multiples of 9.
Find all the factors of 27 and all the factors of 36.

③ Please find the Least Common Multiple of the numbers in ( ).

(1) (3, 5) (2) (6, 10) (3) (12, 20) (4) (14, 42)

④ Please find the Greatest Common Factor of the numbers in ( ).

(1) (4, 6) (2) (8, 20) (3) (6, 18) (4) (16, 30)



▶ Greatest Common Factor

2 Let's think about how to find common factors of 24 and 36!



Sayuri: Write the factors of 24 and 36, and then find which ones are common.

Factors of 24… 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36… 1, 2, 3, 4, 6, 9, 12, 18, 36



Yuji: Among the factors of 24, find those that 36 can be divided by without a remainder.

Factors of 24	1	2	3	4	6	8	12	24
Numbers that can divide 36	○	○	○	○	○	×	○	×

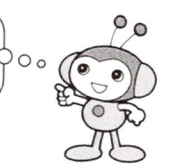
Among common factors, the largest one is called the **Greatest Common Factor** (**GCF**).

1 What is the Greatest Common Factor of 24 and 36?

1 Please find all the common factors of the numbers in ( ). What is the Greatest Common Factor?

(1) (12, 16) (2) (20, 30) (3) (18, 36)

2 Please find the common factors of 6, 9, and 12. Think about an effective way.

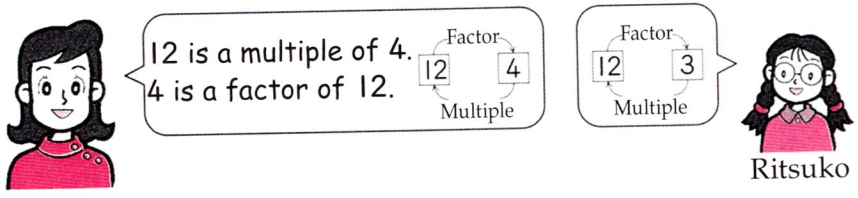


When the lengths of the sides of the square pieces are 1, 2, 3 and 6, the sheet can be covered with no overlap or gaps.

Factors that are the same among the factors of 12 and 18, such as 1, 2, 3, and 6, are **common factors** of 12 and 18.

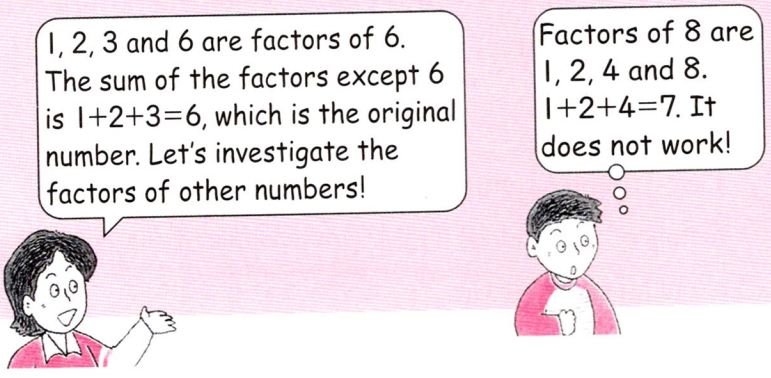
1 List all the factors of 16 from least to greatest.

2 List all common factors of 6 and 9. List all common factors of 18 and 24.

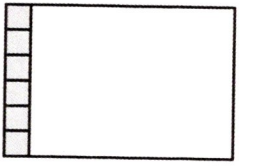


Challenge

Finding Interesting Factors



1 For the square pieces to fill the length of the rectangle completely, how long must the side of the square be?



Length of a Side of Square (cm)	1	2	3	4	5	6	7	8	9	10	11	12
No gaps ... ○ Gaps ... ×	○											

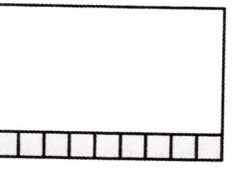
2 What numbers can 12 be divided by without a remainder?

12 can be divided by 1, 2, 3, 4, 6 and 12.
These numbers, 1, 2, 3, 4, 6 and 12, are **factors** of 12.

When the length of the square piece is a factor of 12, you can fill the length of the sheet completely.

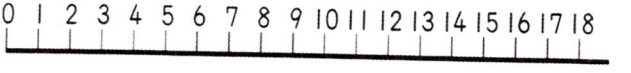


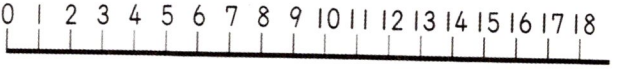
3 For the square pieces to fill the width of the rectangle completely, how long must the side of the square be?



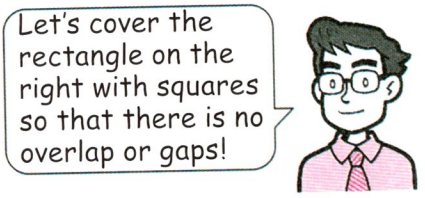
4 What kind of numbers are used for the length of the side of the square, when the square pieces fill the width of the sheet completely?

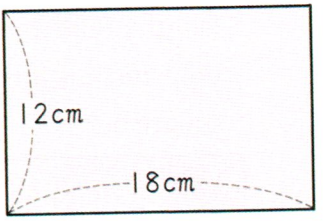
5 Circle the factors of 12 and 18 on the number lines.

Factors of 12 

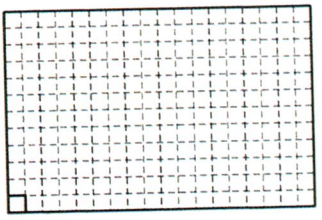
Factors of 18 

3 Factors and Common Factors

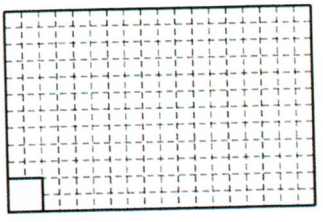




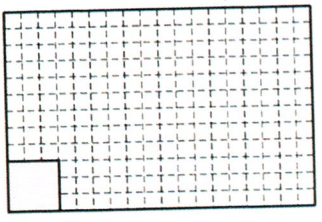




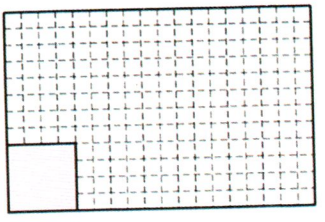
□ 1cm square



□ 2cm square

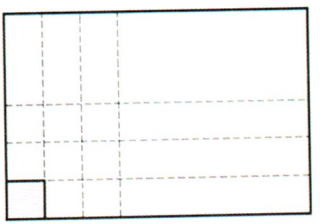


□ 3cm square



□ 4cm square

1 We want to cover a rectangular sheet that is 12cm long and 18cm wide using congruent colored square tiles.
How long should 1 side of the colored square tiles be in order to cover the sheet without any overlap or gaps?



▶ Least Common Multiple

3 Let's think about how we can find common multiples of 4 and 6!



Kazuya: List the multiples of 4, and the multiples of 6.

Multiples of 4 ····· 4, 8, (12), 16, 20, (24), ·····

Multiples of 6 ····· 6, (12), 18, (24), 30, 36, ·····



Mami: Divide the multiples of 6 by 4.

Multiples of 6 ····· 6, (12), 18, (24), 30, (36), ·····

Can be divided by 4 × ○ × ○ × ○

The common multiples of 4 and 6 are those multiples of 6 that result in whole number quotients with no remainder when divided by 4.

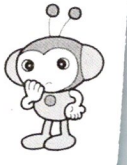
Among all the common multiples, the smallest number is called the **Least Common Multiple** (**LCM**).

1 What is the Least Common Multiple of 4 and 6?

1 Please find the 3 smallest common multiples and the Least Common Multiple for the number in ( ).

(1) (6, 8) (2) (4, 8) (3) (3, 7)

2 Please find the common multiples of 2, 3, and 4.



1 Circle the numbers that are multiples of 2, 3, 4, 5 and 6 on the number lines below.

Also find the common multiples of 3 and 4, and the common multiples of 4 and 5.



2 Let's think about how to find multiples!

1 Which of the following are multiples of 4?

20 34 48 50 56

Multiples of 4 are numbers that can be divided by 4.

1 Which numbers are multiples of 7?

7 16 21 32 65 84

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

2, 4, 6, ⋯ are 1, 2, 3, ⋯ times 2.

Multiples of 2 are numbers that are produced by multiplying 2 with a whole number.

0 is not a multiple of any number.

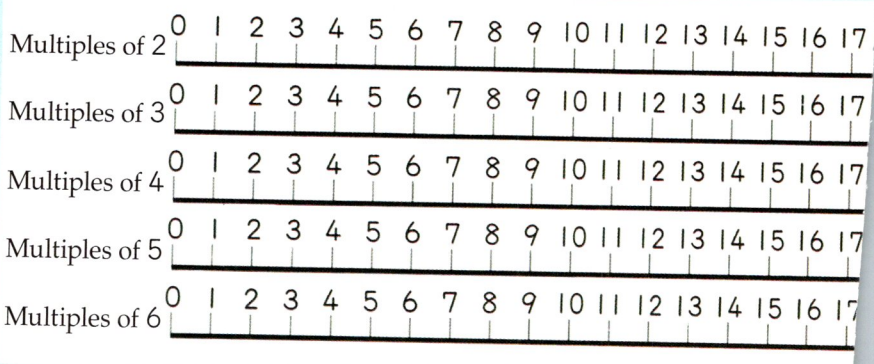
2 The numbers that represent width are multiples of which number?

3 How long is one side of the first square you can form? How long is one side of the square that is formed next?

When the numbers that represent the length of a square are 6, 12, 18, ..., the shapes are squares.

6, 12, 18, ... are multiples that are common to both 2 and 3.

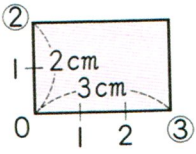
Multiples that are common to both multiples of 2 and 3, such as 6, 12, and 18, are called **common multiples** of 2 and 3.

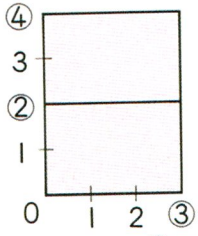


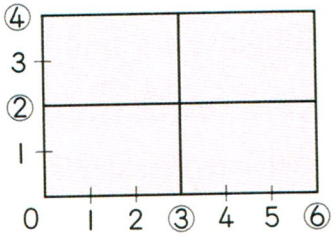
2 Multiples and Common Multiples

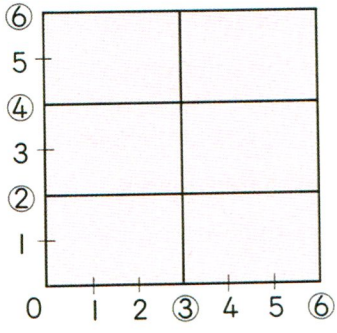


Using the cards shown on the right, make squares. All cards should have the same orientation.











Makoto

1 Rectangular cards that are 2cm long and 3cm wide are arranged with the same orientation. Under what circumstance do you have a square?

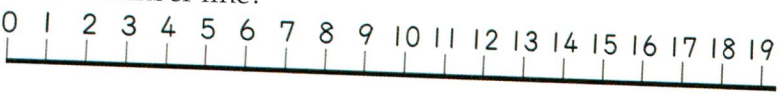
1 How does the length change when the cards are placed one above the other?
How does the width change when the cards are placed side by side?

Length \ Number of cards	1	2	3	4	5	6	7	8
Length (cm)	2	4						
Width (cm)	3	6						

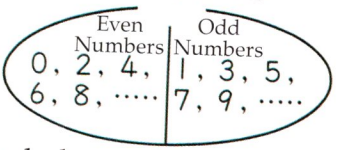
1 What type of whole numbers are on Team B? Let's try to divide them by 2.
What type of whole numbers are in Team A?

A whole number that can be divided by 2 is called an **even number**.
A whole number that cannot be divided by 2 is called an **odd number**.
0 is an even number.

2 How are even numbers and odd numbers arranged on a number line?



Whole numbers can be classified as even numbers or odd numbers.



1 Let's classify the following whole numbers as either even or odd numbers.

41 42 43 50 51 100

2 Please fill in the ☐ with the appropriate numbers.

Even Numbers $8 = 2 \times \square$ $22 = 2 \times \square$

Odd Numbers $9 = 2 \times \square + \square$ $23 = 2 \times \square + \square$

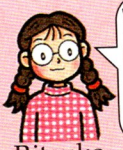


Minoru: Is every number either an even or odd number? Is 1638476 an even number or an odd number?



For playing dodge-ball, I want to split a class of 34 students into two teams. How can we divide them?





We can use our student numbers. Put the numbers 1-17 on one team and 18-34 on the other.

Ritsuko

How about starting with 1 and putting them on alternating teams?



Makoto

Let's investigate whole numbers from new points of view!

1 Even and Odd Numbers

1 A class of 34 students was divided into two teams, A and B.

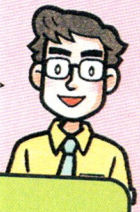
Let's find a rule for how teams A and B were formed!

Team A …①, ③, ⑤, ⑦, ⑨, ⑪, ⑬, …
Team B …②, ④, ⑥, ⑧, ⑩, ⑫, ⑭, …



Which team does number 15 belong to?

8 Characteristics of Whole Numbers

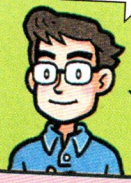


Whole numbers are used for classifying things.

Komachi 11	10:44	Akita	Komachi 24	10:30	Tokyo
Komachi 1	11:24	Akita	Komachi 16	11:01	Tokyo
Komachi 23	12:09	Akita	Komachi 46	12:29	Tokyo
Komachi 13	12:54	Akita	Komachi 4	13:15	Tokyo



I went to a train station to research train numbers and their departure times.

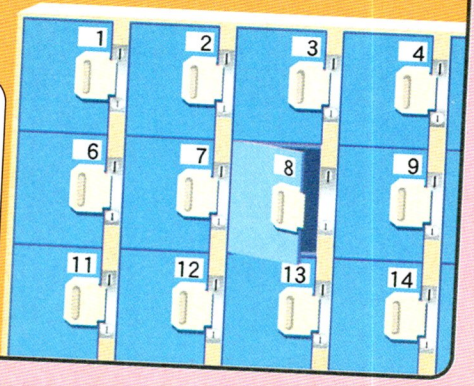


Can you find any rule for the train numbers?



Kazuya

As you go down, the locker numbers are increasing by 5 . . .





9 Per Unit Quantity



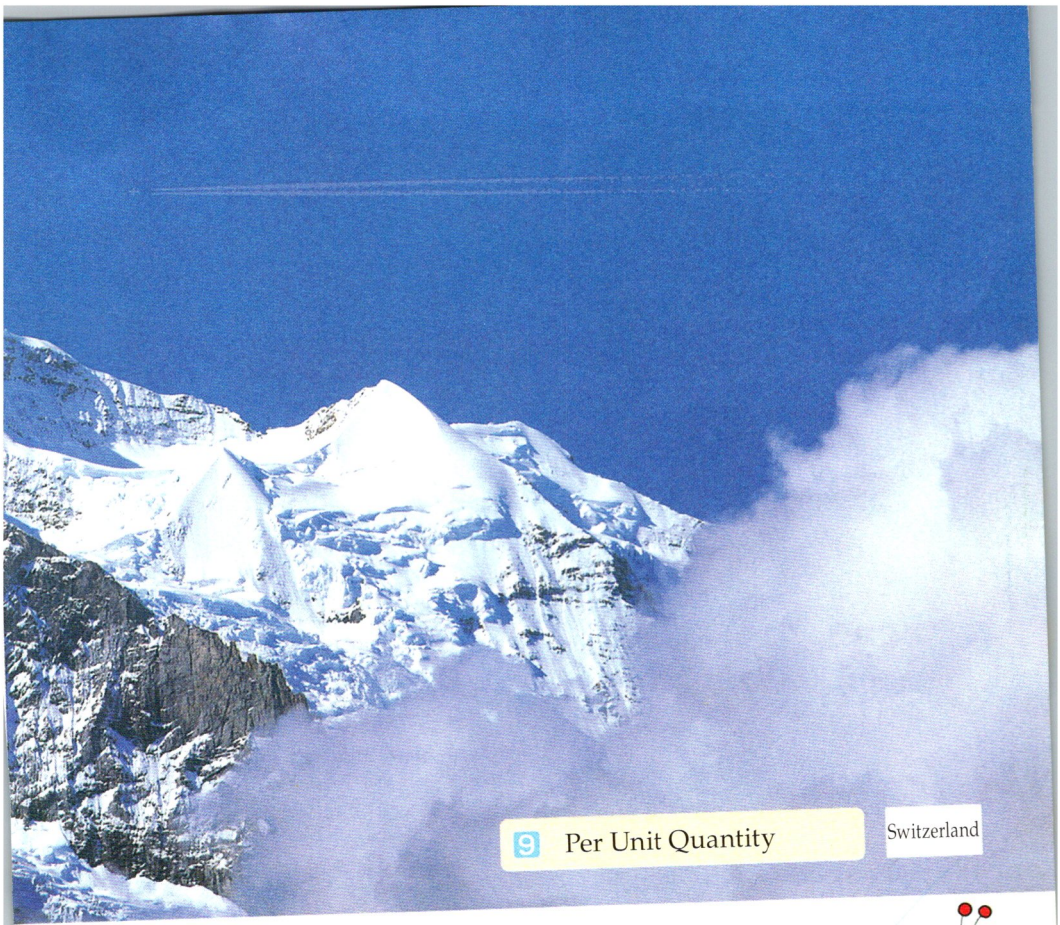


Sapporo,
Hokkaido

13 Circles and Regular Polygons







Our Earth 92

5th Grade Review 94

Here are the symbols used in this textbook.



- ..... New Problem
- ..... Aims of Learning
- ..... Hints for Thinking
- ..... Summary of Learning
- ..... Practice Problems

5 Grade 5A

1 Whole Numbers and Decimal Numbers
2 Volume
3 Multiplication of Decimal Numbers
4 Division of Decimal Numbers
5 Congruent Shapes
6 Area of Quadrilaterals and Triangles
7 Variables and Mathematical Equations

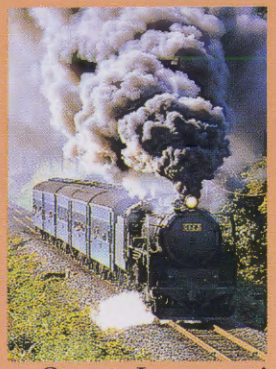
Various Kinds of Speed



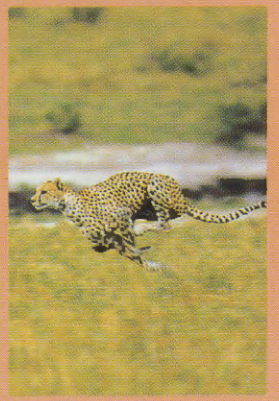
● Bicycle: 13*km* per hour



● Marathon:
16*km* per hour

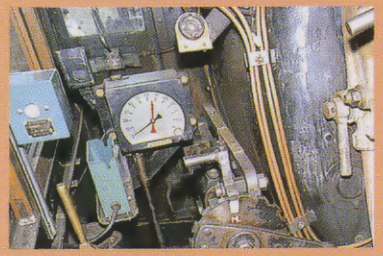


● Steam Locomotive:
50*km* per hour



● Cheetah:
100*km* per hour

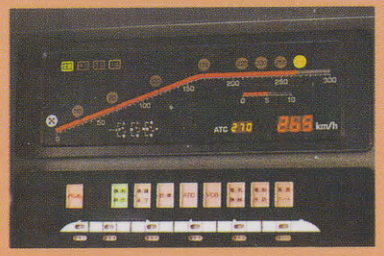
Various Kinds of Speedometers



● Steam Locomotive



● Jet Plane



● Shinkansen
(Bullet Train)

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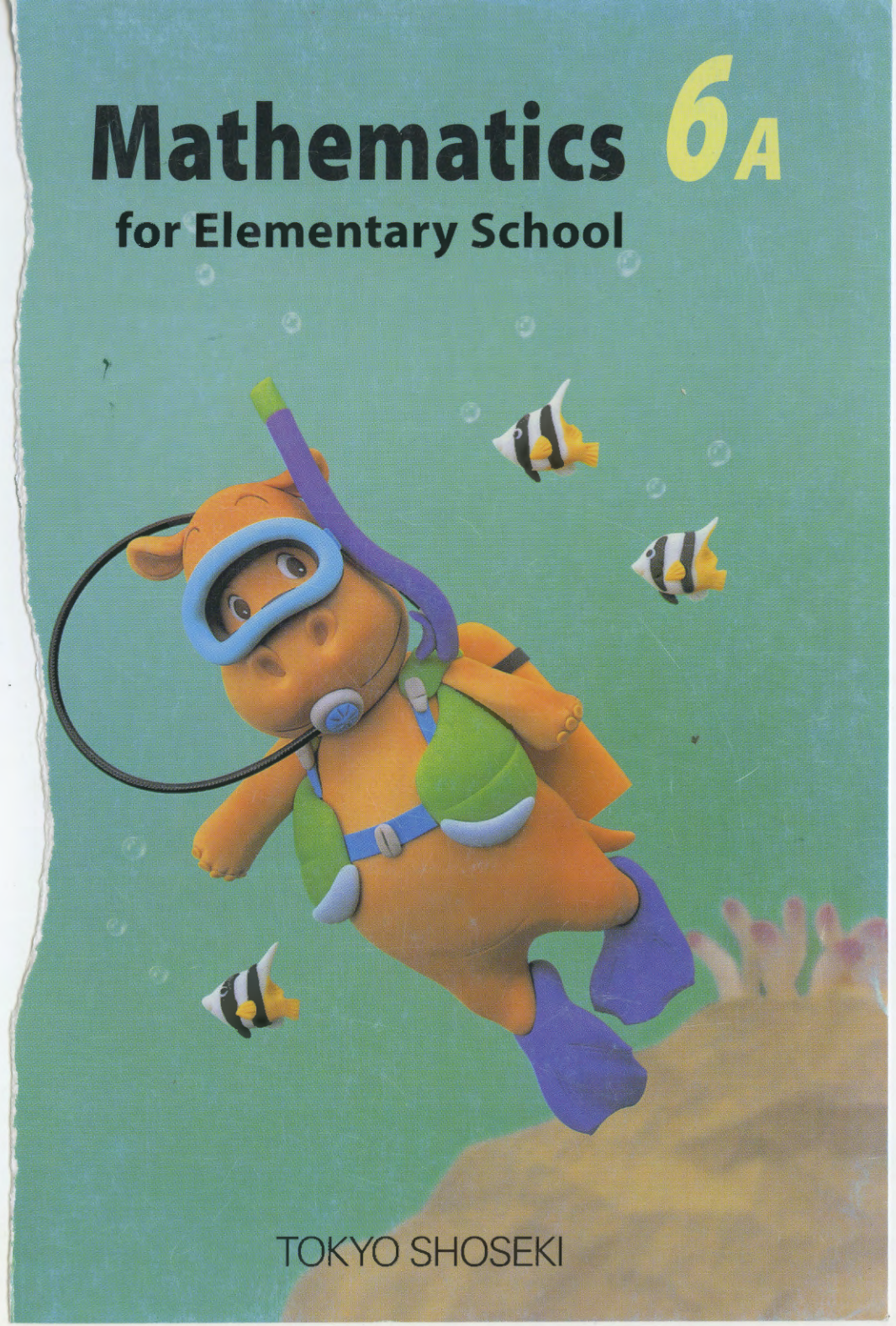
TOKYO SHOSEKI

Mathematics 6A

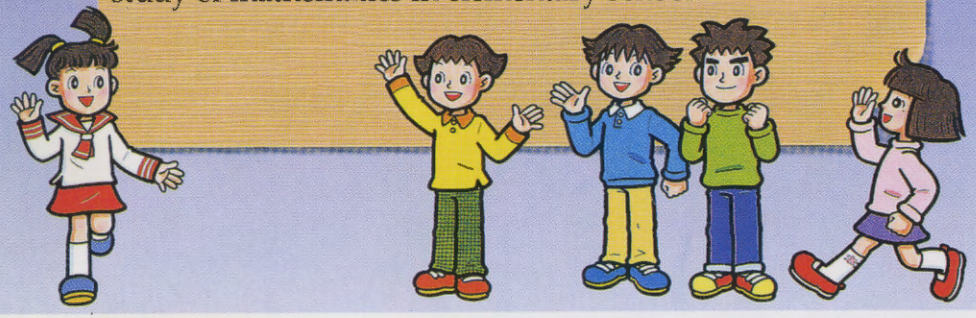
for Elementary School



TOKYO SHOSEKI

There is only one more year left in elementary school. In the 6th grade, you will learn new things in mathematics class. But you also will have the opportunity to reflect on and summarize what you have learned up to the 5th grade. While you are doing this, maybe you will discover new things.

Based on what you learn, it is good to ask yourself, "Why is it like this?" and "What can I use to think about this problem?" with this kind of attitude, let's complete your study of mathematics in elementary school!



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11 Number of Cases

12 How to Analyze Data

13 Units of Measurement

14 Various Graphs

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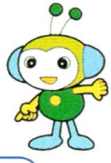


 ..... New Problem

 ..... Aims of Learning

 ..... Hints for Thinking

 ..... Summary of Learning

 ..... Practice Problems

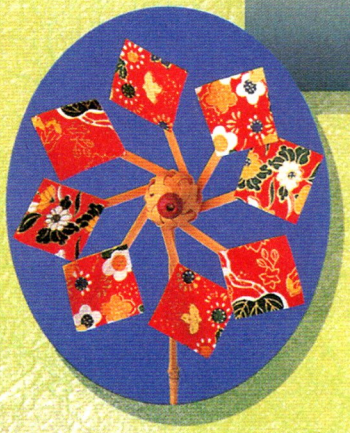


Fukuoka City,
Fukuoka Prefecture

4 Symm



Yamanakako Village,
Yamanashi Prefecture





Osaka City,
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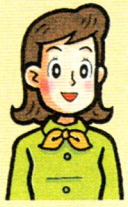
Shapes



Urawa City,
Saitama Prefecture

1 Multiplication and Division of Fractions



What kind of calculations have you learned so far?



Naoko

$946 + 758$ $\quad 1235 - 764$ $\quad 278 \times 175$ $\quad 342 \div 21$

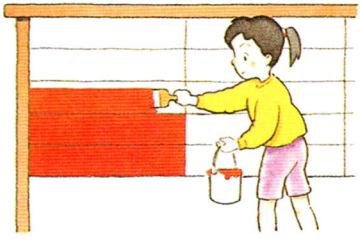
$2.64 + 1.73$ $\quad 3.64 - 2.76$ $\quad 2.17 \times 2.8$ $\quad 8.28 \div 3.6$

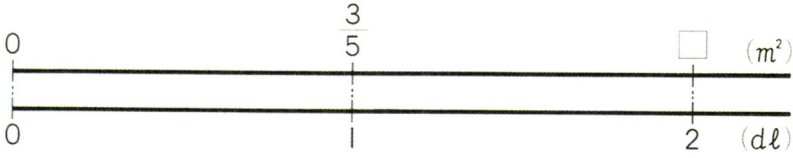
$2\frac{1}{3} + 1\frac{4}{5}$ $\quad 2\frac{1}{6} - 1\frac{2}{3}$



Makoto

1 With $1d\ell$ of paint you can paint $\frac{3}{5}m^2$ of boards. How many m^2 can you paint with $2d\ell$ of paint?

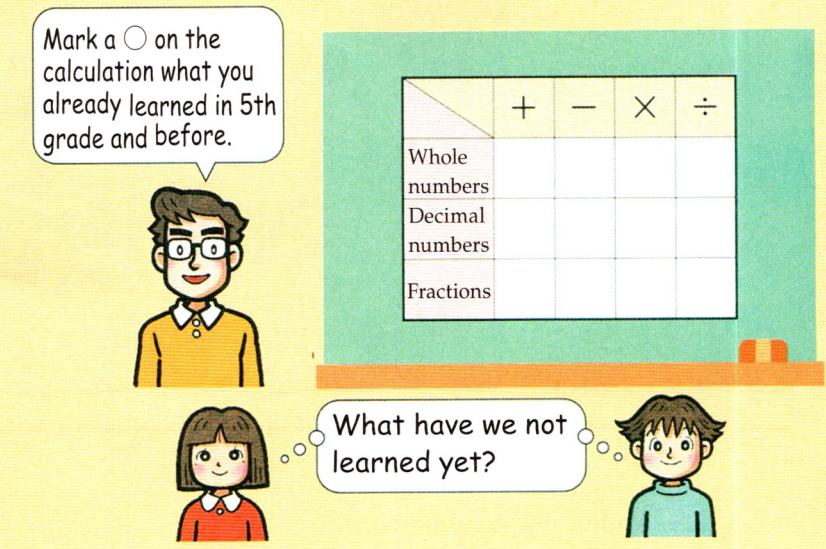




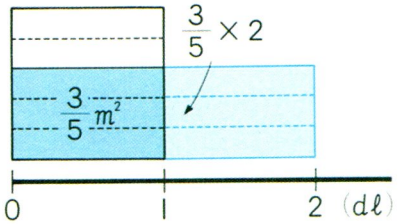
1 What kind of math sentence should you write?

Math Sentence

Let's think about how to calculate this!



Let's think about multiplication and division of fractions!



$$\frac{3}{5} \times 2 = \frac{3 \times 2}{5}$$

$$= \frac{6}{5} = 1\frac{1}{5}$$

Answer: $1\frac{1}{5}\ m^2$

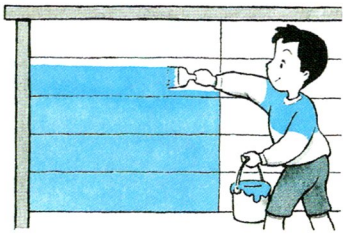
When you multiply a fraction by a whole number, keep the denominator the same and multiply the numerator by the whole number.

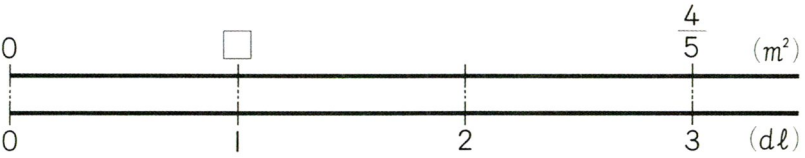
$$\frac{b}{a} \times c = \frac{b \times c}{a}$$

1 (1) $\frac{1}{5} \times 3$ (2) $\frac{2}{9} \times 4$ (3) $\frac{6}{7} \times 2$ (4) $\frac{3}{10} \times 9$

2

With $3d\ell$ of paint you can paint $\frac{4}{5}m^2$ of boards.
How many m^2 can you paint with $1d\ell$ of paint?

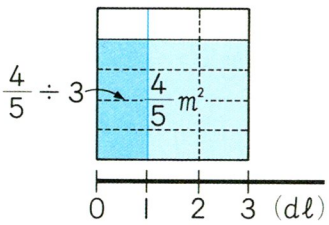




1 What kind of math sentence should you write?

Math Sentence ▶ [ ]

Let's think about how to calculate!



$$\frac{4}{5} \div 3 = \frac{4}{5 \times 3}$$

$$= \frac{4}{15}$$

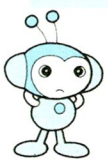
Answer: $\frac{4}{15}m^2$

When you divide a fraction by a whole number, keep the numerator the same and multiply the denominator by the whole number.

$$\frac{b}{a} \div c = \frac{b}{a \times c}$$

1

(1) $\frac{2}{3} \div 5$ (2) $\frac{1}{6} \div 6$ (3) $\frac{3}{4} \div 8$ (4) $\frac{7}{10} \div 4$

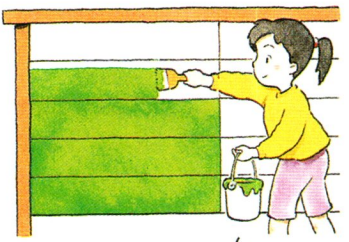


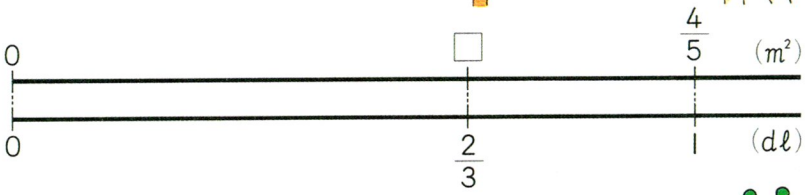
Next, we will think about multiplying and dividing two fractions.

2 Multiplication of Fractions

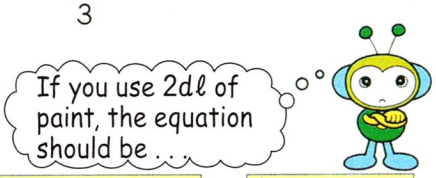
1 Multiplying by a Fraction

1 With $1d\ell$ of paint you can paint $\frac{4}{5}m^2$ of boards.
How many m^2 can you paint with $\frac{2}{3}d\ell$ of paint?





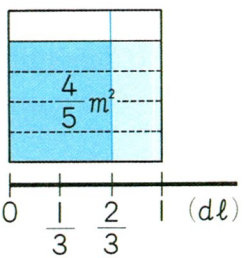
1 What kind of math sentence should you write?



Area that can be painted with $1d\ell$	×	Amount you use (in $d\ell$)	=	Area you can paint

Math Sentence ▶ ☐

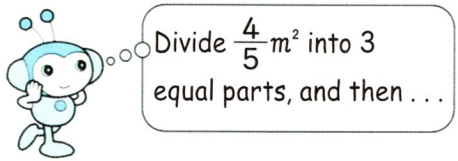
Even though the amount used is expressed as a fraction, to find the area you can paint, multiply in the same way as with whole numbers.

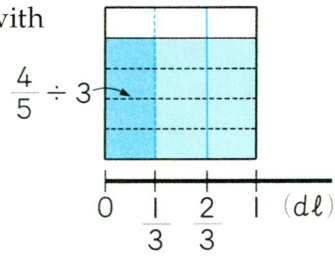


2 Let's think about how to calculate this!

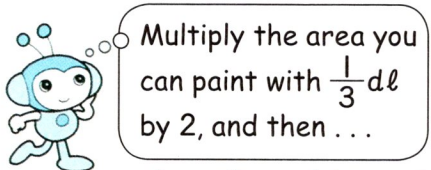
You can find the area that can be painted with $\frac{2}{3}$ dℓ of paint by first finding the area that can be painted with $\frac{1}{3}$ dℓ and then multiplying it by 2.

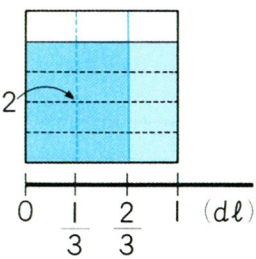
2 How many m^2 can you paint with $\frac{1}{3}$ dℓ?





3 How many m^2 can you paint with $\frac{2}{3}$ dℓ?





$$\frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2$$

$$= \frac{4}{5 \times 3} \times 2$$

$$= \frac{\square \times \square}{\square \times \square} = \square$$

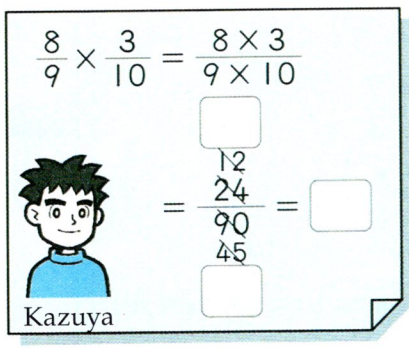
Answer: $\square$ m^2

When you multiply a fraction by another fraction, multiply the two denominators together and the two numerators together.

$$\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$$

1 (1) $\frac{7}{8} \times \frac{3}{4}$ (2) $\frac{4}{9} \times \frac{1}{3}$ (3) $\frac{1}{4} \times \frac{3}{5}$ (4) $\frac{9}{7} \times \frac{5}{8}$

2 Let's think about a better way to calculate $\frac{8}{9} \times \frac{3}{10}$!



When you can simplify fractions in the middle of the calculation, it is easier to simplify first and then continue with the calculation.

1 (1) $\frac{6}{7} \times \frac{1}{4}$ (2) $\frac{3}{14} \times \frac{7}{8}$ (3) $\frac{3}{8} \times \frac{4}{9}$ (4) $\frac{5}{12} \times \frac{9}{10}$

3 Let's think about how to calculate $3 \times \frac{2}{5}$ and $\frac{2}{7} \times 5$!

$$3 \times \frac{2}{5} = \frac{3}{1} \times \frac{2}{5}$$
$$= \frac{3 \times 2}{1 \times 5}$$
$$= \frac{6}{5} = 1\frac{1}{5}$$



$$\frac{2}{7} \times 5 = \frac{2}{7} \times \frac{5}{1}$$
$$= \frac{2 \times 5}{7 \times 1}$$
$$= \frac{10}{7} = 1\frac{3}{7}$$

1 (1) $5 \times \frac{3}{8}$ (2) $12 \times \frac{8}{15}$ (3) $\frac{2}{7} \times 4$ (4) $\frac{2}{9} \times 6$

4 Let's think about how to calculate $1\frac{3}{4} \times 2\frac{1}{3}$!

$$1\frac{3}{4} \times 2\frac{1}{3} = \frac{7}{4} \times \frac{7}{3}$$

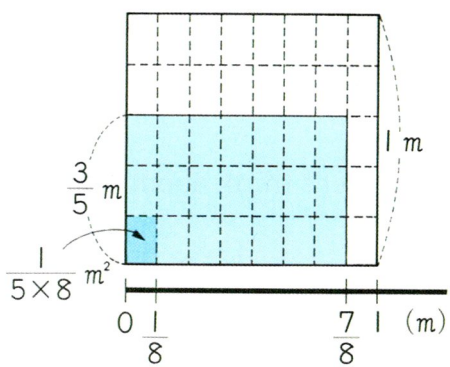
$$= \frac{7 \times 7}{4 \times 3}$$

$$= \frac{49}{12} = \square\frac{\square}{12}$$

To multiply mixed numbers, first convert mixed numbers to improper fractions and then multiply them the same way as you multiply proper fractions.

1 (1) $1\frac{1}{4} \times 2\frac{1}{6}$ (2) $1\frac{2}{5} \times \frac{7}{8}$ (3) $\frac{5}{7} \times 4\frac{2}{3}$

5 A rectangular sheet of paper is $\frac{3}{5}$ m long and $\frac{7}{8}$ m wide. Let's find the area of this paper!

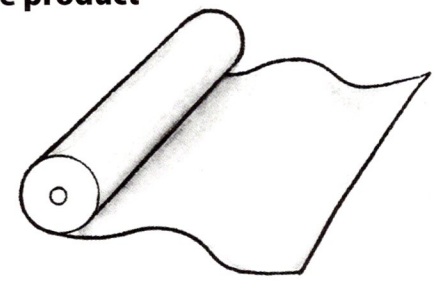


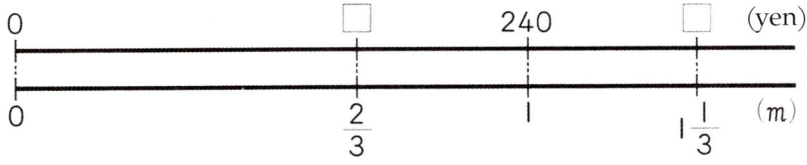
$$\frac{3}{5} \times \frac{7}{8} = \square\ (m^2)$$

Even if the length of each side is expressed as a fraction, the area of a rectangle or a square can be found by multiplying the same way as you multiply whole numbers.

▶ Multipliers and the size of the product

6 1m of cloth costs 240 yen. What is the price of $1\frac{1}{3}$ m of the same cloth? What is the price of $\frac{2}{3}$ m?





1 Please write a math sentence and find the answer.

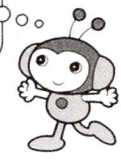
$240 \times 1\frac{1}{3} =$ ☐ $\quad 240 \times \frac{2}{3} =$ ☐

? Let's investigate the relationship between the multiplier and the size of the product!

2 In the 2 equations above, which one has a product that is less than the multiplicand of 240?

! When multiplying by a fraction, if the multiplier is less than 1, then the product will be less than the multiplicand.

Multiplier >1 → Product $>$ Multiplicand
Multiplier <1 → Product $<$ Multiplicand



1 Which math sentences will have a product that is less than a?

Ⓐ $a \times \frac{3}{4}$ Ⓑ $a \times \frac{7}{5}$ Ⓒ $a \times \frac{1}{2}$ Ⓓ $a \times 2\frac{5}{6}$

▶ Multiplication of 3 fractions

7 Let's think about how to calculate $\frac{3}{4} \times \frac{5}{9} \times \frac{2}{5}$!

$$\frac{3}{4} \times \frac{5}{9} \times \frac{2}{5} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}} \times \underset{1}{\cancel{5}}}$$

$$= \square$$

When multiplying several fractions, you can multiply all of the denominators together and all of the numerators together.

1 (1) $\frac{3}{4} \times \frac{5}{6} \times \frac{2}{3}$ (2) $3\frac{1}{7} \times \frac{5}{11} \times \frac{4}{5}$

(3) $\frac{2}{7} \times 5 \times 1\frac{1}{15}$

I can think of 5 as $\frac{5}{1}$.



▶ Properties of calculation

8 Please investigate whether the same properties of calculation that we found for multiplication of whole numbers hold for fractions.

(1) $a \times b = b \times a$

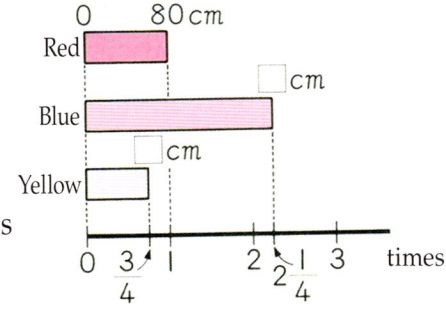
(2) $(a \times b) \times c = a \times (b \times c)$

(3) $(a + b) \times c = a \times c + b \times c$

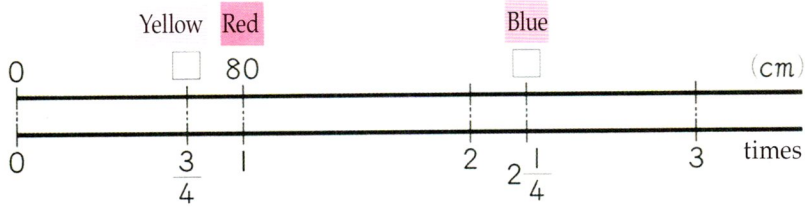
1 Please investigate the properties by substituting fractions for a, b, and c.

2 Times as Much and Multiplication

1 There are red, blue, and yellow ribbons. The red ribbon is 80cm long. The blue ribbon is $2\frac{1}{4}$ times as long as the red ribbon, and the yellow ribbon is $\frac{3}{4}$ times as long as the red ribbon. What is the length of the blue and yellow ribbons in cm?



Let's think about what kinds of calculations you need to do!



Blue $80 \times 2\frac{1}{4} = \square$ Answer: $\square$ cm

Yellow $80 \times \frac{3}{4} = \square$ Answer: $\square$ cm

If we consider 80cm as 1, the length that corresponds to $2\frac{1}{4}$ is 180cm and the length that corresponds to $\frac{3}{4}$ is 60cm.

Also in the case of fractions, how many times as much (or many) can be found by multiplication.

1 How many kg is $3\frac{3}{5}$ times 10kg?

Practice

① Please calculate the following.

(1) $\frac{3}{5} \times \frac{1}{7}$ (2) $\frac{4}{7} \times \frac{3}{16}$ (3) $\frac{5}{6} \times \frac{12}{25}$ (4) $\frac{3}{8} \times \frac{7}{12}$

(5) $\frac{8}{9} \times \frac{3}{7}$ (6) $\frac{6}{7} \times \frac{14}{15}$ (7) $3 \times \frac{3}{7}$ (8) $1\frac{3}{16} \times 24$

(9) $2\frac{6}{11} \times 4\frac{1}{8}$ (10) $3\frac{1}{3} \times \frac{5}{6}$ (11) $\frac{5}{12} \times 1\frac{4}{5}$

(12) $\frac{1}{3} + \frac{7}{9} \times \frac{3}{14}$ (13) $4\frac{2}{3} - 1\frac{2}{7} \times 1\frac{1}{6}$

② $1kg$ of rice contains about $\frac{3}{4}kg$ of starch.
How many kilograms of starch are in $2\frac{4}{5}kg$ of rice?

③ Takeshi had 840 yen. He spent $\frac{4}{7}$ of his money on a sketchbook.
How much did the sketchbook cost?



④ $1m$ of pipe weighs $1\frac{6}{7}kg$. Let's use this information to make a problem for the math sentence $1\frac{6}{7} \times \frac{7}{8}$!

Challenge

Let's Create Fraction Multiplication Sentences!

Please complete the math sentence below by using the cards on the right.

$\frac{\square}{\square} \times \frac{\square}{\square} = 6$



Check

1 Please fill in the ☐ with the appropriate numbers.

$$1\frac{2}{3} \times \frac{2}{7} = \frac{\square}{3} \times \frac{\square}{7}$$

$$= \frac{\square \times \square}{3 \times 7} = \square$$

2 Which one has a product less than 3?

$3 \times \frac{2}{7}$ $3 \times 1\frac{1}{2}$ $3 \times \frac{5}{4}$ $3 \times \frac{14}{15}$

3 Please calculate the following.

(1) $\frac{1}{2} \times \frac{3}{5}$ (2) $\frac{3}{7} \times \frac{8}{7}$ (3) $\frac{5}{8} \times \frac{4}{15}$ (4) $\frac{5}{9} \times \frac{3}{10}$

(5) $1\frac{1}{3} \times 2\frac{2}{5}$ (6) $1\frac{5}{7} \times \frac{4}{5}$ (7) $6 \times 3\frac{1}{9}$



At the end of the lesson:

Please write down the following points in your notebook.

- ◎ What you understood and what was good
- ◎ What resembled things that you already knew
- ◎ Your friends' ideas that you thought were good
- ◎ What you want to learn in the future



Review・1

① Please convert the decimals to fractions and the fractions to decimals.

(1) 0.7 (2) 2.3 (3) $\frac{4}{5}$ (4) $\frac{3}{8}$ (5) $\frac{6}{25}$

② Please calculate the following.

(1) $\frac{3}{5}+\frac{1}{3}$ (2) $\frac{5}{7}+\frac{2}{9}$ (3) $1\frac{1}{6}+\frac{7}{8}$ (4) $2\frac{5}{6}+1\frac{7}{10}$

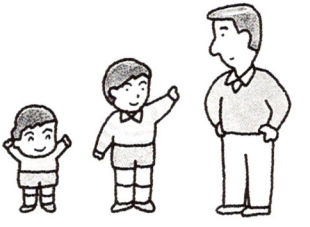
(5) $\frac{6}{7}-\frac{2}{5}$ (6) $1\frac{1}{6}-\frac{8}{9}$ (7) $2\frac{2}{3}-1\frac{8}{9}$ (8) $3\frac{1}{2}-1\frac{5}{6}$

③ Please calculate the following.

(1) $\frac{5}{7}\times 6$ (2) $\frac{4}{5}\times 2$ (3) $\frac{3}{8}\times 4$ (4) $\frac{7}{6}\times 9$

(5) $\frac{3}{5}\div 7$ (6) $\frac{3}{7}\div 4$ (7) $\frac{2}{5}\div 6$ (8) $\frac{8}{5}\div 10$

④ Kenta's height is 150cm. His older brother is 175cm tall and his younger brother is 125cm tall.
How many times as tall are Kenta's older and younger brothers as Kenta? Express these heights in fractions.



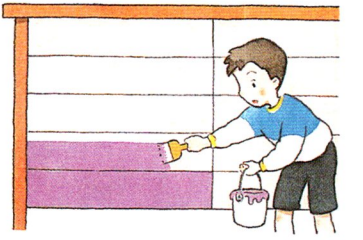
⑤ Noriko's household used 198.6m^3 of water last month. This month, they used 165.5m^3.
What is the relative value of last month's usage compared to this month's usage?

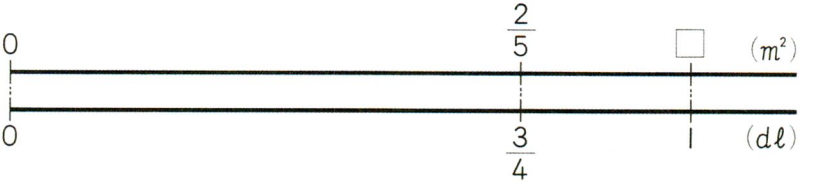


3 Division of Fractions

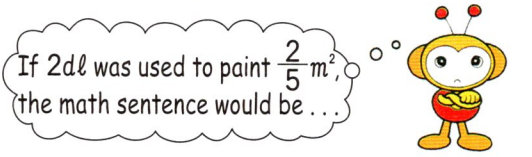
1 Dividing by a Fraction

1 With $\frac{3}{4}$ dℓ of paint you can paint $\frac{2}{5}$ m^2 of board.
How many m^2 can you paint with 1 dℓ of paint?





1 What math sentence should we write?

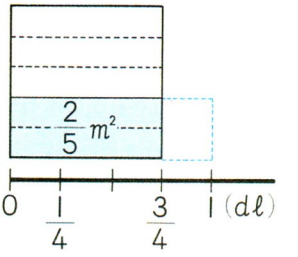


Area painted	÷	Amount used (dℓ)	=	Area that can be painted with 1dℓ

Math Sentence

To find the area you can paint with 1 dℓ, you can divide the same way as with whole numbers even though the amount is expressed as fraction.

Let's think about how to calculate!

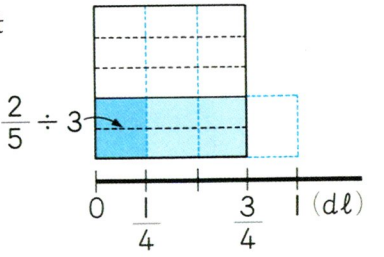


You can calculate the area that can be painted with $1\,d\ell$ by first finding the area that can be painted with $\frac{1}{4}\,d\ell$ and then multiplying it by 4.

2 How many m^2 can you paint with $\frac{1}{4}\,d\ell$?



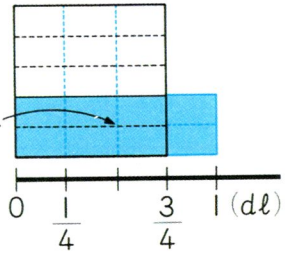
Divide $\frac{2}{5}\,m^2$ into 3 equal parts and then . . .



3 How many m^2 can you paint with $1\,d\ell$?



Multiply the area that you can paint with $\frac{1}{4}\,d\ell$ of paint by 4, then . . .



$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4$$

$$= \frac{2}{5 \times 3} \times 4$$

$$= \frac{\square \times \square}{\square \times \square} = \square$$

Answer: $\square$ m^2

4 What fraction can you multiply $\frac{2}{5}$ by to get the same calculation as above?

When you divide by a fraction, invert the denominator and the numerator of the divisor and then multiply the dividend by the resulting new fraction.

$$\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \times \frac{c}{d}$$

$$= \frac{b \times c}{a \times d}$$

1 (1) $\frac{5}{8} \div \frac{6}{7}$ (2) $\frac{1}{7} \div \frac{5}{6}$ (3) $\frac{4}{5} \div \frac{3}{8}$ (4) $\frac{3}{2} \div \frac{1}{3}$

2 Let's think about how to calculate $\frac{9}{10} \div \frac{3}{4}$!



Sayuri

$$\frac{9}{10} \div \frac{3}{4} = \frac{9}{10} \times \frac{4}{3}$$

$$= \frac{\overset{3}{\cancel{9}} \times \overset{2}{\cancel{4}}}{\underset{5}{\cancel{10}} \times \underset{1}{\cancel{3}}}$$

$$= \frac{\square}{5} = \square\frac{\square}{5}$$



1 (1) $\frac{6}{7} \div \frac{3}{5}$ (2) $\frac{9}{10} \div \frac{7}{15}$ (3) $\frac{4}{9} \div \frac{8}{15}$ (4) $\frac{3}{8} \div \frac{9}{14}$

3 Let's think about how to calculate $5 \div \frac{2}{3}$ and $\frac{5}{7} \div 3$!

$$5 \div \frac{2}{3} = \frac{5}{1} \div \frac{2}{3}$$

$$= \frac{5}{1} \times \frac{3}{2}$$

$$= \frac{15}{2} = 7\frac{1}{2}$$

$$\frac{5}{7} \div 3 = \frac{5}{7} \div \frac{3}{1}$$

$$= \frac{5}{7} \times \frac{1}{3}$$

$$= \frac{5}{21}$$



1 (1) $6 \div \frac{5}{7}$ (2) $15 \div \frac{10}{13}$ (3) $\frac{7}{8} \div 4$ (4) $\frac{3}{5} \div 6$

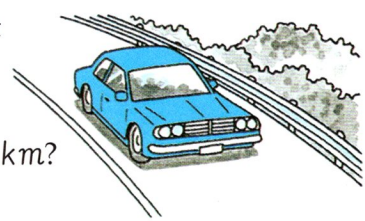
4 Let's think about how to calculate $1\frac{4}{5} \div 2\frac{1}{2}$!

$$1\frac{4}{5} \div 2\frac{1}{2} = \frac{9}{5} \div \frac{5}{2}$$
$$= \frac{9}{5} \times \frac{2}{5}$$
$$= \frac{9 \times 2}{5 \times 5} = \square$$

When dividing with mixed numbers, convert them to improper fractions and then divide the same way as with proper fractions.

1 (1) $2\frac{2}{3} \div 2\frac{1}{5}$ (2) $2\frac{1}{3} \div \frac{2}{5}$ (3) $\frac{5}{9} \div 2\frac{1}{7}$

2 How many minutes does it take for a car traveling $\frac{3}{5}$ km per minute to go $2\frac{7}{10}$ km?



Challenge

Let's Create Division Sentences with Fractions!

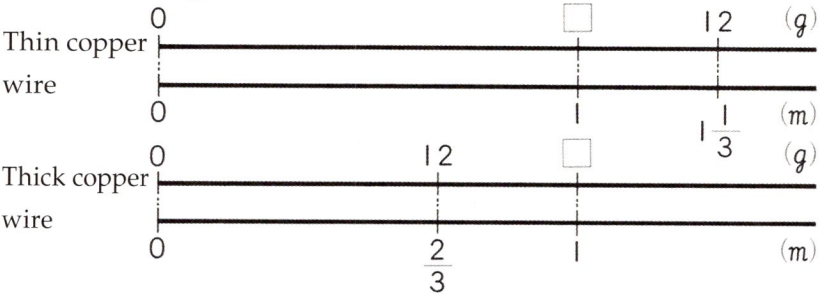
Let's complete the math sentence below by using the cards on the right!

$$\frac{\square}{\square} \div \frac{9}{\square} = \frac{\square}{\square}$$



▶ Divisors and the size of the quotient

5 $1\frac{1}{3}$ m of thin copper wire weighs $12g$ and $\frac{2}{3}$ m of thick copper wire weighs $12g$. How many g does $1m$ of each kind of wire weigh?



1 Please write math sentences and find the answers.

$12 \div 1\frac{1}{3} = \square$ $12 \div \frac{2}{3} = \square$

? Let's investigate the relationship between the divisor and the size of the quotient!

2 Of the two math sentences above, which one will have a quotient that is greater than the dividend 12?

! Even when dividing with fractions, if you divide by a number less than 1, the quotient will be greater than the dividend.

Divisor > 1 → Quotient < Dividend
Divisor < 1 → Quotient > Dividend



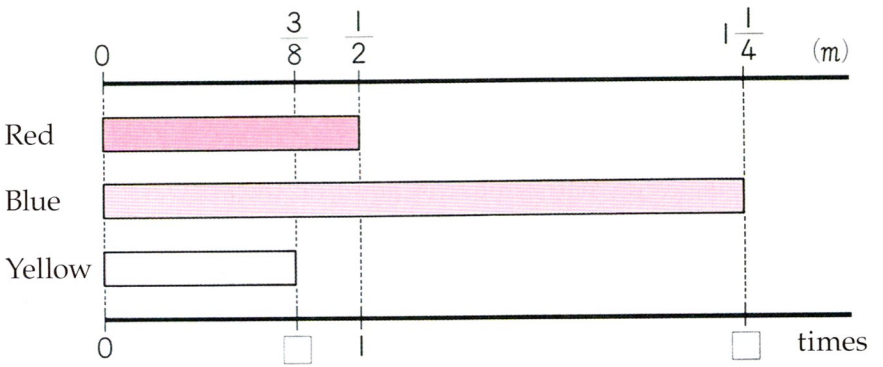
1 In which math sentences will the quotient be greater than a?

Ⓐ $a \div \frac{3}{7}$ Ⓑ $a \div \frac{9}{4}$ Ⓒ $a \div \frac{1}{5}$ Ⓓ $a \div 2\frac{2}{3}$

2 Times as Much and Division

1 There are red, blue, and yellow ribbons. The red ribbon is $\frac{1}{2}$ m long, the blue ribbon is $1\frac{1}{4}$ m, and the yellow ribbon is $\frac{3}{8}$ m.
How many times as long as the red ribbon are the blue ribbon and the yellow ribbon?

Let's think about what kinds of calculations you need to do!



Blue $1\frac{1}{4} \div \frac{1}{2} =$ ☐ Answer: ☐ times

Yellow $\frac{3}{8} \div \frac{1}{2} =$ ☐ Answer: ☐ times

If you consider $\frac{1}{2}$ m as 1, then $2\frac{1}{2}$ is the relative value for $1\frac{1}{4}$ m and $\frac{3}{4}$ is the relative value for $\frac{3}{8}$ m.

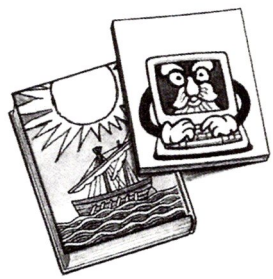
To determine how many times as much (or many) one quantity is of another, we can use division, even in the case of fractions.

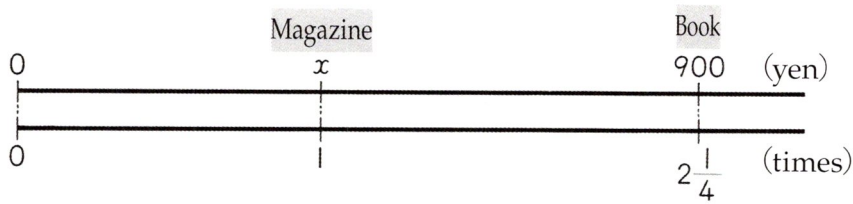
1 Please answer the following questions.

(1) How many times as much as $\frac{1}{3}$ kg is $1\frac{2}{9}$ kg?

(2) If you consider $1\frac{7}{8}$ ℓ as 1, what is the relative value of $4\frac{1}{6}$ ℓ?

2 Akira paid 900 yen to buy a book. The book's price is $2\frac{1}{4}$ times as much as the price of a magazine. How much does the magazine cost?





1 Let's consider the price of the magazine as x yen and write a multiplication sentence.

$$x \times 2\frac{1}{4} = 900$$

2 From the math sentence above, find the number that should be used for x.

1 You have milk and juice. The amount of juice is $4\frac{1}{2}$ dℓ and it is $\frac{3}{5}$ as much as the amount of milk. How many deciliters of milk do you have?

Practice

① Please fill in the ☐ with the symbols =, <, or >.

(1) $\frac{2}{5} \div 1 \square \frac{2}{5} \div \frac{5}{6}$ (2) $6 \div 1\frac{2}{7} \square \frac{6}{1} \div \frac{9}{7}$

② Please calculate the following.

(1) $\frac{2}{9} \div \frac{3}{8}$ (2) $\frac{5}{6} \div \frac{7}{12}$ (3) $\frac{2}{3} \div \frac{8}{9}$ (4) $\frac{2}{7} \div \frac{4}{9}$

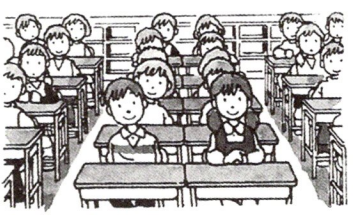
(5) $\frac{1}{2} \div \frac{7}{10}$ (6) $\frac{3}{4} \div \frac{2}{3}$ (7) $14 \div \frac{7}{15}$ (8) $3\frac{1}{2} \div 8$

(9) $1\frac{13}{15} \div 4\frac{1}{5}$ (10) $1\frac{5}{6} \div \frac{5}{9}$ (11) $\frac{4}{5} \div 1\frac{5}{7}$

(12) $\frac{6}{7} \div \frac{1}{5} + \frac{4}{5}$ (13) $8\frac{1}{6} \div 1\frac{2}{5} + 2\frac{1}{10}$

③ $4\frac{1}{5}\ell$ of rice weighs $3\frac{1}{2}kg$.
How many kg does 1ℓ of rice weigh?

④ There are 18 boys in Hiroshi's class. This is $\frac{9}{17}$ of the whole class.
How many students are in Hiroshi's class?



⑤ A clock runs 20 seconds too fast every week.
How many weeks will it take until this clock is running 5 minutes fast?

⑥ A pipe weighs $1\frac{6}{7}kg$. Please make a problem that uses the math sentence $1\frac{6}{7} \div \frac{7}{8}$.

3 Reciprocals and Fraction Calculation

▶ Reciprocals

1 $\frac{5}{6} \times \frac{6}{5} = 1$. In the same way, find the numbers below whose products are 1.

$\frac{2}{3}$	$\frac{7}{5}$	$\frac{1}{4}$	$\frac{5}{7}$	$\frac{3}{2}$	4

When the product of 2 numbers is 1, as in $\frac{2}{3}$ and $\frac{3}{2}$, $\frac{7}{5}$ and $\frac{5}{7}$, and $\frac{1}{4}$ and 4, we say that one number is the **reciprocal** of the other.

$\frac{2}{3} \times \frac{3}{2} = 1$	$\frac{7}{5} \times \frac{5}{7} = 1$	$\frac{1}{4} \times 4 = 1$

Reciprocals of proper fractions and improper fractions are fractions in which the denominators and numerators are switched.

The reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.
The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.



You can calculate $1 \div a$ to find the reciprocal of a.

1 Please verify this using $\frac{2}{3}$, $\frac{5}{7}$, and 4.

1 Please find the reciprocals of the following numbers.

(1) $\frac{5}{8}$ (2) $\frac{1}{3}$ (3) $\frac{13}{9}$ (4) $1\frac{2}{7}$ (5) 6 (6) 0.8

2 Find a fraction that fits in the ☐.

$$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \square$$

1 In the math sentence above, compare the divisor and the multiplier.

Dividing by the divisor is the same as multiplying by the reciprocal of the divisor.

1 Please fill in the ☐ with the appropriate numbers.

(1) $\frac{7}{8} \div \frac{4}{5} = \frac{7}{8} \times \frac{\square}{\square}$

(2) $\frac{3}{5} \div 7 = \frac{3}{5} \times \frac{\square}{\square}$

Story of Math

Changing Division of Fractions into Multiplication

150 ÷ 50 = 3

Divide by 10 Divide by 10 Equal

15 ÷ 5 = 3

150 ÷ 50 = 3

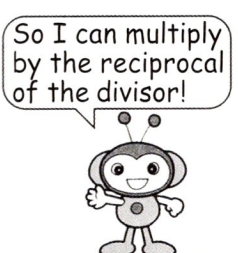
Multiply by 10 Multiply by 10 Equal

15 ÷ 5 = 3

Division has the properties shown above.

In $\frac{7}{8} \div \frac{3}{5}$, to make the divisor equal to 1, multiply both $\frac{7}{8}$ and $\frac{3}{5}$ by the reciprocal of $\frac{3}{5}$.

$$\begin{aligned} \frac{7}{8} \div \frac{3}{5} &= \left(\frac{7}{8} \times \frac{5}{3}\right) \div \left(\frac{3}{5} \times \frac{5}{3}\right) \\ &= \left(\frac{7}{8} \times \frac{5}{3}\right) \div 1 \\ &= \frac{7}{8} \times \frac{5}{3} \end{aligned}$$



3 Let's figure out a way to calculate $\frac{3}{4} \div \frac{2}{3} \times \frac{1}{5}$ more efficiently!

$$\frac{3}{4} \div \frac{2}{3} \times \frac{1}{5} = \frac{3}{4} \times \frac{3}{2} \times \frac{1}{5}$$

$$= \frac{3 \times 3 \times 1}{4 \times 2 \times 5} = \square$$

If an equation contains both multiplication and division of fractions, you can convert the divisor into its reciprocal to write a math sentence with only multiplication.

1 (1) $\frac{2}{9} \div \frac{4}{7} \div \frac{5}{6}$ (2) $\frac{2}{3} \times \frac{1}{6} \div 1\frac{5}{9}$ (3) $2\frac{2}{7} \div 9 \times \frac{3}{8}$

4 Let's calculate $0.9 \div 8 \times 4 \div 2.1$ by first converting the decimals and whole numbers into fractions!

$$0.9 \div 8 \times 4 \div 2.1 = \frac{9}{10} \div \frac{8}{1} \times \frac{4}{1} \div \frac{21}{10}$$

$$= \frac{9}{10} \times \frac{1}{8} \times \frac{4}{1} \times \frac{10}{21}$$

$$= \frac{\overset{3}{\cancel{9}} \times \overset{1}{\cancel{4}} \times \overset{1}{\cancel{10}}}{\underset{1}{\cancel{10}} \times \underset{2}{\cancel{8}} \times \underset{7}{\cancel{21}}} = \square$$

If a math sentence contains both multiplication and division of decimal numbers and whole numbers, sometimes it is easier if you convert it into a fraction multiplication sentence.

1 Please convert the decimal numbers and whole numbers to fractions and then calculate.

(1) $2.7 \div 1.5$ (2) $3.75 \div 7 \times 4.2$ (3) $42 \times 4 \div 8 \div 3$

Check

1 Please fill in the ☐ with the appropriate numbers.

$$\frac{4}{7} \div 1\frac{2}{3} = \frac{4}{7} \div \frac{\square}{3}$$

$$= \frac{4}{7} \times \frac{\square}{\square}$$

$$= \frac{4 \times \square}{7 \times \square} = \square$$

2 Which one has a quotient that is greater than 3?

Ⓐ $3 \div \frac{2}{7}$ Ⓑ $3 \div 1\frac{1}{2}$ Ⓒ $3 \div \frac{5}{4}$ Ⓓ $3 \div \frac{14}{15}$

3 Please calculate the following.

(1) $\frac{1}{2} \div \frac{3}{5}$ (2) $\frac{5}{9} \div \frac{7}{15}$ (3) $\frac{3}{8} \div \frac{5}{6}$ (4) $\frac{4}{7} \div \frac{8}{21}$

(5) $2\frac{1}{7} \div 6\frac{1}{4}$ (6) $1\frac{7}{8} \div \frac{3}{4}$ (7) $15 \div 1\frac{1}{3}$

Challenge

Let's Make Math Sentences!

Please use 3 of the cards on the right and +, −, ×, ÷ and ( ) to make these math sentences.



☐ ☐ ☐ = 1 ☐ ☐ ☐ = 4

☐ ☐ ☐ = 2 ☐ ☐ ☐ = 5

☐ ☐ ☐ = 3 ☐ ☐ ☐ = 6

What Calculation Are We Going to Use?

Kiyoshi and his uncle went for a ride.

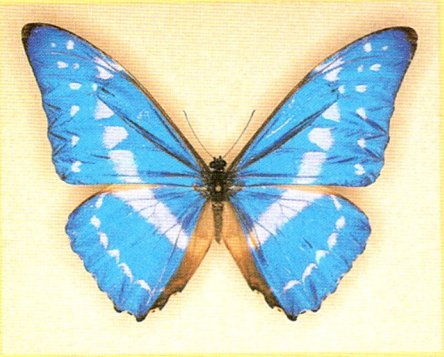
① The car can go $9\frac{1}{5}$ km on 1ℓ of gasoline. When the trip started, the car had 20ℓ of gasoline. How far could they go?

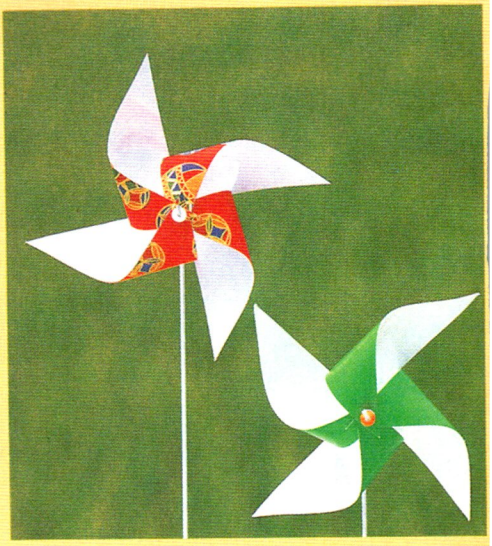
② By the time they reached C, they had used $6\frac{2}{3}$ ℓ of gas. How many km did they travel on 1ℓ of gas between A and C?

③ Between C and D, they traveled at a rate of $1\frac{1}{3}$ km per minute. How many minutes did it take?

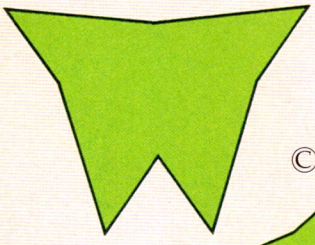
④ From D, they traveled at a rate of $1\frac{1}{4}$ km per minute for 48 minutes. How far did they go?

4 Symmetric Shapes

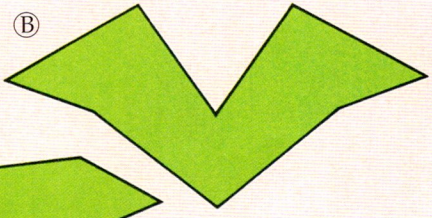




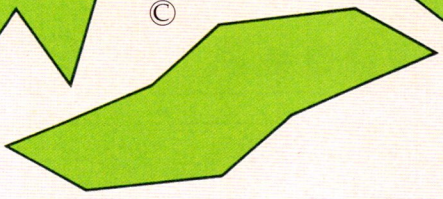
Ⓐ

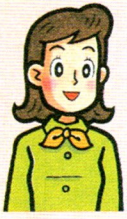


Ⓑ



Ⓒ

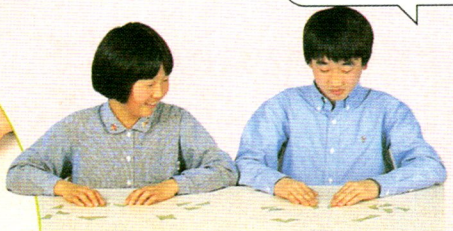




Cut out the congruent shapes on page 98. Then put the two congruent shapes together by matching the sides with the same lengths to make different shapes.

Let's sort the shapes you made into two groups!

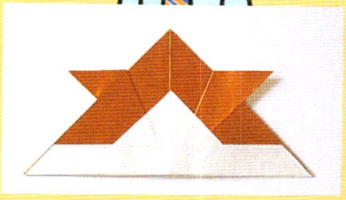


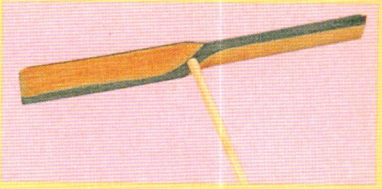


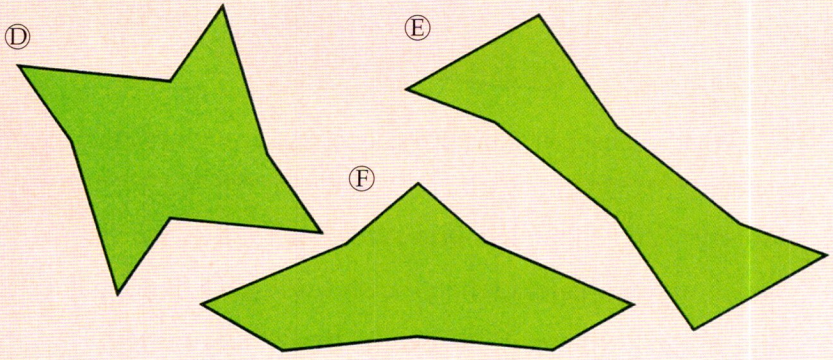


You can find many things around you whose shapes are balanced and beautiful.





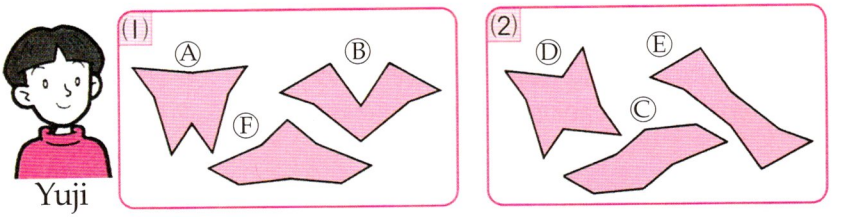




(1) Ⓐ

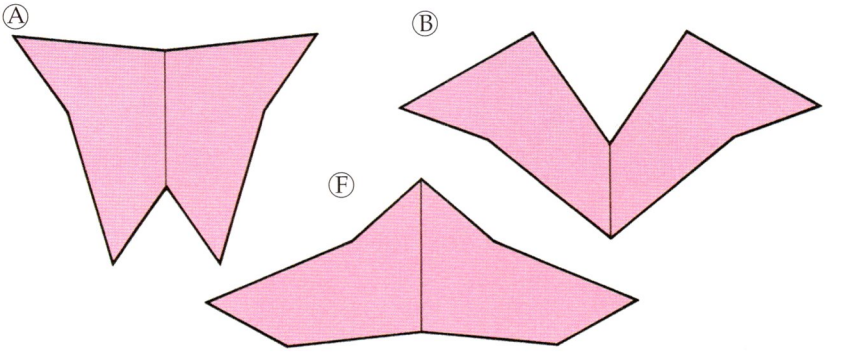
(2) Ⓓ

Let's explore the characteristics of shapes that are balanced and beautiful!



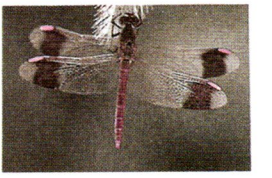
1 Line Symmetry

1 Let's investigate the characteristics of the shapes in group (1)!



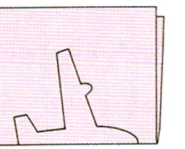
1 What will happen to the two sides of the shapes if you fold them?

We say that a shape has **line symmetry** if the two sides match up completely when the shape is folded along a line. We call this straight line the **line of symmetry.**



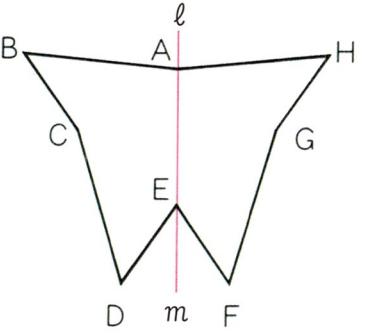
1 Let's find some shapes that have line symmetry around you!

2 Let's draw half shapes on sheets of paper that are folded once and then cut out the shapes to make different shapes with line symmetry!



Properties of shapes with line symmetry

2 The diagram on the right shows a shape with line symmetry. Fold it along the line of symmetry ℓm and investigate the vertices, sides and angles that match up.



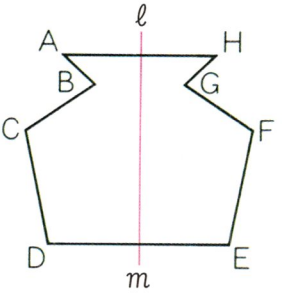
1 Which vertex will match up with B? Also, which side will match up with BC?

In line symmetry, we call the vertices, sides and angles that match up, corresponding vertices, corresponding sides, and corresponding angles.

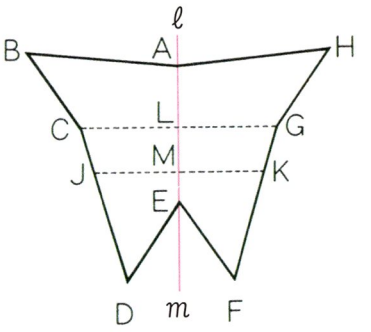
2 In the shape above, please compare the lengths of the corresponding sides and the measures of the corresponding angles.

In figures with line symmetry, the lengths of corresponding sides are equal and the measures of corresponding angles are equal.

1 The diagram on the right shows a shape that has line symmetry. Let's identify the corresponding vertices and the corresponding sides!



3 The diagram below shows a shape that has line symmetry. Let's explore how lines connecting corresponding vertices will intersect the line of symmetry!



1 How do line CG and line JK intersect with the line of symmetry ℓm?

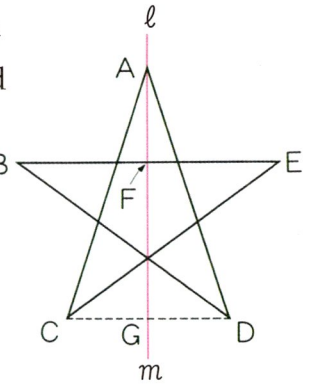
2 Let's compare the length of CL with the length of GL! Also, let's compare the length of JM with the length of KM!

In a shape that has line symmetry, lines connecting corresponding vertices intersect the line of symmetry perpendicularly. The distances from the point of intersection to the corresponding vertices are equal.

1 The diagram on the right shows a shape that has line symmetry and whose line of symmetry is ℓm.

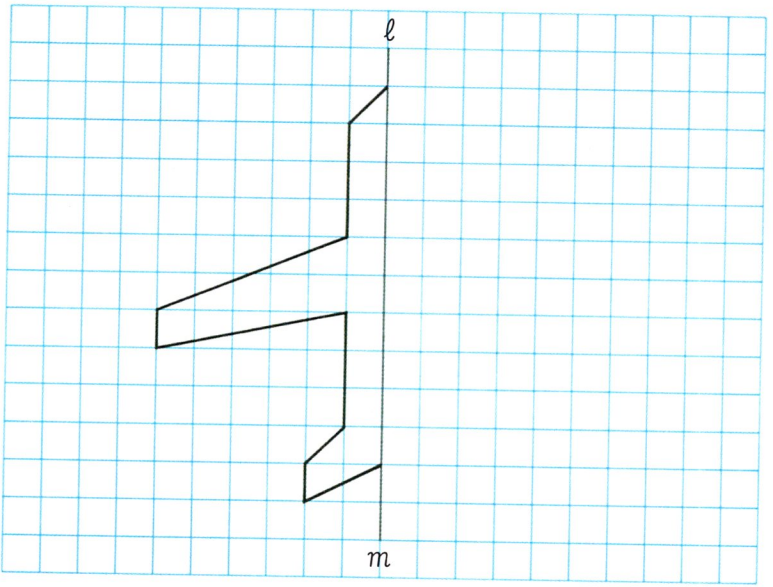
What segments are equal in length to BF and DG?

Also, how many lines of symmetry are there besides ℓm?



How to draw a shape with line symmetry

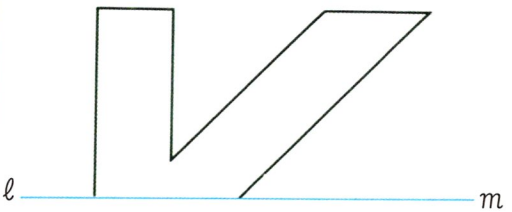
4 On the graph below, draw a shape with line symmetry using ℓm as the line of symmetry.



1 Using ℓm as the line of symmetry, draw a shape with line symmetry.

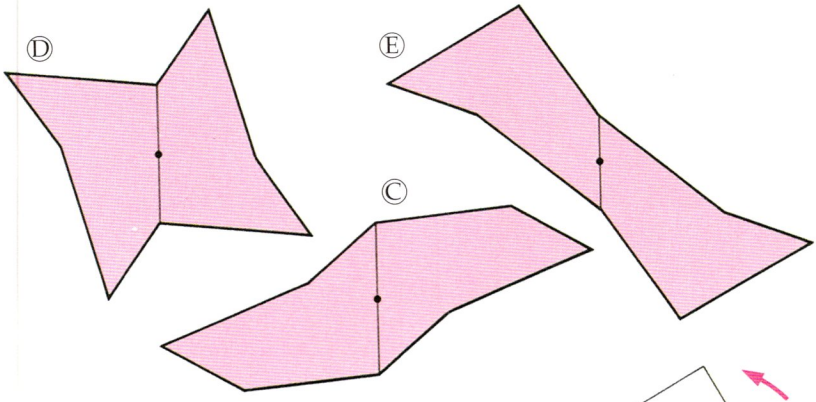
To find the corresponding vertex, draw a straight line perpendicular to the line of symmetry from the original vertex and . . .



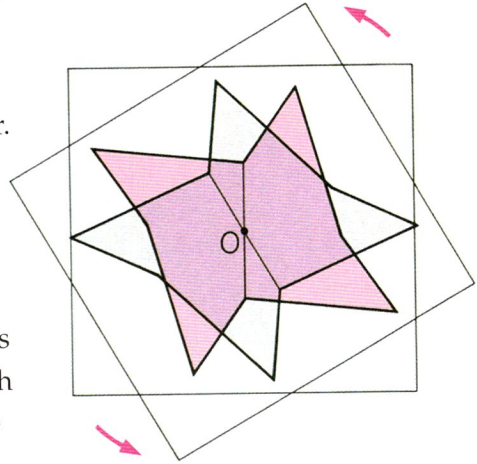


2 Point Symmetry

1 Let's explore the characteristics of the shapes in group (2) on page 32!



1 Copy one of the shapes above on a tracing paper. Superimpose it over the original shape, and rotate the paper around point O.
How many degrees does it need to rotate to match up completely with the original shape?



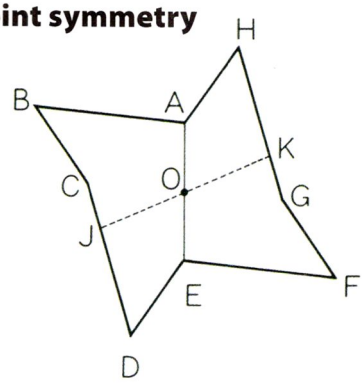
We say that a figure has **point symmetry** when the figure is rotated 180 degrees around a point and it matches up exactly with the original figure. We call this point the **center of symmetry.**

1 Let's find things around us that have point symmetry!



Properties of shapes with point symmetry

2 The diagram on the right shows a shape with point symmetry. Rotate the figure 180 degrees around point O and investigate the vertices, sides, and angles that match up.



1 Which vertex will match up with B? Which side will match up with BC?

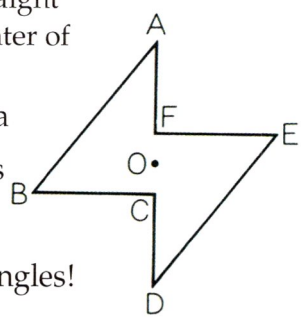
In point symmetry, we call the matching vertices, sides, and angles, corresponding vertices, corresponding sides, and corresponding angles, respectively.

2 In the shape above, please compare the lengths of the corresponding sides and the measures of the corresponding angles.

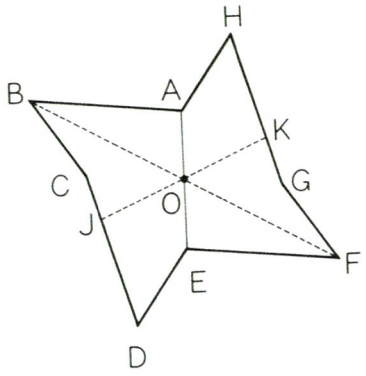
In point symmetry, the lengths of corresponding sides are equal and the measures of corresponding angles are equal.

3 In the shape above, are the two shapes created by cutting it with the straight line JK that goes through the center of symmetry, point O, congruent?

1 The diagram on the right shows a shape with point symmetry. Let's identify the corresponding vertices and the corresponding angles!



3 Let's investigate the relationship between the segments that connect the corresponding vertices and the center of symmetry!

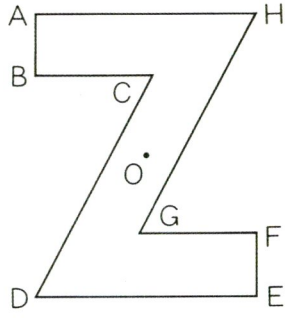


1 Where does line BF cross line JK?

2 Let's compare the lengths of OB and OF! Also, compare the lengths of OJ and OK.

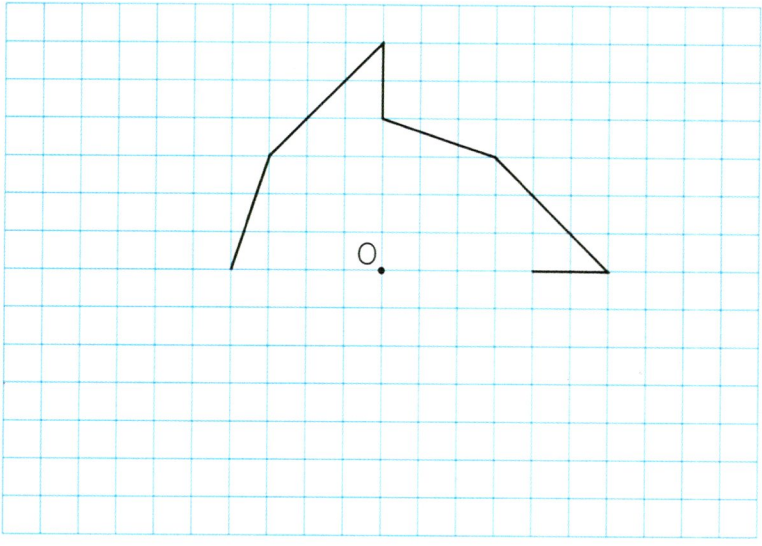
In point symmetry, lines that connect corresponding vertices run through the center of symmetry.
Also, the distance of these vertices from the center of symmetry is equal.

1 The figure on the right has point symmetry.
How can you find the position of the center of symmetry, O?
Which line has the same length as line OB?

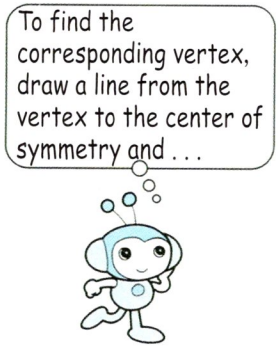


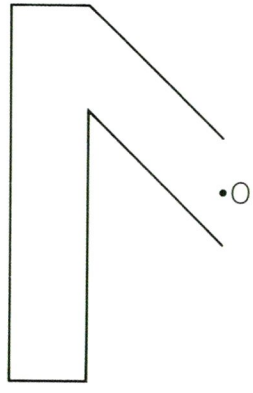
How to draw a shape with point symmetry

4 Using O as the center of symmetry, let's draw a shape that has point symmetry on the graph below!



1 Please draw a shape that has point symmetry with O as the center of symmetry.

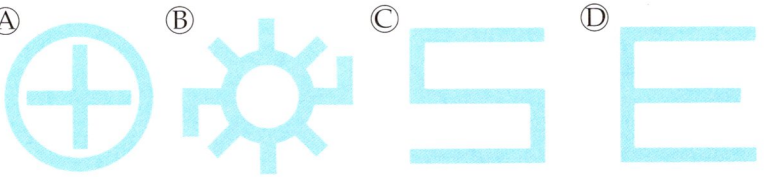




Practice

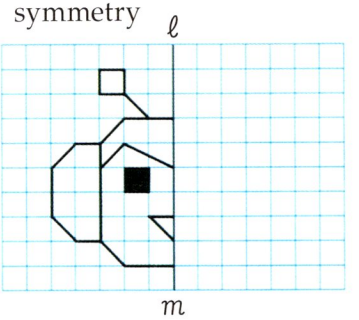
① Which figures below have line symmetry? Which ones have point symmetry?
In each shape with line symmetry, how many lines of symmetry are there?

Ⓐ Ⓑ Ⓒ Ⓓ

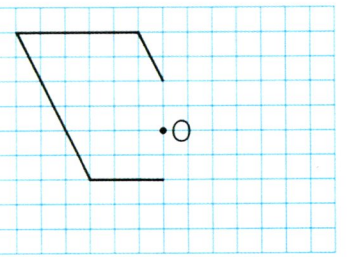


② Draw a shape with line symmetry and a shape with point symmetry:

(1) that has ℓm as the line of symmetry



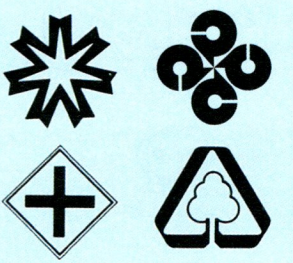
(2) that has point O as the center of symmetry

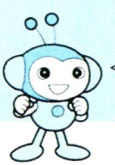


Challenge

Let's Find Symbols with Symmetry!

From the symbols on page 99, find the shapes with line symmetry and point symmetry.

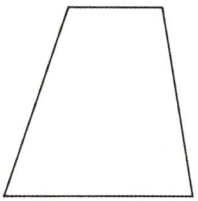




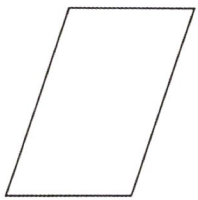
You will find a lot more around you.

3 Polygons and Symmetry

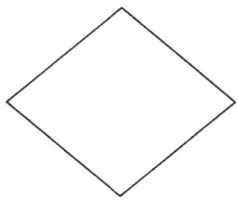
1 For each quadrilateral below, let's investigate if it has line symmetry or point symmetry!



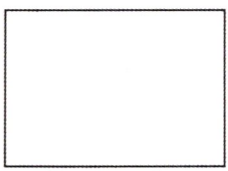
Trapezoid



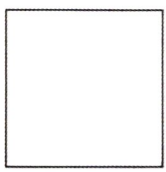
Parallelogram



Rhombus



Rectangle

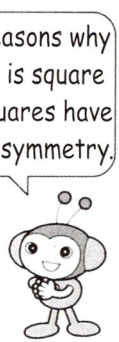


Square

1 Which ones have line symmetry? Please draw the line of symmetry.

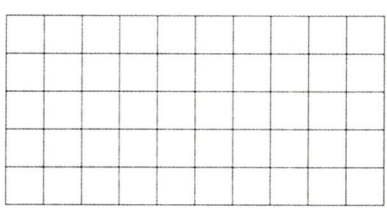
2 Which ones have point symmetry? Please draw the center of symmetry.

3 Which ones have both line and point symmetry?

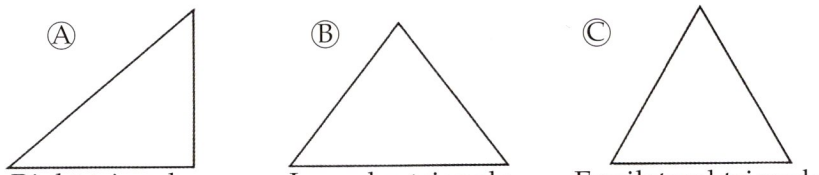


① Which quadrilaterals have line symmetry with two diagonal lines as lines of symmetry?

② Please draw a trapezoid that has line symmetry.



2 For each triangle below, let's investigate if it has line symmetry or point symmetry.

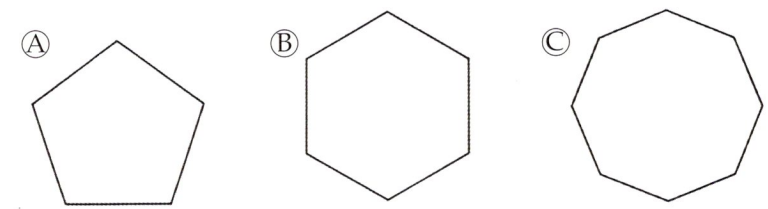


Right triangle　　Isosceles triangle　　Equilateral triangle

1 Which ones have line symmetry?
Please draw the line of symmetry.

2 Do any have point symmetry?

3 For each regular polygon below, let's investigate if it has line symmetry or point symmetry.



Regular pentagon　　Regular hexagon　　Regular octagon

1 Which ones have line symmetry?
How many lines of symmetry are there?
Please draw the lines of symmetry.

2 Which ones have point symmetry?
Please draw the center of symmetry.

3 Which ones have both line and point symmetry?

(1) Does a circle have line symmetry? Does it have point symmetry?

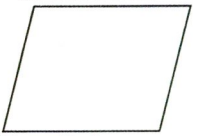
Check

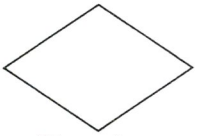
1 Please fill in the ☐ with the appropriate words.

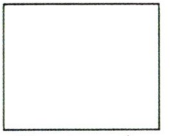
(1) In a figure with line symmetry, a segment connecting corresponding vertices is intersected perpendicularly by ☐ and the segment is equally divided in length at this point of intersection.

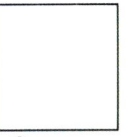
(2) In a figure with point symmetry, a segment connecting correspoinding vertices is bisected at ☐.

2 Summarize the characteristics of the following four quadrilaterals in the table.


Parallelogram


Rhombus


Rectangle


Square

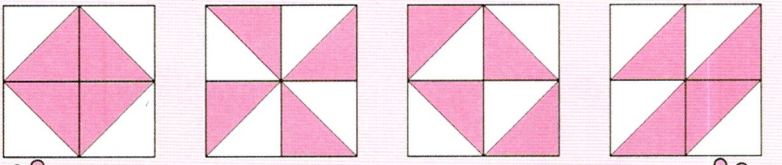
	Parallelogram	Rhombus	Rectangle	Square
Line symmetry	×			
Number of line of symmetry	0			
Point symmetry	○			

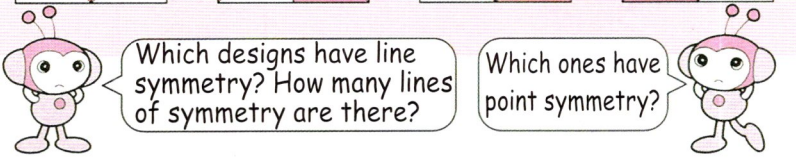
Challenge

Let's Make Designs with Symmetry!

Cut out the shapes on page 97. Then make various designs using four of them like the designs below.



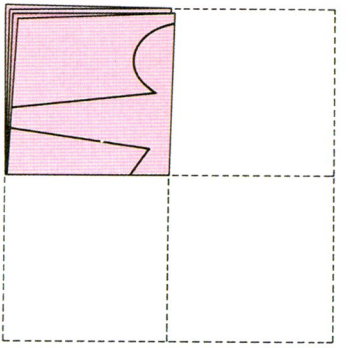




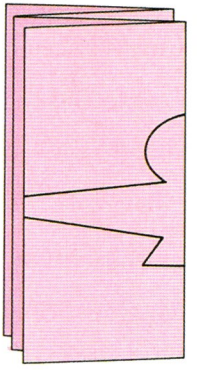
Math Plaza

Cutting Paper

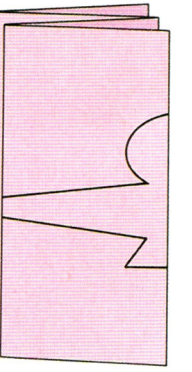
(1)



(2)



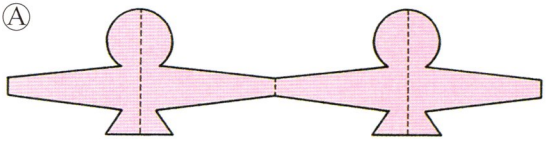
(3)



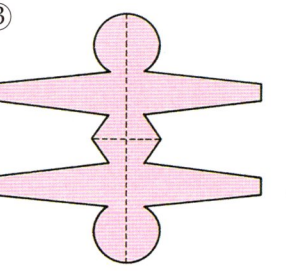


If you cut out shapes (1), (2), and (3) on folded paper and then unfold them, what shape will be made from each paper? Let's check by folding the sheets and cutting out the shapes!

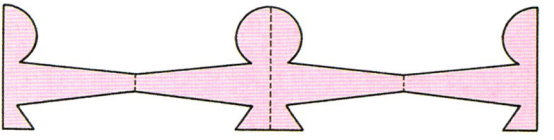
Ⓐ



Ⓑ



Ⓒ

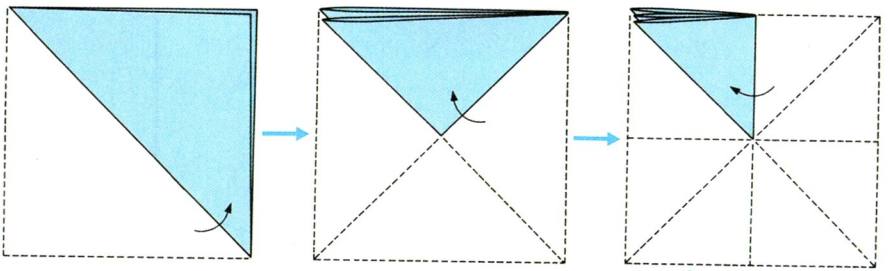


Which ones have line symmetry?



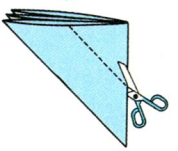
Which ones have point symmetry?



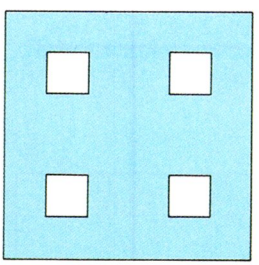




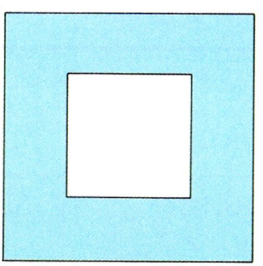
As shown above, fold a square sheet of paper so that you get eight layers. Then cut along the dotted line as shown on the right. Which shape from below do you get?



Ⓐ



Ⓑ



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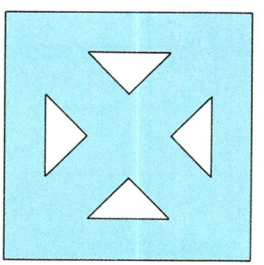
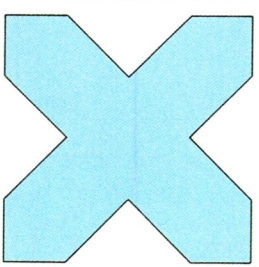
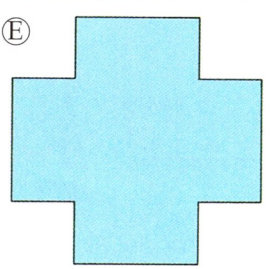


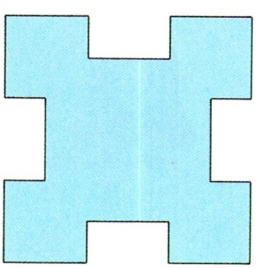
Ⓓ



Ⓔ

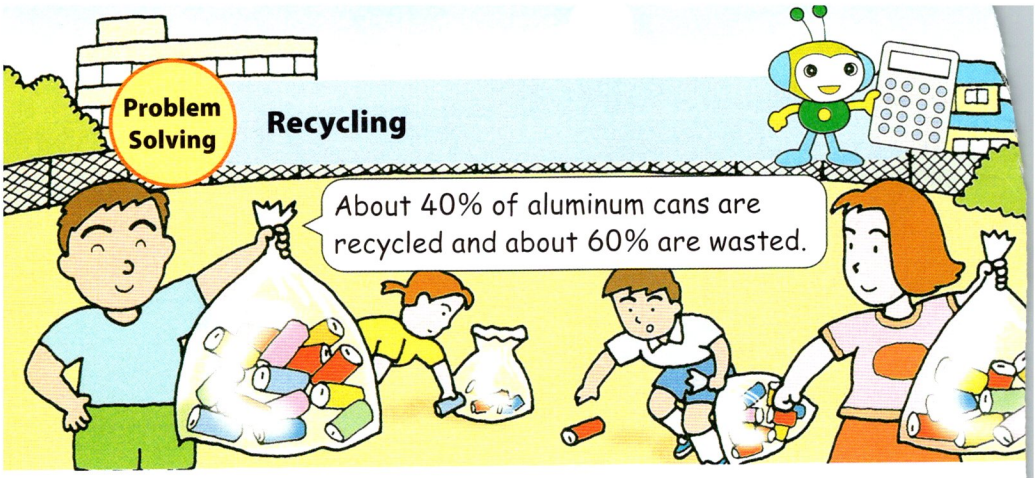


Ⓕ



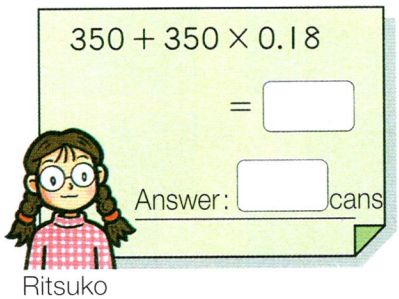


To make the other five shapes, which part should you cut out?

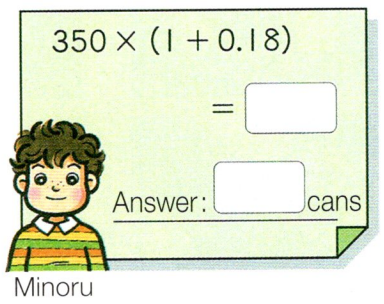


1 **Naoko's school collects aluminum cans. Last month they collected 350 cans. This month they collected 18% more than last month. How many aluminum cans were collected this month?**





Ritsuko



Minoru

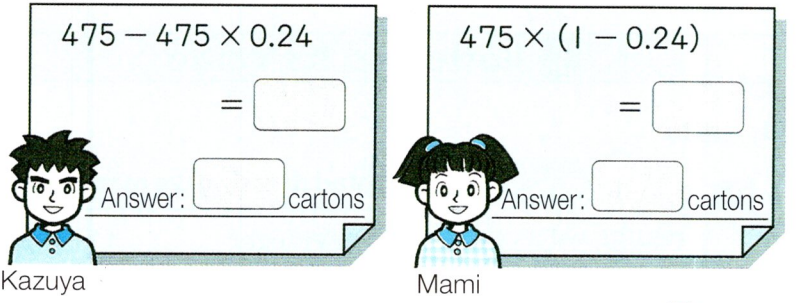
① Store A sells a plastic model at 2160 yen. Store B sells at a price that is $\frac{1}{9}$ higher than store A's price. How much does the plastic model cost at store B?





2 Naoki's school collects milk cartons. Last month they collected 475 cartons. This month they collected 24% fewer than last month. How many cartons were collected this month?





① Store A sells a stuffed animal for 1875 yen. Store B sells the stuffed animal at a price that is $\frac{2}{15}$ lower than the store A's price.

How much does the stuffed animal cost at store B?



5 Ratio and Its Value

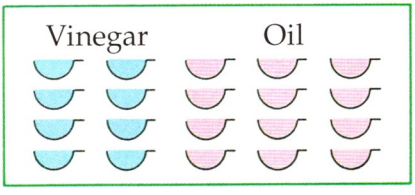


1 Ratio and Its Value

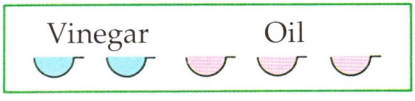
▶ Ratio

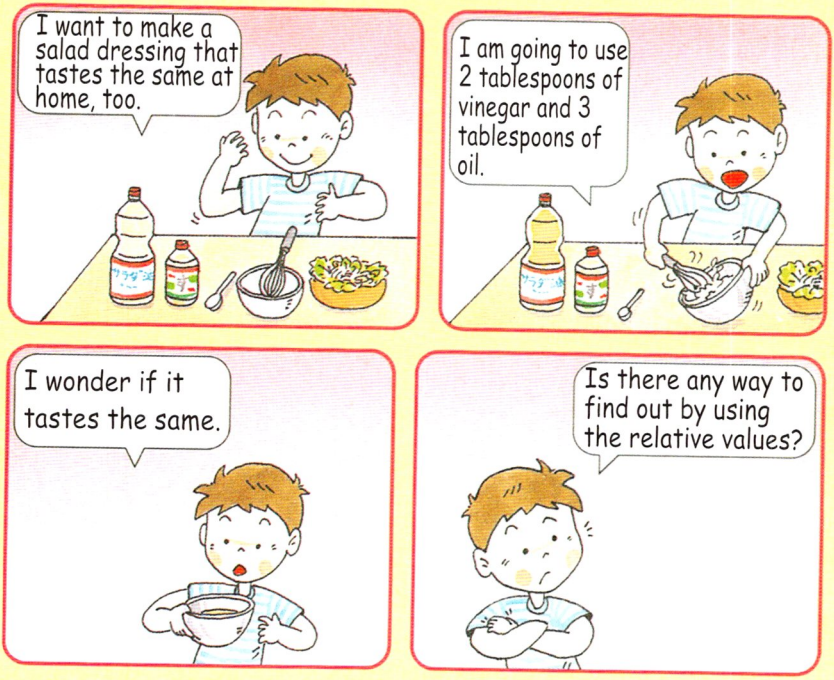
1 Does the salad dressing we made at school taste the same as the one we made at home?
Let's think about how to express the relative values of the amount of vinegar and oil!

Salad dressing we made at school



Salad dressing we made at home

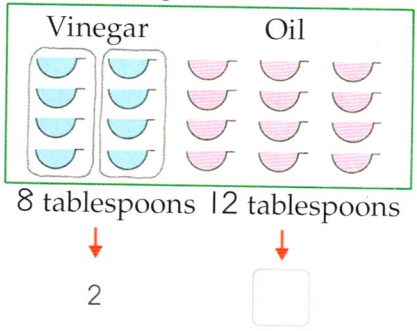




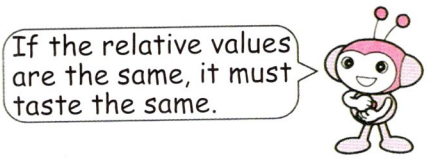
Let's think about various ways to express proportions!

1 If we consider the amount of vinegar we used at school as 2, what would the amount of oil be?

Salad dressing we made at school



In both dressings, the relative values of the amount of vinegar and the amount of oil are 2 and 3.



The relative value of 2 and 3 can be expressed as 2 : 3 by using the symbol " : ".

2 : 3 is read as "2 to 3."

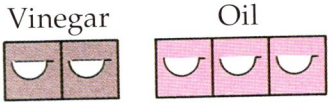
A relative value expressed in this way is called a **ratio**.

2 : 3 can also be read as "the ratio of 2 and 3."

1 For the dressing made at school described on page 48, if we make the amount of vinegar 8, what would the ratio of vinegar to oil be?

▶ The value of a ratio

2 In the salad dressing we made at home, let's think about what fraction the amount of vinegar is compared to the amount of oil.



Ratio 2 : 3 ➡ $2 \div 3 = \frac{2}{3}$

In the ratio $a : b$, the, quotient $\frac{a}{b}$ (a divided by b) is called **the value of the ratio**. In the ratio $2 : 3$, the value of the ratio is $\frac{2}{3}$.

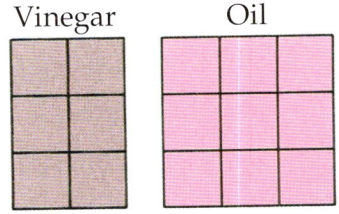
The value of the ratio of $a : b$ is the value of a when b is considered to be 1.

1 Let's find the values of the ratios!

(1) $3 : 4$ (2) $9 : 6$ (3) $2.1 : 0.7$ (4) $\frac{1}{2} : \frac{2}{5}$

Equivalent ratio

3 Kiyoshi made a salad dressing with a ratio of vinegar to oil of $6 : 9$. Let's find the value of this ratio!



Ratio $6 : 9$ ➡ $6 \div 9 = \frac{6}{9} = \square$

1 Let's compare the values of the ratios $2 : 3$ and $6 : 9$!

When the values of the ratios are equal in 2 ratios, we say that the ratios are equivalent and express it in the following way:

$$2 : 3 = 6 : 9$$

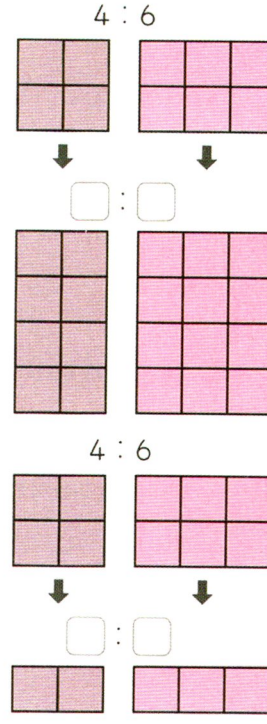
1 Let's find the values of the ratios and equivalent ratios!

$2 : 4$ $6 : 8$ $2.5 : 5$ $3.6 : 6$

2 Properties of Ratios

Properties of ratios

1 Let's think about how to make ratios that are equivalent to 4 : 6!



1 In 4 : 6, please write a new ratio by multiplying both 4 and 6 by 2. Also, find the values of these ratios and compare them.

2 In 4 : 6, please write a new ratio by dividing both 4 and 6 by 2. Also, find the values of these ratios and compare them.

$\times 2$

$4 : 6 = 8 : 12$

$\times 2$

$\div 2$

$4 : 6 = 2 : 3$

$\div 2$

! In $a : b$, if we multiply or divide a and b by the same number, the resulting ratios are all equivalent to each other.

We use this property when we make equivalent ratios.

1 Let's write 3 ratios that are equivalent to 4 : 10!

▶ Simplifying ratios

2 Let's find a ratio that is equivalent to 49 : 63 but with the smallest whole numbers!

$$49 : 63 = (49 \div 7) : (63 \div 7)$$
$$= \square : \square$$

$\frac{\overset{7}{\cancel{49}}}{\underset{9}{\cancel{63}}} = \frac{7}{9}$



Changing the ratio to an equivalent ratio with the smallest possible whole numbers is called "simplifying the ratio."

3 Let's simplify 0.9 : 1.5 and $\frac{2}{3} : \frac{4}{5}$!

(1) $0.9 : 1.5 = (0.9 \times 10) : (1.5 \times 10)$
$= 9 : 15$
$= \square : \square$

In both problems, it is easy if we convert them to whole numbers.

(2) $\frac{2}{3} : \frac{4}{5} = \left(\frac{2}{3} \times 15\right) : \left(\frac{4}{5} \times 15\right)$
$= 10 : 12$
$= \square : \square$

$\frac{2}{3} \cdot \frac{4}{5} = \frac{10}{15} \cdot \frac{12}{15}$
You can also use a common denominator.



1 Please simplify the ratios.

(1) 12 : 9 (2) 8 : 20 (3) 18 : 42 (4) 14 : 49

(5) 0.4 : 0.6 (6) 2.5 : 3 (7) $\frac{2}{3} : \frac{2}{9}$ (8) $2\frac{3}{5} : 7$

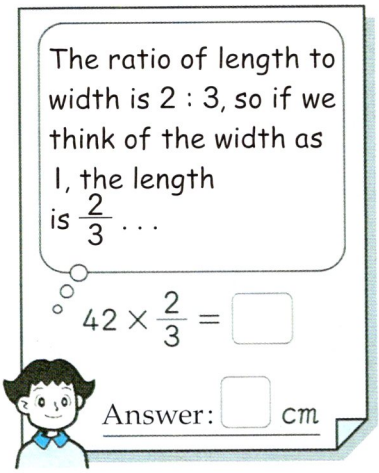
3 Using Ratios

▶ Finding one value in a ratio

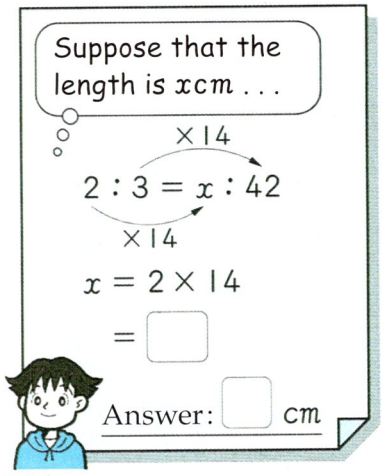
1 The ratio of the length to the width of the flag of the United Nations is 2 : 3.
If the width is 42cm, what is the length?



1 Let's explain the 2 students' ideas!



Sayuri



Makoto

1 Akira's school has 722 students. The ratio of the number of boys to the number of all the students is 10 : 19.
How many boys are there in Akira's school?

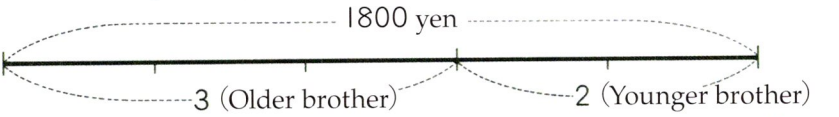
2 Please find the number represented by x.

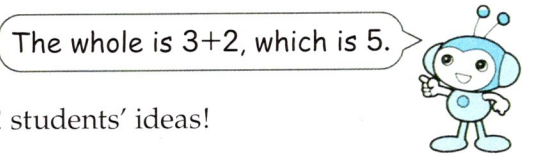
(1) $15 : 10 = x : 2$ (2) $7.5 : 5 = 3 : x$

▶ Dividing the whole into a given ratio

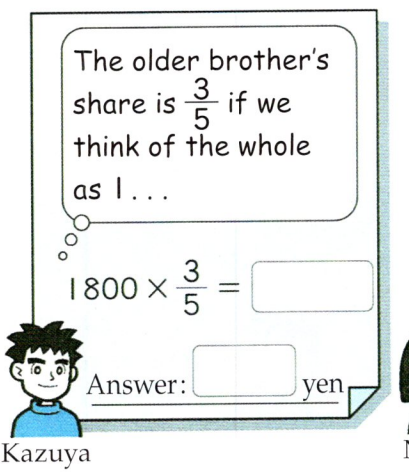
2 Two brothers are going to divide 1800 yen between them. They want the ratio of the older brother's amount to the younger brother's amount to be 3 : 2.
How much should the older brother get?



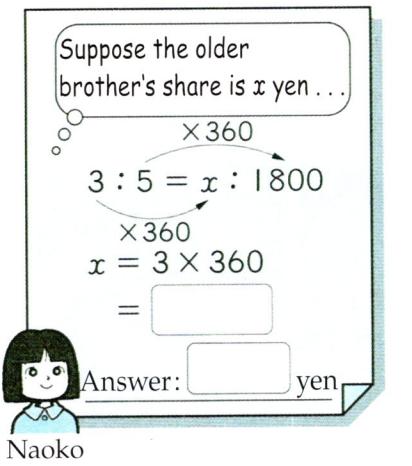




1 Let's explain the 2 students' ideas!

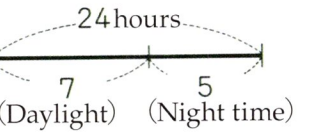


Kazuya



Naoko

1 On May 14th, the ratio of daylight to night time was 7 : 5. How many hours of daylight were there?



Practice

① Please find the values of the ratios.

(1) $3 : 1$ (2) $0.6 : 1.8$ (3) $2\frac{2}{3} : 3$ (4) $1 : 1\frac{2}{5}$

② Please simplify the ratios.

(1) $48 : 42$ (2) $1 : 0.8$ (3) $0.65 : 1.5$ (4) $\frac{2}{3} : 1\frac{2}{5}$

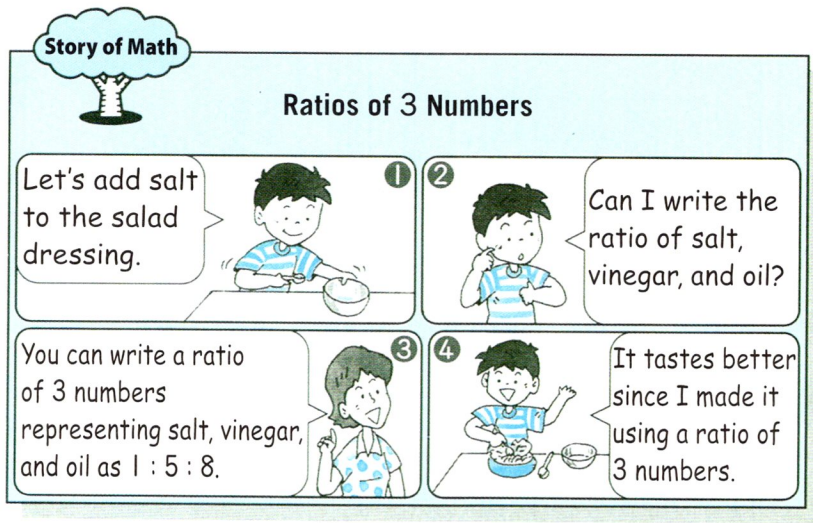
③ For each of the following values of the ratios, please find the simplest whole number ratio.

(1) $\frac{2}{7}$ (2) $2\frac{1}{3}$ (3) 1.4 (4) 0.375 (5) 1.05

④ Please find the number represented by x.

(1) $3.6 : 2.7 = x : 3$ (2) $\frac{4}{5} : 0.7 = x : 7$

⑤ $\frac{2}{5}$ of a $2m$ long tape was used. Please write the ratio of the length that was used to the length that was left over.



Check

1 Please fill in the ☐ with the appropriate numbers.

The ratio of 5 and 9 is ☐ : ☐.

2 Please find the values of the ratios.

(1) 3 : 5 (2) 25 : 45 (3) 0.9 : 1.2 (4) 0.8 : 2

3 Please simplify the ratios.

(1) 40 : 120 (2) 5.6 : 2.1 (3) $\frac{1}{5} : \frac{1}{3}$ (4) $\frac{5}{6} : 1\frac{2}{5}$

4 Please find the number represented by x.

(1) $10 : 4 = 5 : x$ (2) $2 : 0.5 = x : 2$

Challenge

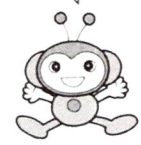
What Symbol Can You See?

Let's color the ratios that are equal to 6:15 and move toward the exit!

↓ Entrance

6 : 15	0.8 : 2	2.1 : 5.2	1.4 : 3.5	2 : 5	→ Exit
$\frac{14}{9} : \frac{34}{9}$	$\frac{12}{5} : 6$	0.5 : 1.5	5 : $12\frac{1}{2}$	value of ratio $\frac{11}{25}$	
1 : 2.5	15 : 37.5	5.5 : 14	0.4 : 1	4.4 : 10	
1.8 : 4.5	1.6 : 4.6	11 : 28	value of ratio $\frac{12}{30}$	13 : 32	
3 : 7.5	17 : 42	3 : 7	$\frac{9}{10} : 2\frac{1}{4}$	1.2 : 1.8	
1.2 : 3	value of ratio $\frac{8}{20}$	$1\frac{3}{5} : 4$	7 : $\frac{35}{2}$	$4\frac{3}{5} : 12$	

You will start seeing a Japanes Hiragana character. After you finish the next review, you will have summer vacation!



Review · 2

① Please calculate the following.

(1) $\frac{2}{5} \times 3$ (2) $\frac{5}{16} \times 12$ (3) $1\frac{3}{7} \times 5$ (4) $2\frac{2}{9} \times 6$

(5) $\frac{3}{4} \times \frac{5}{7}$ (6) $\frac{2}{9} \times \frac{7}{8}$ (7) $\frac{5}{6} \times \frac{2}{5}$ (8) $\frac{8}{9} \times \frac{3}{14}$

(9) $2\frac{1}{3} \times \frac{2}{9}$ (10) $4\frac{2}{3} \times \frac{1}{12}$ (11) $\frac{5}{9} \times 1\frac{1}{8}$ (12) $1\frac{7}{8} \times 2\frac{2}{3}$

② Please calculate the following.

(1) $\frac{2}{3} \div 8$ (2) $\frac{4}{9} \div 6$ (3) $1\frac{1}{6} \div 3$ (4) $3\frac{4}{7} \div 10$

(5) $\frac{3}{5} \div \frac{4}{9}$ (6) $\frac{2}{5} \div \frac{7}{10}$ (7) $\frac{9}{14} \div \frac{3}{28}$ (8) $\frac{12}{25} \div \frac{8}{15}$

(9) $2\frac{1}{3} \div \frac{3}{7}$ (10) $1\frac{1}{9} \div \frac{5}{7}$ (11) $\frac{3}{8} \div 5\frac{1}{4}$ (12) $1\frac{1}{6} \div 2\frac{5}{8}$

(13) $3\frac{1}{2} \div \frac{1}{2} \times \frac{6}{7}$ (14) $3\frac{1}{5} \times 2\frac{1}{2} \div \frac{2}{7}$

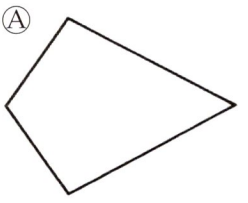
③ Please find the reciprocals of the following numbers.

(1) $\frac{1}{3}$ (2) $\frac{7}{4}$ (3) $1\frac{3}{8}$ (4) 3 (5) 10 (6) 0.7

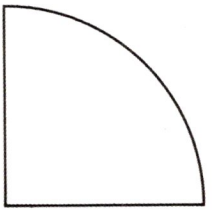
④ In the following diagrams, which figures have line symmetry? Which ones have point symmetry?

How many lines of symmetry are there for each figure that has line symmetry?

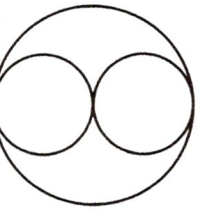
Ⓐ



Ⓑ



Ⓒ



⑤ Please convert the whole numbers and decimal numbers into fractions and then calculate.

(1) $5.25 + 2\frac{1}{3}$ (2) $1\frac{3}{5} - 0.12$ (3) $4 - \frac{1}{2} - 1.45$

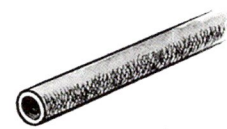
(4) $\frac{3}{8} \times 0.25$ (5) $2.7 \div \frac{3}{5}$ (6) $1\frac{1}{3} \div \frac{2}{3} \div 7$

(7) $4 \div 7 \times 14 \div 6$ (8) $21 \times 15 \div 27 \div 7$

⑥ A steel pipe weighs $1.2kg$ and its length is $1\frac{1}{3}m$.

(1) Find the weight of $1m$ of this pipe.

(2) Find the length of $1kg$ of this pipe.



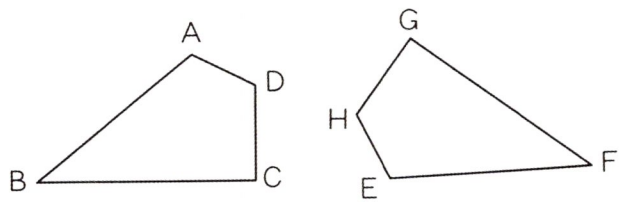
⑦ Please answer the following questions.

(1) If the relative value of $\frac{4}{5}g$ is $\frac{1}{2}$, what is the weight of 1?

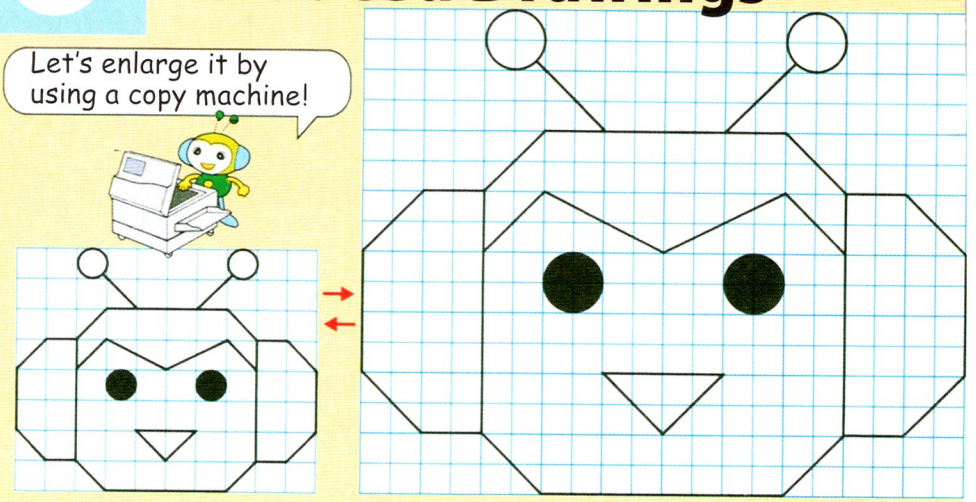
(2) How much is $\frac{2}{3}$ of $\frac{4}{5}\ell$?

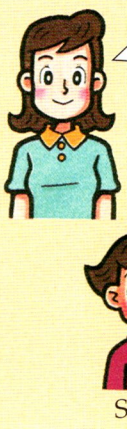
(3) How many times as long as $\frac{3}{4}m$ is $\frac{8}{9}m$?

⑧ In the diagram below, the 2 quadrilaterals are congruent. Please name the corresponding vertices, corresponding sides, and corresponding angles.



6 Enlarged Drawings and Reduced Drawings





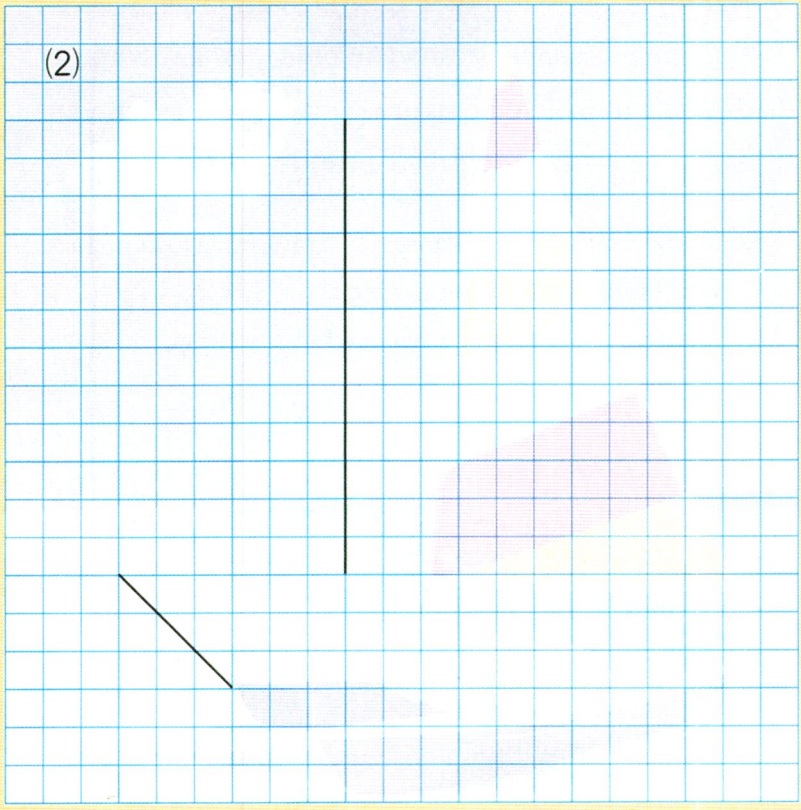
(1)



Sayuri

1 Enlarged Drawings and Reduced Drawings

1 Let's find the relationship between the lengths and the widths of picture (1) above and picture (2) on the next page!



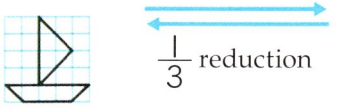
Let's explore figures in which we change the size without changing the shape!

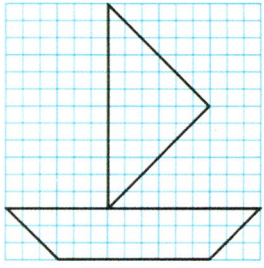
1 How many times as long are the length and width of (2) as the length and width of (1)?

2 The length and width of (1) are what fraction of the length and width of (2)?

A drawing in which the figure is made larger without changing its shape is called an **enlarged drawing.** A drawing in which the figure is made smaller without changing its shape is called a **reduced drawing.**

3 How many times as large as drawing (1) on page 60 is drawing (2) on page 61?
Drawing (1) on page 60 is what fraction of drawing (2) on page 61?



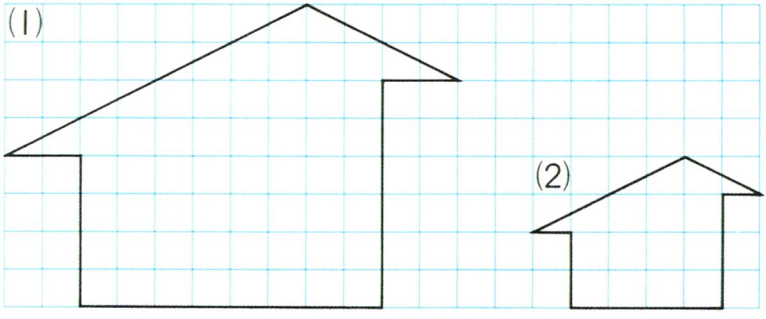






(1) Let's find enlarged and reduced drawings!

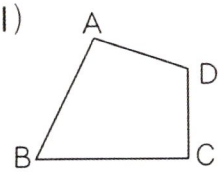
(2) Drawing (2) below is a $\frac{1}{2}$ reduced drawing of (1). Let's verify this!



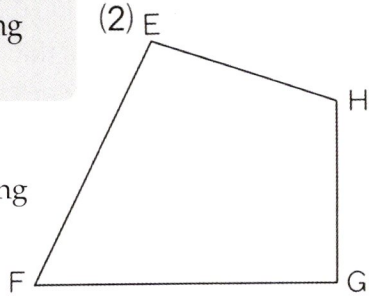
▶ Properties of enlarged and reduced drawings

2 Quadrilateral (2) on the right is an enlarged drawing of (1). Let's investigate the lengths of the corresponding segments and the sizes of the corresponding angles!

(1)



(2)

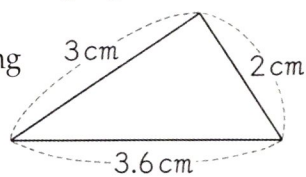


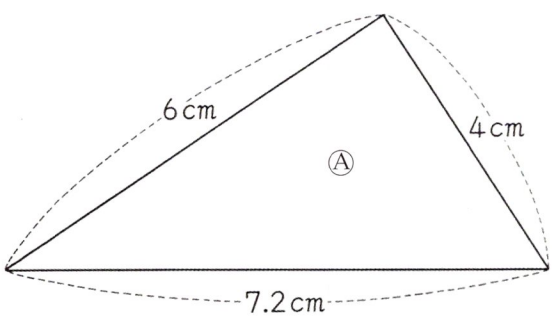
1 Please find the ratio of the lengths of the corresponding sides and compare the measures of the corresponding angles.

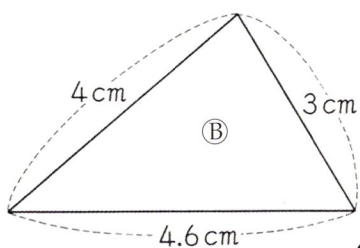
If you compare the ratios of the lengths of the corresponding sides of an enlarged or a reduced drawing to its original drawing, you will find that all the ratios are equivalent. Also, you will see that the measures of the corresponding angles are equal.

2 If you compare quadrilateral (1) to a quadrilateral that is conguent to (1), what are the ratios of the lengths of the corresponding sides? What are the measures of the corresponding angles?

1 Which one is an enlarged drawing of the triangle on the right?

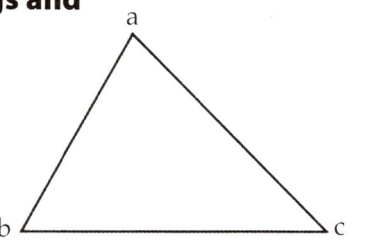






How to draw enlarged drawings and reduced drawings

3 Let's think about a way to draw triangle ABC, which is an enlargement that is twice as big as the triangle on the right!



1 Let's draw a line BC that is twice as long as the length of segment bc!

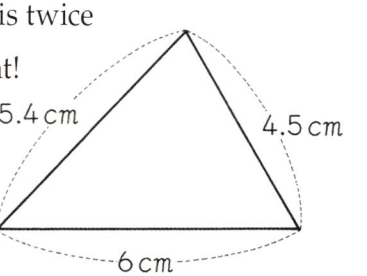


B ---- 2-times the length of segment bc. ---- C

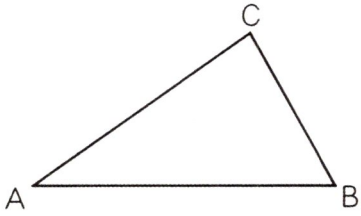
2 How can we find the position of vertex A, which corresponds to vertex a?

3 Let's see if the enlarged triangle ABC is twice as big as triangle abc!

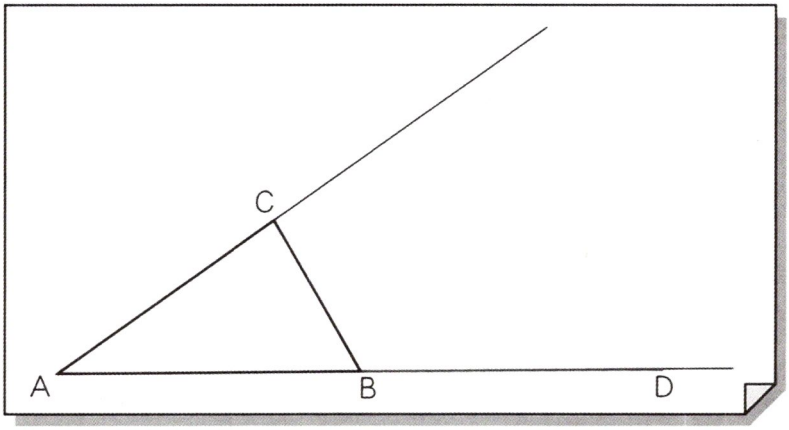
1 Let's draw an enlargement that is twice as big as the triangle on the right! Also, let's draw a $\frac{1}{3}$ reduced drawing of the triangle on the right!



4 Let's think of a way to draw triangle ADE that is an enlargement twice as big as triangle ABC and centered at A!

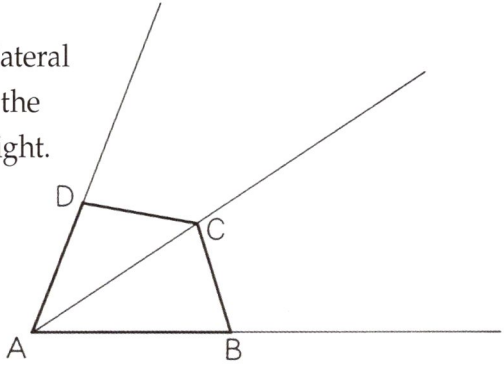


1 Extend segment AB by doubling its length to determine the position of vertex D. Then, find the position of E in the same way.



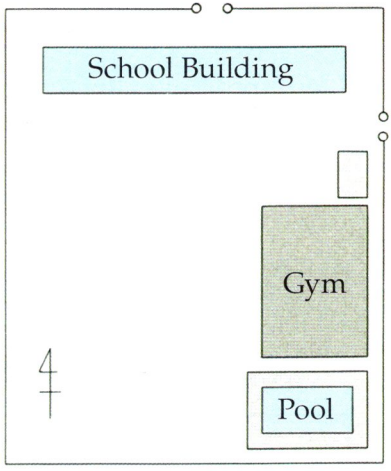
2 Let's check to see if triangle ADE is twice as big as triangle ABC!

1 Please draw a quadrilateral that is twice as big as the quadrilateral on the right. Also, draw a $\frac{1}{2}$ reduced drawing of the quadrilateral on the right.



2 Using Reduced Drawings

1 The diagram on the right is a reduced drawing of Kiyoshi's school.
The gym, that has an actual length of 40m, is represented as 2cm. Let's investigate what the actual width of the school building is!



Let's find the ratio of reduction in this drawing!

1 Let's represent the reduction as a fraction.
Also, let's represent it as ratio!

A ratio of reduction based on the original length is called a **scale**. Scales can be represented in the following ways:

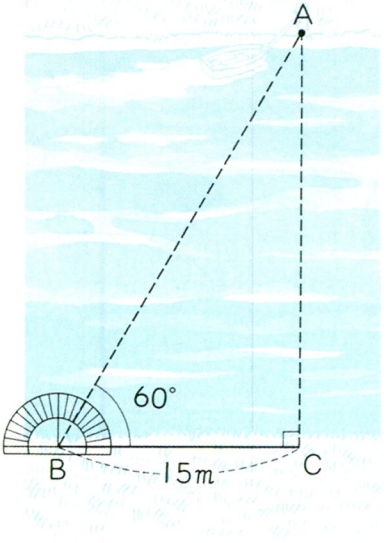
(1) $\frac{1}{2000}$ (2) 1 : 2000 (3) 0 20 40 60m

2 Please find the actual width of the school building.

1 The size of a classroom is 9m × 7m. To draw the classroom with a $\frac{1}{200}$ scale, how many cm should each side of the classroom on the reduced drawing be?

Using reduced drawings

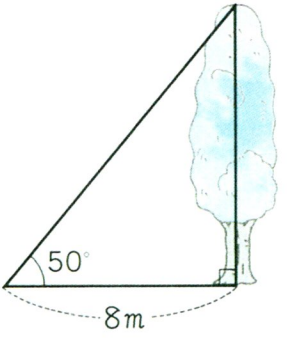
2 Let's think about how to determine the actual width AC of the river by using a reduced drawing!



1. Draw a line BC, parallel to the shore.
   Determine the position of A on the other side of the river by extending a perpendicular line to line BC from point C.
2. Measure the length of line BC and the size of angle B.
3. Draw a reduced drawing of right triangle ABC.
4. From this reduced drawing, find the actual width of the river, AC.

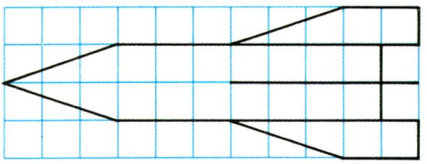
1 Let's draw a $\frac{1}{200}$ reduced drawing of right triangle ABC to find the width of the river!

1 What is the height of the tree in the diagram on the right?
Please draw a reduced drawing and then find the actual height.



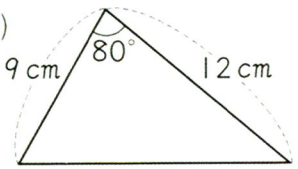
Practice

① Using grid paper, please draw an enlarged drawing that is 3 times as big as the figure on the right.

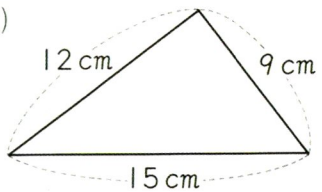


② Please draw a $\frac{1}{3}$ reduced drawing of each triangle below.

(1)



(2)



③ If a distance is represented as 8cm on a $\frac{1}{50000}$ map, how many cm will represent the same distance on a $\frac{1}{200000}$ map?

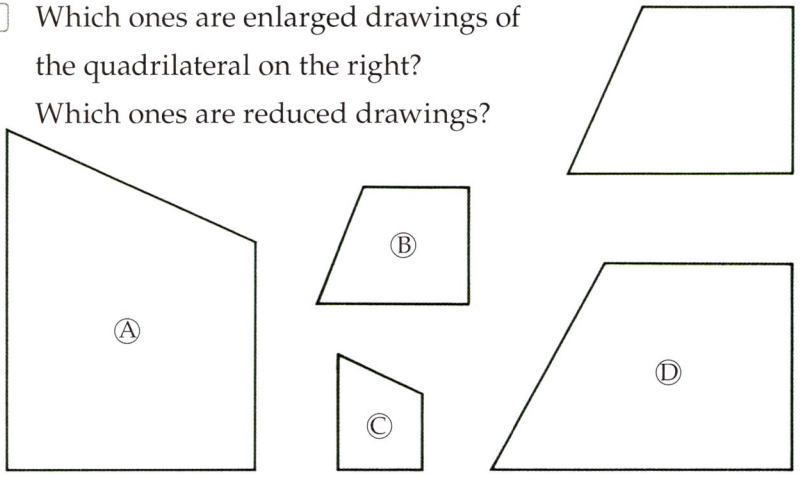
Challenge

Finding the Distance on a Map



Check

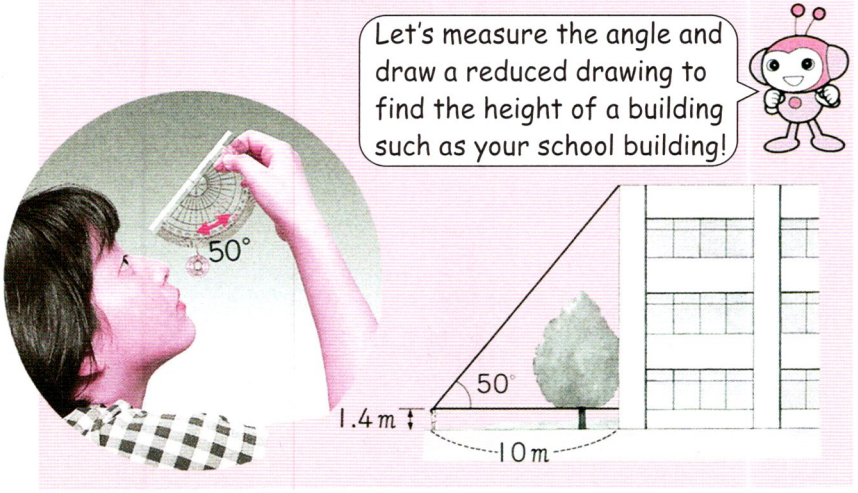
1 Which ones are enlarged drawings of the quadrilateral on the right?
Which ones are reduced drawings?



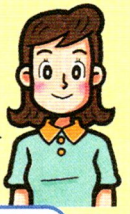
2 A distance of $3km$ is represented as $6cm$ on a map.
Please express the scale of this map as a fraction and as a ratio.

Challenge

Let's Measure the Height of Your School Building!

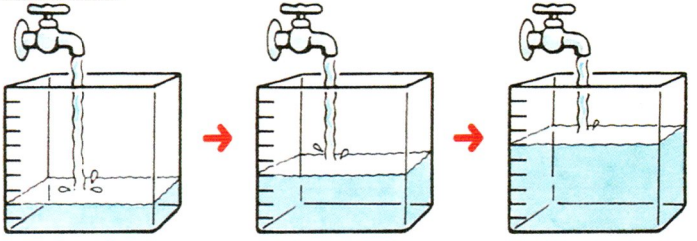


7 Proportional Relationships

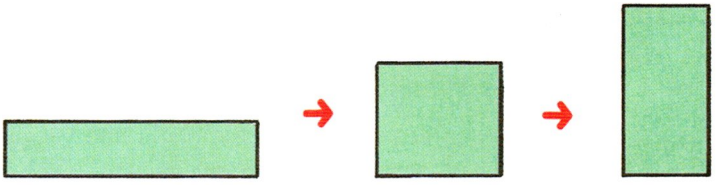


There are many situations around you in which one quantity changes as another quantity changes.

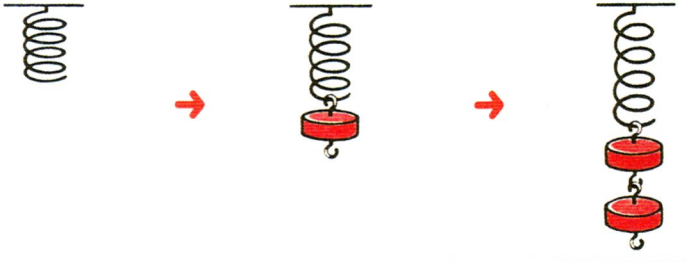
Ⓐ As the amount of water increases, the depth of water will [ ].



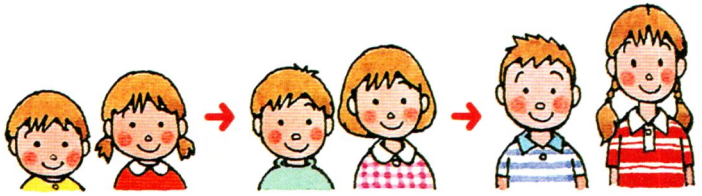
Ⓑ For rectangles that have the same area, if the length increases, the width will [ ].



Ⓒ As the weight increases, the length of the spring will [ ].



Ⓓ If a brother and a sister have the same birthday, when the brother's age increases, the sister's age will ☐.



Ⓔ As the amount of time a candle is burning increases, its height will ☐.



Let's explore how two quantities change together and their relationship to each other!

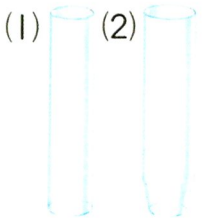
1 Different Ways of Changing Quantities

1 In situations Ⓐ, Ⓑ, Ⓒ, Ⓓ, and Ⓔ above, let's see how the second quantity changes as the amount of the first quantity increases!

1 In which example does the second quantity increase as the first one increases? In which example does the second quantity decrease as the first one increases?

2 Properties of Proportional Relationships





1 The depth of water is measured in the 2 containers shown above.
Let's investigate how the depth of water changes as the amount of water increases!

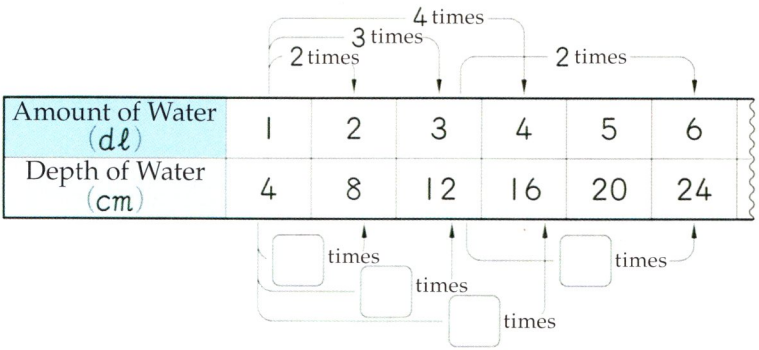
(1)

Amount of Water (dℓ)	1	2	3	4	5	6
Depth of Water (cm)	4	8	12	16	20	24

(2)

Amount of Water (dℓ)	1	2	3	4	5	6
Depth of Water (cm)	4	7	10	13	16	19

1 In Table (1), how does the depth of water change as the amount of water doubles, triples, and so on?



2 Let's explore Table (2) in the same way!

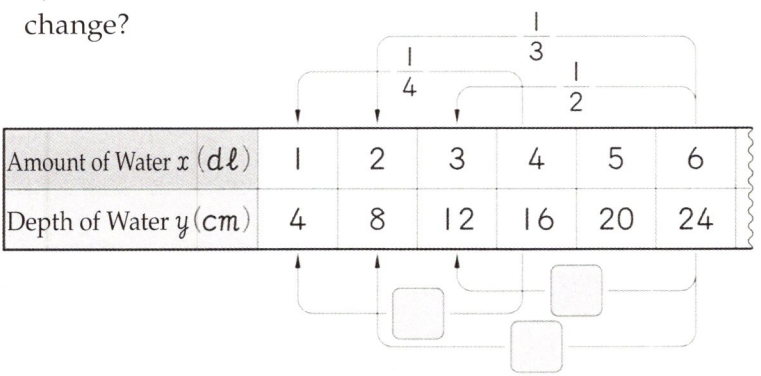
Suppose the amount of water is represented as x dℓ and the depth of water as y cm. In Table (1), as x doubles, triples, ... , so does y.

If two quantities x and y change in such a way that y increases by a factor of 2, 3, ... , as x increases by a factor of 2, 3, ... , we say that "y is proportional to x."

In container (1), the depth of water is proportional to the amount of water.

3 In container (2), is the depth of water proportional to the amount of water?

4 In Table (1), as the amount of water becomes $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of the original amount, how does the depth of water change?

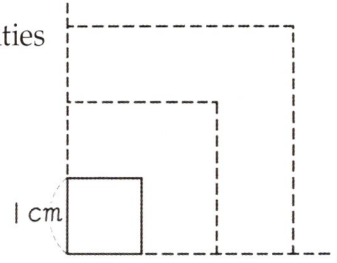


Amount of Water x (dℓ)	1	2	3	4	5	6
Depth of Water y (cm)	4	8	12	16	20	24

If y is proportional to x, the amount of y becomes $\frac{1}{2}$, $\frac{1}{3}$, ... , of the original amount as the amount of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... , of the original amount.

1 In which case are the two quantities proportional to each other?

(1) The length of a side of a square and its perimeter.



Length of Side (cm)	1	2	3	4	5	6
Perimeter (cm)	4					

(2) The length of a side of a square and its area.

Length of Side (cm)	1	2	3	4	5	6
Area (cm^2)	1					

2 The table below shows the amount of gasoline used and the distance traveled by a car traveling at a constant speed. Is the distance traveled proportional to the amount of gasoline used?



Amount of gas used (ℓ)	3	6	9	12	15
Distance traveled (km)	37.5	75	112.5	150	187.5



3 Equations of Proportional Relationships and Their Graphs

▶ The equation of a proportional relationship

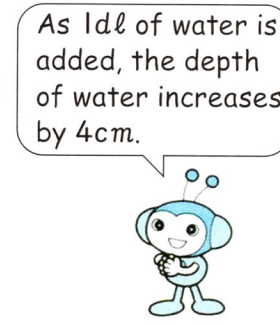
1 Using Table (1) in 1 on page 72, let's express the relationship between the depth of water and the amount of water with an equation!

1 Write the quotient of y (depth of water) divided by x (amount of water) in the table below and compare.

Amount of water x ($d\ell$)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24
The quotient of y divided by x						

The table above shows that the quotients of y divided by x are always the same and that they are equal to 4. This means that the depth of water per $1d\ell$ of water is fixed and is $4cm$.

2 Write equations that express the relationship between the amount of water ($xd\ell$) and the depth of water (ycm).



$$y \div x = 4$$

$$y = 4 \times x$$

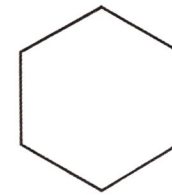
If equations like these are used, the value of y can be calculated when the value of x is known.

3 Please find the value of y when x is 0, 9, and 15.

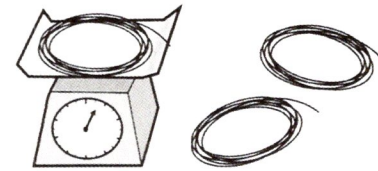
If y is proportional to x, the quotient of the value of y divided by the corresponding value of x is always the same number. Also, the following equation holds true:

$$y = \textbf{fixed number} \times x$$

1 Suppose the length of a side of a regular hexagon is $x$cm. Write an equation to represent the perimeter, $y$cm.



2 The table below shows various lengths of wire and their corresponding weights.



Lengths and Weights of Wire

Length x (m)	2	3	4	5	6	7
Weight y (g)	18	27	36	45	54	63

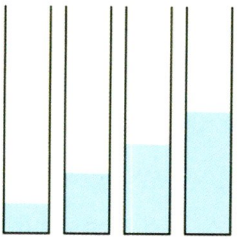
(1) Find the quotients of the weight of the wire (y) divided by the length of the wire (x) and compare them.

(2) Is the weight of the wire proportional to its length?

(3) Suppose the length of the wire is xm and the weight is yg. Write an equation to represent y.

▶ The graph of a proportional relationship

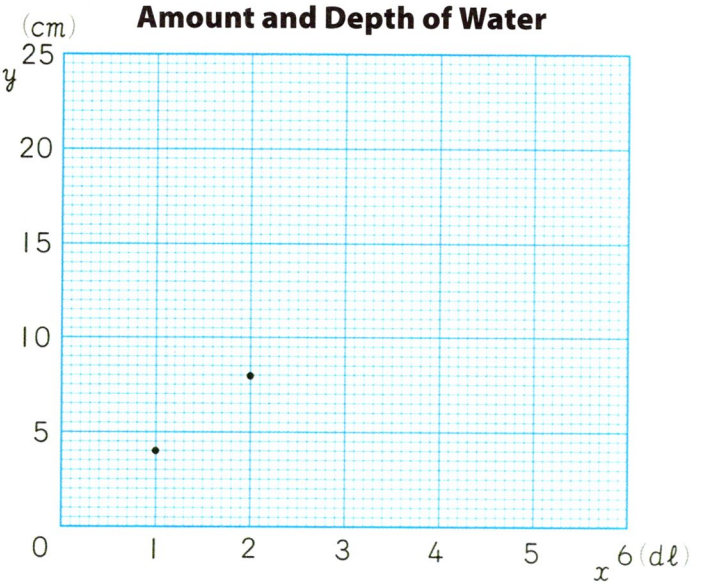
2 Using Table (1) in **1** on page 72, let's draw a graph that expresses the proportional relationship between the depth of water and the amount of water and explore its characteristics!



Amount and Depth of Water

Amount of water x (dℓ)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

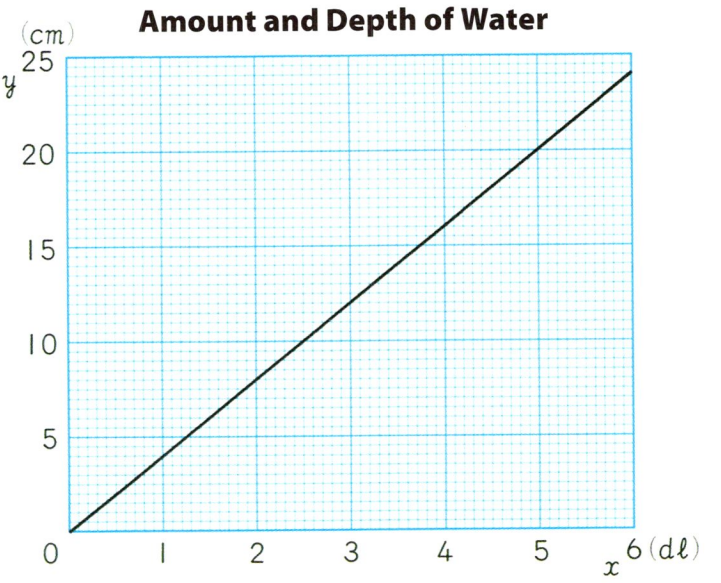
1 Plot points for x (amount of water) and y (depth of water) on the graph.



2 How are these points arranged on the graph?

The graph representing the relationship between the amount of water ($x d\ell$) and the depth of water ($y cm$) is a straight line as shown below.

3 Please find the value of y when x is 0, 1.5, and 4.6 in the equation $y = 4 \times x$. Also, verify that the points representing these values are actually on the straight line of the graph.



A graph representing two quantities that are proportional to each other is a straight line that goes through the point where both quantities are 0.

4 From the graph above, please find the value of y when the value of x is 2.5.

Also, find the value of x when the value of y is 22.

Practice

① For each table below, please determine if y is proportional to x.

(1)

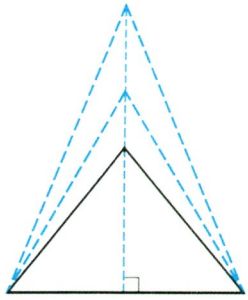
x (ℓ)	2	4	6	8
y (kg)	4.4	6.4	8.4	10.4

(2)

x (m)	0.4	0.6	0.8	1
y (m)	1	1.5	2	2.5

② Please find the area in m^2 of a circle with a radius of $0.5m$, $1.5m$, and $2m$.
Is the area proportional to the radius?

③ The area of a triangle with a fixed base will change as its height changes.
The table below shows this relationship.

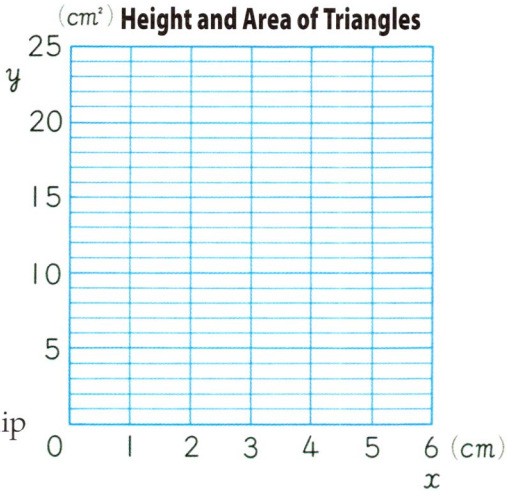


Height (cm)	2	3	4	5	6
Area (cm^2)	8	12	16	20	24

(1) Is the area of a triangle proportional to its height?

(2) Suppose the height is x cm and the area is y cm^2. Please write an equation to represent y.

(3) Please draw a graph to represent the relationship between x and y.



4 Using Proportional Relationships

1 15 nails of the same kind weigh 27g.
How many g will 75 of these nails weigh?



1 The weight of the nails is proportional to the number of nails. Based on this, let's think about the way to solve this problem!

Number of nails	15	75
Weight (g)	27	□

2 Explain the thinking of the 2 students.

Minoru

Fixed Number

$27 \div 15 = \frac{27}{15}$

$= \frac{9}{5}$

Weight of 75 nails

$\frac{9}{5} \times 75 =$ □

Answer: □ g

Mami

Number of nails	15	75
Weight (g)	27	□

5 Times ↑ ↑ 5 Times

$75 \div 15 = 5$

Weight of 75 nails

$27 \times 5 =$ □

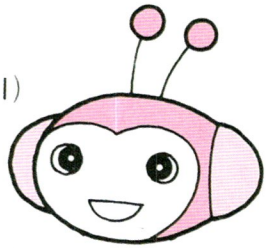
Answer: □ g

1 4 T-shirts can be made from 17 recycled plastic bottles.
How many T-shirts can be made from 119 recycled plastic bottles?

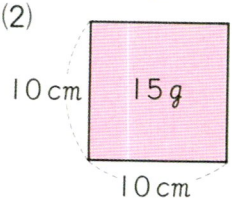


2 Figure (1) on the right was cut out of construction paper. It weighs 24g. The square (2) made from the same paper weighs 15g. What is the area of Figure (1) in cm^2?

(1)



(2)



1 What kind of relationship exists between the area and the weight of the construction paper?

2 Based on the fact that the weight of the construction paper is proportional to its area, find the area of Figure (1).

Area (cm^2)	100	☐
Weight (g)	15	24

Challenge

About How Many Minutes Later

On the Tokaido Bullet Line, Mt. Fuji can be seen from Shin-Fuji Station. If you get on the Nozomi Express at Tokyo Station, about how many minutes later will you be able to see Mt. Fuji?

	Tokyo to Shin-Fuji	Tokyo to Nagoya
Distance (km)	146	366
Time (minutes)	☐	96



Check

1 In which cases are the two quantities in a proportional relationship?

Ⓐ The number of candies costing 30 yen each and the total cost

Ⓑ The age of a father and the age of a child whose birthday is the same as the father's

Ⓒ The length of a side of an equilateral triangle and its perimeter

2 Fill in the ☐ with the appropriate number, word, or symbol.

(1) $1m$ of ribbon costs 120 yen. If we suppose that $x m$ of the same ribbon costs y yen, the relationship between x and y can be expressed as:

$y =$ ☐ $\times$ ☐ . So y is ☐ to x.

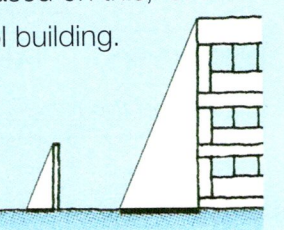
(2) A circle has a diameter of $1.5m$. If the diameter is doubled, tripled, ... , its circumference is also ☐ , ☐ , ... So the circumference of a circle is ☐ to its diameter.

Challenge

Let's Use a Proportional Relationship!

If you use a proportional relationship, you can measure things, such as the height of a building, that are otherwise difficult to measure. The length of the shadow of an object is proportional to the object's height. Based on this, let's measure the height of a tree or a school building.

		Stick	School building
Height	(m)		
Length of shadow	(m)		



Review・3

① Please calculate.

(1) $\frac{5}{7} \times \frac{3}{4}$ (2) $\frac{3}{5} \times \frac{2}{9}$ (3) $\frac{5}{6} \times \frac{3}{8}$ (4) $2\frac{4}{5} \times 2\frac{1}{7}$

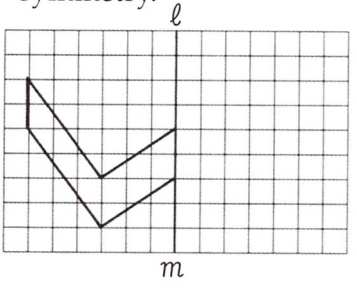
(5) $\frac{3}{8} \div \frac{1}{9}$ (6) $\frac{4}{5} \div \frac{2}{15}$ (7) $1\frac{5}{9} \div \frac{7}{8}$ (8) $4\frac{1}{6} \div 1\frac{2}{3}$

② Please find the number represented by x in the following equations.

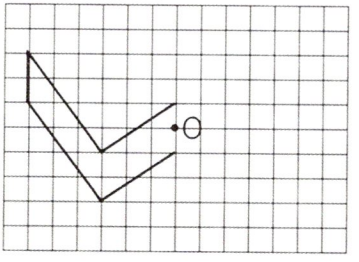
(1) $2 : 3 = 6 : x$ (2) $2.4 : 1.6 = x : 4$ (3) $\frac{1}{3} : \frac{1}{4} = x : 3$

③ Please draw a figure that has line symmetry and a figure that has point symmetry.

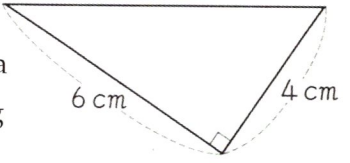
(1) Line ℓm is the line of symmetry.



(2) Point O is the center of symmetry.



④ Please draw an enlarged drawing that is twice as big and a reduced drawing that is $\frac{1}{2}$ as big as the triangle on the right.



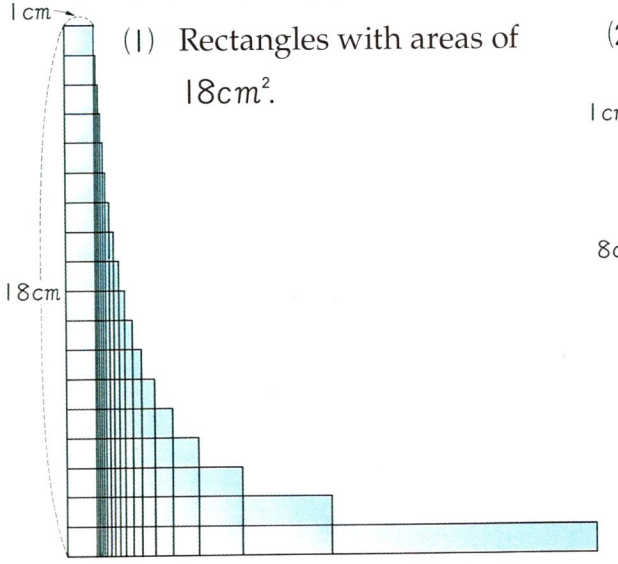
⑤ The distance from Higashida City to Nishikawa City is 180km.

Please find the time it takes to drive this distance at a speed of 30km/hour, 45km/hour, and 60km/hour, respectively.

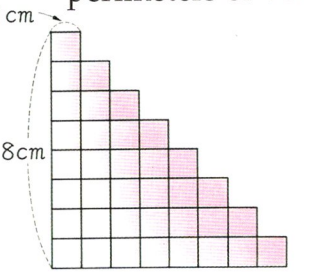
8 Inverse Proportions

1 Properties of Inverse Proportions

(1) Rectangles with areas of $18cm^2$.



(2) Rectangles with perimeters of $18cm$.



1 Draw rectangles with areas of $18cm^2$ and rectangles with perimeters of $18cm$.
Let's find out what happens to the width when the length increases!

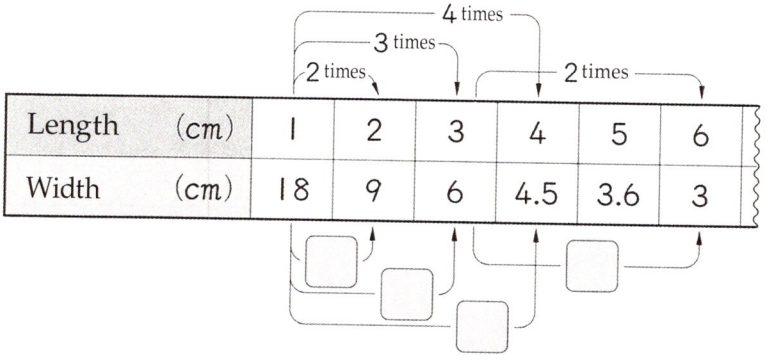
(1)

Length (cm)	1	2	3	4	5	6
Width (cm)	18	9	6	4.5	3.6	3

(2)

Length (cm)	1	2	3	4	5	6
Width (cm)	8	7	6	5	4	3

1 In Table (1), if the length doubles, triples, ... , how does the width change?



Length (cm)	1	2	3	4	5	6
Width (cm)	18	9	6	4.5	3.6	3

2 Explore Table (2) in the same way.

Suppose the length is x cm and the width is y cm. In Table (1), if the value of x doubles, triples, ... , the value of y becomes $\frac{1}{2}$, $\frac{1}{3}$, ...

If there is a relationship between 2 quantities, x and y, that when the value of x doubles, triples, ... , the value of y becomes $\frac{1}{2}$, $\frac{1}{3}$, ... , then we say that "y is **inversely proportional** to x."

In Table (1), the rectangle's width is inversely proportional to its length.

3 In Table (2), is the rectangle's width inversely proportional to its length?

4 In Table (1), if the length becomes $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, how does the corresponding width change?

Length x (cm)	1	2	3	4	5	6
Width y (cm)	18	9	6	4.5	3.6	3

$\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$

□ times, □ times, □ times

In an inversely proportional relationship between x and y, if the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... , the value of y doubles, triples, ...

1 The table below shows the different speeds that a car traveled from Higashigawa City to Nishida City and the corresponding times it took. Is the time taken inversely proportional to the speed?



Speed x (km/h)	10	20	30	40	50	60
Time y (hours)	18	9	6	$4\frac{1}{2}$	$3\frac{3}{5}$	3

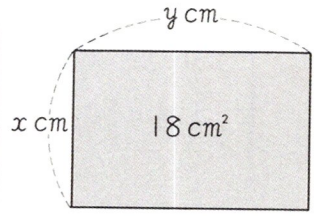
2 times, $\frac{1}{6}$

□ times

2 Equations of Inverse Proportions and Their Graphs

▶ The equation of an inverse proportion

1 Express the inverse proportional relationship between the width and the length of the rectangles shown in Table (1) in 1 on page 84 with an equation.



1 For each value of x (length) and the corresponding value of y (width), find the product. What do you notice?

Length x (cm)	1	2	3	4	5	6
Width y (cm)	18	9	6	4.5	3.6	3
Product of x and y						

The table above shows that the product of x and y is always the same and equals 18.

This means that the areas of the rectangles are fixed and that they equal $18cm^2$.

2 Write an equation to represent the relationship between the length, xcm, and the width, ycm.

$$x \times y = 18$$

$$y = 18 \div x$$

If you write equations such as these, the corresponding value of y can be calculated when the value of x is known.

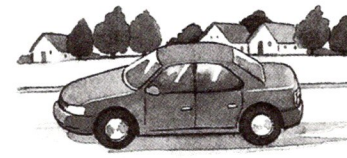
3 Find the values of y when x is 2.5, 8, and 10.

If y is inversely proportional to x, the products of the values of x and the corresponding values of y are always the same.

The following equation holds true:

$$y = \textbf{fixed value} \div x$$

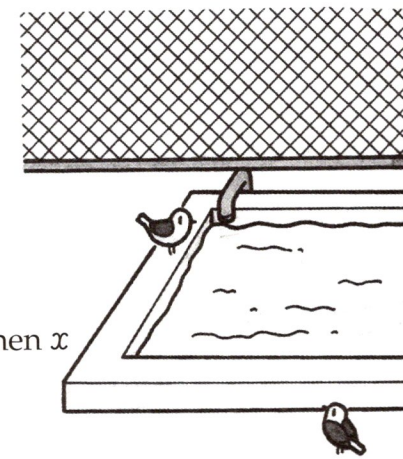
1. The distance between Higashigawa City and Nishida City is $180km$ in problem ① on page 86. Suppose the car's speed is xkm per hour and the time it takes to travel the distance is y hours. Write an equation to represent y.



2. Water is poured into a container that has a capacity of $24m^3$.

   (1) Suppose the amount of water poured in 1 hour is xm^3 and the time it takes to completely fill the container is y hours. Write an equation to represnt y.

   (2) Please find the value of y when x is 2, 5, and 10.



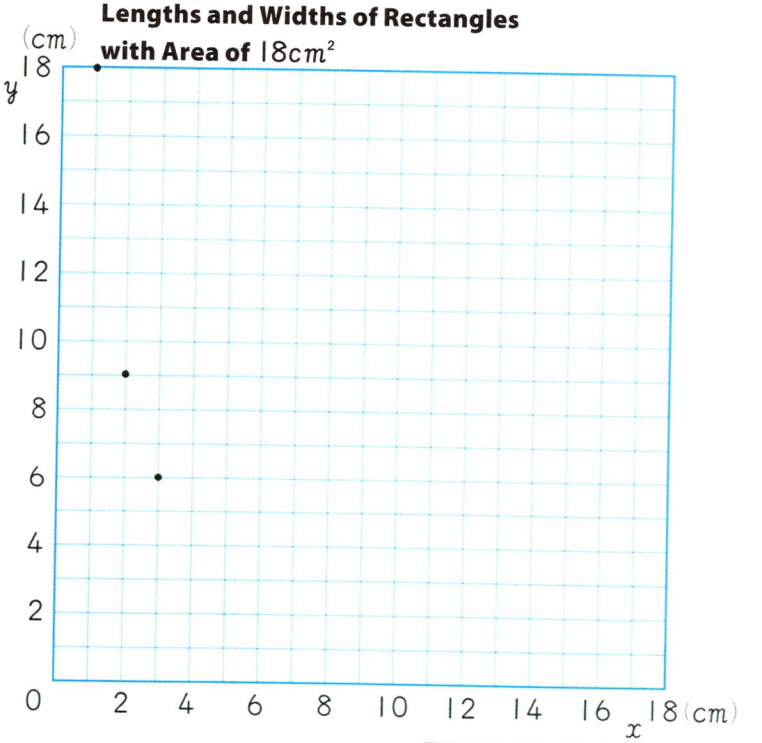
▶ The graph of an inverse proportion

2 Using Table (1) in [1] on page 84, draw a graph representing the inverse proportional relationship between the width and length of the rectangles and explore its characteristics.

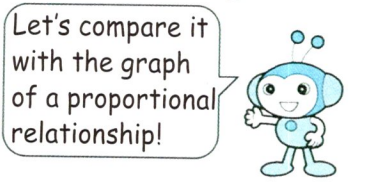
Lengths and Widths of Rectangles with Area of $18cm^2$

Length x (cm)	1	2	3	4	5	6	9	18
Width y (cm)	18	9	6	4.5	3.6	3	2	1

1 Plot points for x (length) and y (width) on the graph.

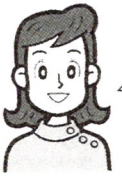


2 What are the characteristics of the graph?



Story of Math

Formulas and Proportion and Inverse Proportion



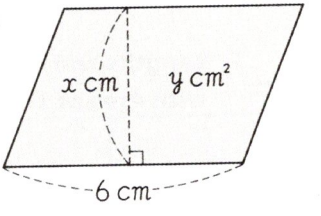
What kind of relationships are there between the area, base, and height of a parallelogram?

Area of Parallelogram = Base × Height



Mami

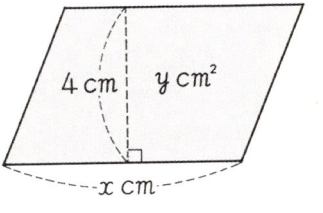
(1) Suppose the height is $x$cm and the area is $y$cm^2. When the length of the base is 6cm, the equation is $y = 6 \times x$ and the area is proprotional to the height.





Yuji

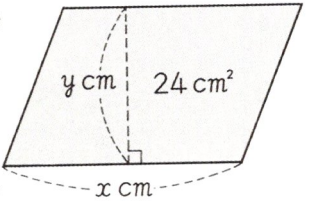
(2) Suppose the base is $x$cm and the area is $y$cm^2. If the height is fixed at 4cm, the equation is $y = x \times 4$ and the area is proportional to the length of the base.





Ritsuko

(3) Suppose the base is $x$cm and the height is $y$cm. If the area is fixed at 24cm^2, the equation is $24 = x \times y$ and the height is inversely proportional to the length of the base.



Check

1 In which cases are two quantities inversely proportional?

① The number of pages read and the number of pages remaining in a book

② The number of bottles and the amount in each bottle when 6ℓ of oil is divided equally

③ The number of items of the same price and their total cost

2 Please fill in the ☐ with the appropriate number, word, or symbol.

(1) It takes y minutes to fill a tank with water when the water is poured in at a rate of $x\ell$ per minute. If the value of x is doubled, the value of y is ☐.

(2) It took y hours to travel $90km$ at a speed of xkm per hour. The relationship between x and y can be expressed as: $y =$ ☐ $\div$ ☐, so y is ☐ to x.

Challenge

Let's Use Inverse Proportions!

Let's find the time it takes to travel from Tokyo Station to Hakata Station in different ways!



	Nozomi	Hikari	Airplane	Car	Bicycle
Speed (km/h)	232			80	13
Time	5h04m	6h18m	1h45m		

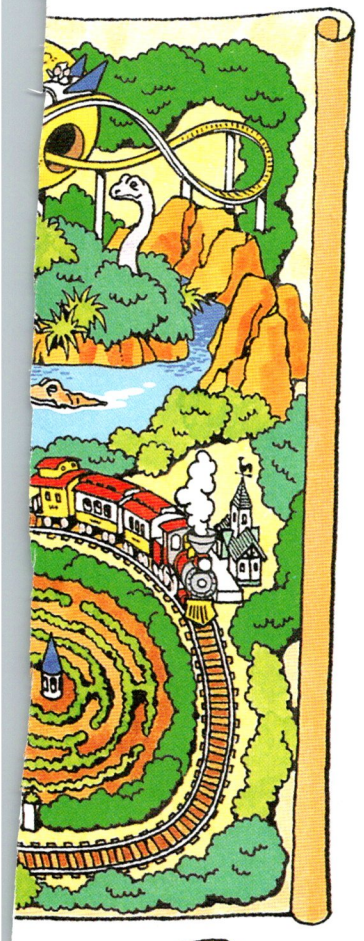
Life and Math

Amusement Park: Dreamland



Hitomi's group is at the amusement park, Dreamland.
1 adult and 4 children are in her group.

1 If they want to go on 4 rides, how should they buy tickets to keep the cost as low as possible?



Price Chart

Entrance Fee	Adult 500 yen Child 300 yen
Ride Tickets	1 Ticket 200 yen 10 Tickets 1600 yen

Ticket Package (Entrance Fee + 5-ride Ticket)	Adult 1400 yen Child 1200 yen

Ride	Ticket
1. Mysterious Coffee Cup	1 Ticket
2. Downstream Canoe Adventure	1 Ticket
3. Ferris Wheel in the Sky	1 Ticket
4. Dream Merry-Go-Round	1 Ticket
5. Circuit Race	2 Tickets
6. Exploring the Castle	2 Tickets
7. Maze Garden	2 Tickets
8. Locomotive in the Forest	3 Tickets
9. Jungle Cruise	3 Tickets
10. Roller Coaster in Space	3 Tickets



Let's ride 2, 4, 5, and 6!

Let's ride 5, 6, 7, and 10!



① Please choose what you want to ride, and think about which tickets you should buy.

Calculation Practice

① (1) $\frac{2}{9} \times \frac{5}{6}$ (2) $\frac{5}{18} \times \frac{6}{25}$ (3) $\frac{3}{4} \times \frac{2}{7}$ (4) $\frac{9}{16} \times \frac{8}{27}$

(5) $\frac{3}{5} \times \frac{4}{7}$ (6) $\frac{3}{10} \times \frac{4}{9}$ (7) $\frac{8}{15} \times \frac{10}{13}$ (8) $\frac{6}{11} \times \frac{22}{27}$

(9) $2\frac{3}{8} \times \frac{5}{7}$ (10) $\frac{3}{4} \times 6$ (11) $\frac{4}{7} \times 5\frac{1}{4}$ (12) $8 \times \frac{5}{12}$

(13) $\frac{2}{3} \times 3\frac{1}{2}$ (14) $15 \times 2\frac{3}{10}$ (15) $3\frac{5}{9} \times 5\frac{1}{16}$

(16) $2\frac{2}{9} \times 12$ (17) $4\frac{3}{8} \times 1\frac{3}{14}$ (18) $4\frac{7}{12} \times \frac{1}{22}$

② (1) $\frac{3}{7} \div \frac{2}{3}$ (2) $\frac{12}{25} \div \frac{9}{10}$ (3) $\frac{5}{6} \div \frac{7}{8}$ (4) $\frac{9}{14} \div \frac{5}{21}$

(5) $4\frac{5}{6} \div \frac{4}{9}$ (6) $5 \div \frac{3}{11}$ (7) $\frac{3}{8} \div 2\frac{1}{4}$ (8) $\frac{6}{7} \div 9$

(9) $\frac{2}{5} \div 1\frac{1}{7}$ (10) $2\frac{8}{11} \div 25$ (11) $5\frac{5}{6} \div 1\frac{7}{8}$

(12) $2\frac{5}{6} \div \frac{8}{15}$ (13) $16 \div 6\frac{6}{7}$ (14) $3\frac{3}{16} \div 3\frac{3}{8}$

①	$\frac{1}{6}$	$\frac{2}{15}$	$26\frac{2}{3}$	$\frac{12}{35}$	3	$\frac{5}{27}$	$5\frac{5}{16}$	$\frac{1}{15}$	$2\frac{1}{3}$
	$1\frac{39}{56}$	$34\frac{1}{2}$	$\frac{3}{14}$	$\frac{5}{24}$	18	$4\frac{1}{2}$	$\frac{16}{39}$	$\frac{4}{9}$	$3\frac{1}{3}$
②	$\frac{20}{21}$	$5\frac{5}{16}$	$10\frac{7}{8}$	$\frac{2}{21}$	$\frac{8}{15}$	$\frac{17}{18}$	$\frac{1}{6}$		
	$\frac{7}{20}$	$2\frac{1}{3}$	$\frac{9}{14}$	$18\frac{1}{3}$	$2\frac{7}{10}$	$3\frac{1}{9}$	$\frac{6}{55}$		

Please color the answers.



③ (1) $\frac{1}{6} \times 2\frac{3}{11} \times 2\frac{4}{5}$ (2) $3\frac{3}{4} \times 6 \times 2\frac{2}{9}$

(3) $\frac{1}{12} \div 1\frac{1}{3} \div \frac{1}{16}$ (4) $14 \div 1\frac{3}{5} \div \frac{7}{9}$

(5) $4\frac{1}{6} \times 14 \div \frac{7}{9}$ (6) $2\frac{6}{7} \times \frac{24}{25} \div 1\frac{1}{3}$

(7) $\frac{10}{21} + \frac{3}{10} \times \frac{5}{9}$ (8) $4\frac{4}{5} - \frac{5}{6} \div \frac{2}{3}$

(9) $\frac{4}{9} \times \left(\frac{17}{20} - \frac{5}{8}\right)$ (10) $\left(2\frac{1}{4} - 1\frac{5}{16}\right) \div \frac{3}{4}$

(11) $2\frac{3}{8} \div \left(\frac{2}{3} + 7\frac{1}{2} \times \frac{1}{3}\right)$

④ Please convert all numbers to fractions and calculate.

(1) $0.8 \times \frac{4}{9}$ (2) $1.25 \times \frac{2}{5}$ (3) $0.6 \div \frac{3}{7}$ (4) $1\frac{1}{3} \div 1.6$

(5) $3.75 \div \frac{5}{12} \times 4.5$ (6) $\frac{7}{15} - 1\frac{7}{9} \div 6 \times 0.75$

(7) $8 \div 6 \div 15 \times 9 \div 4 \times 5$ (8) $14 \div 4 \times 9 \times 5 \div 21 \div 6$

③	$1\frac{1}{4}$	1	$\frac{9}{14}$	50	$\frac{3}{4}$	$11\frac{1}{4}$	$1\frac{2}{33}$	75	$3\frac{11}{20}$	$\frac{1}{10}$	$2\frac{2}{35}$
④	$1\frac{1}{4}$	1	$\frac{1}{2}$	$40\frac{1}{2}$	$\frac{5}{6}$	$\frac{16}{45}$	$\frac{11}{45}$	$1\frac{2}{5}$			

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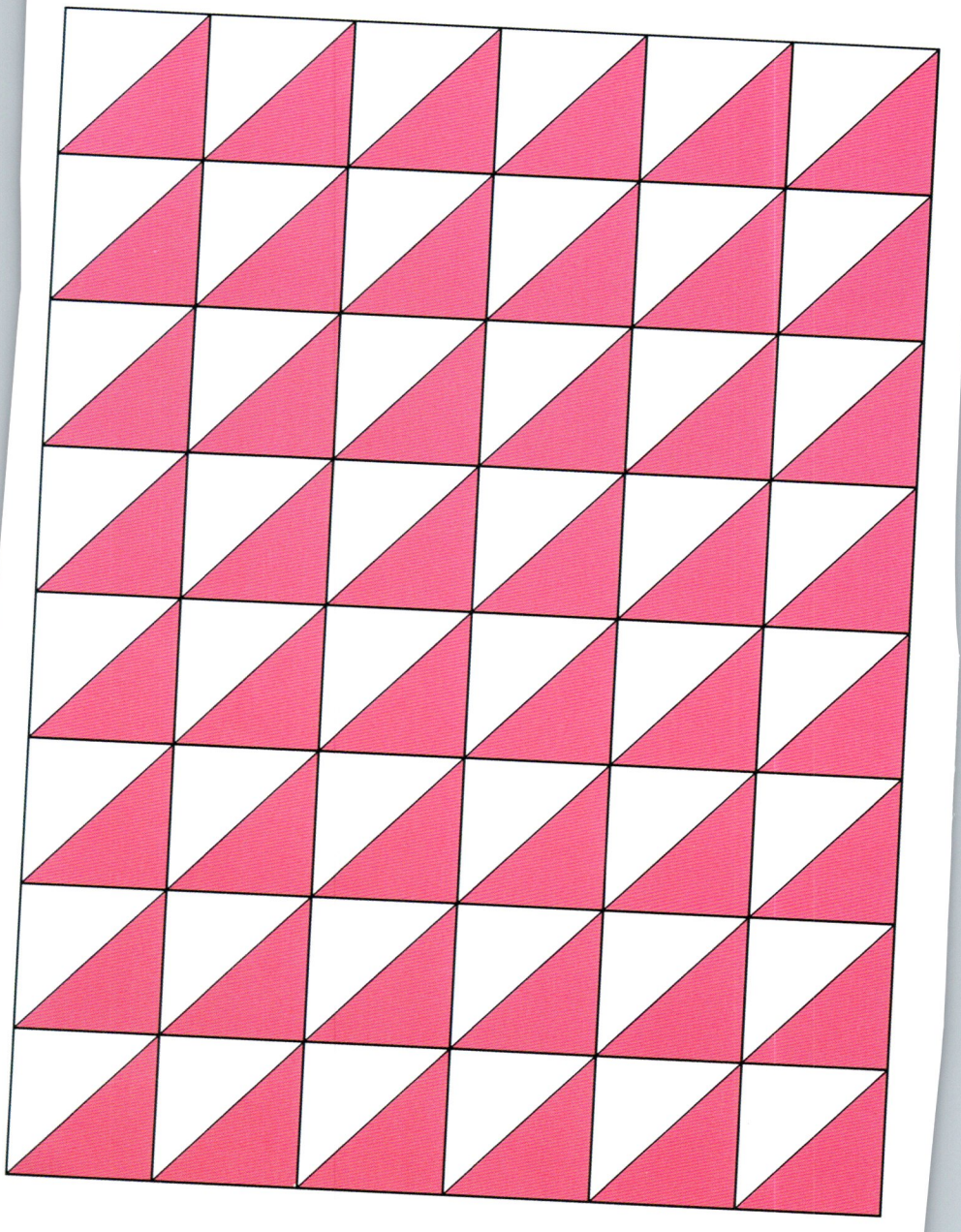

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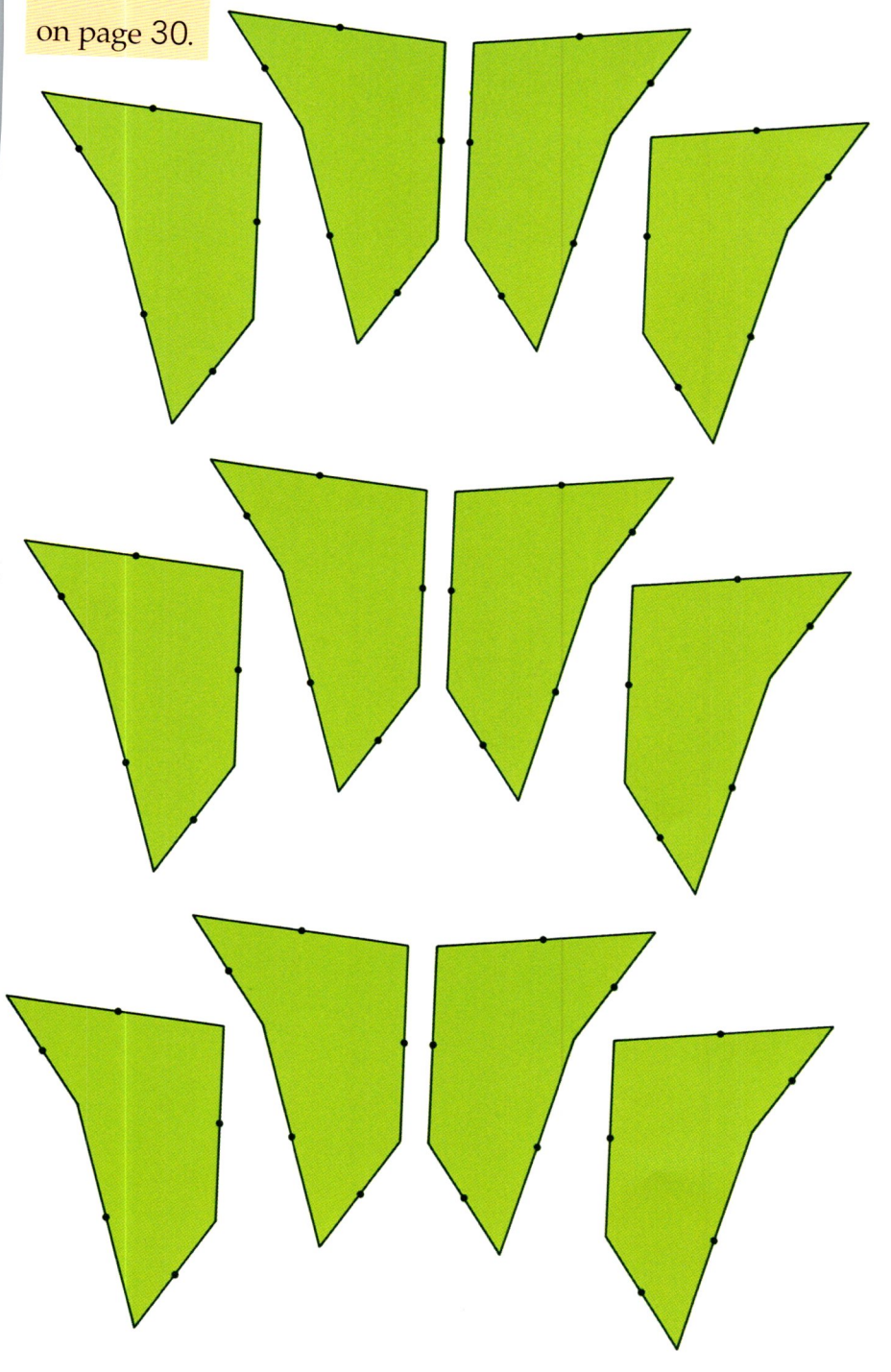
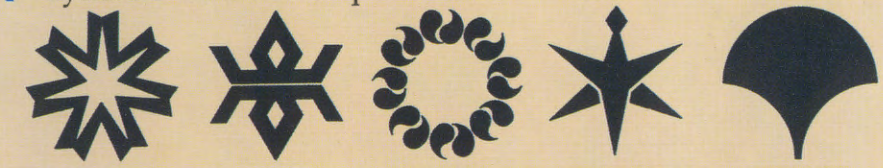


Symbols with Symmetric Shapes

♠ Symbols for different prefectures



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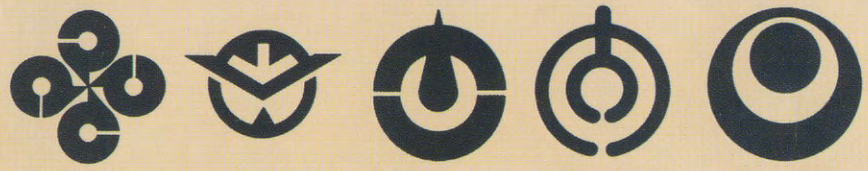

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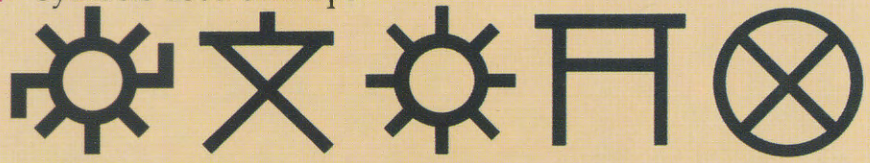
♣ Symbols used on maps



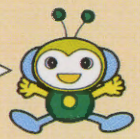
◆ Traffic signs



♥ Others



Which ones have line or point symmetry? How many lines of symmetry are there?



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Mathematics 6B

for Elementary School



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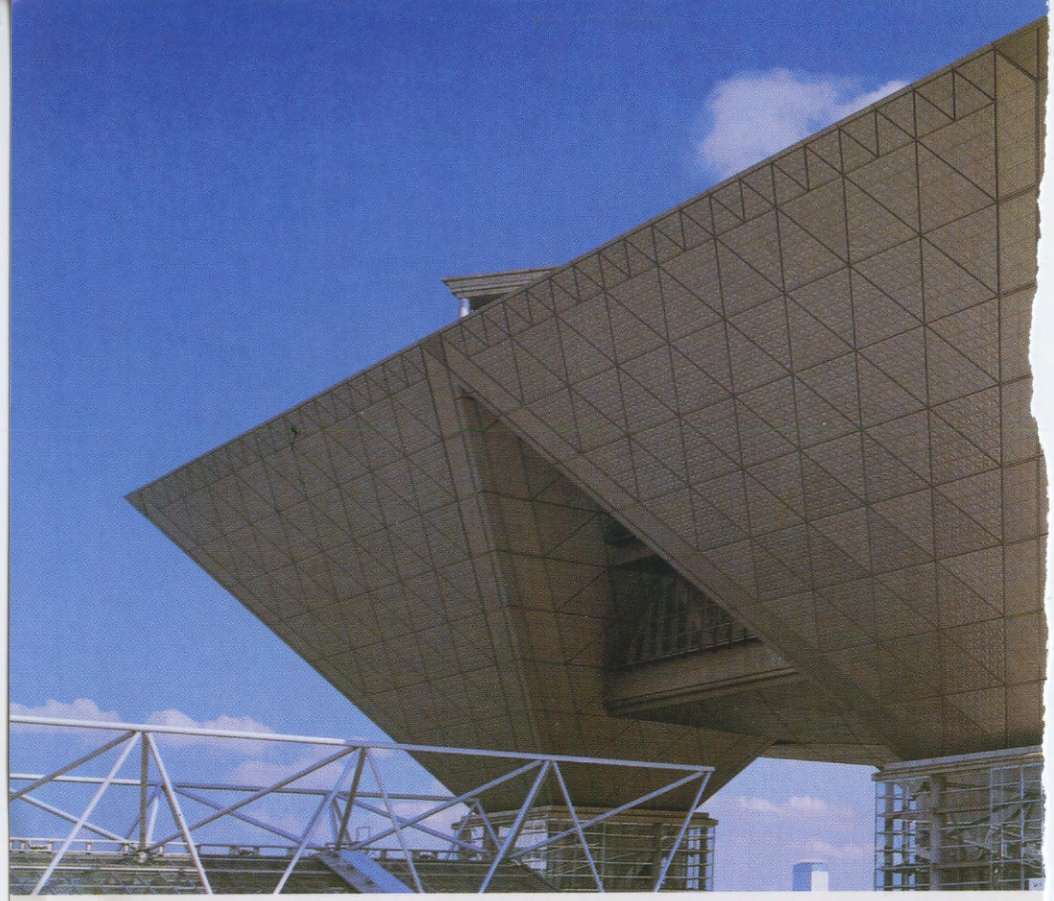

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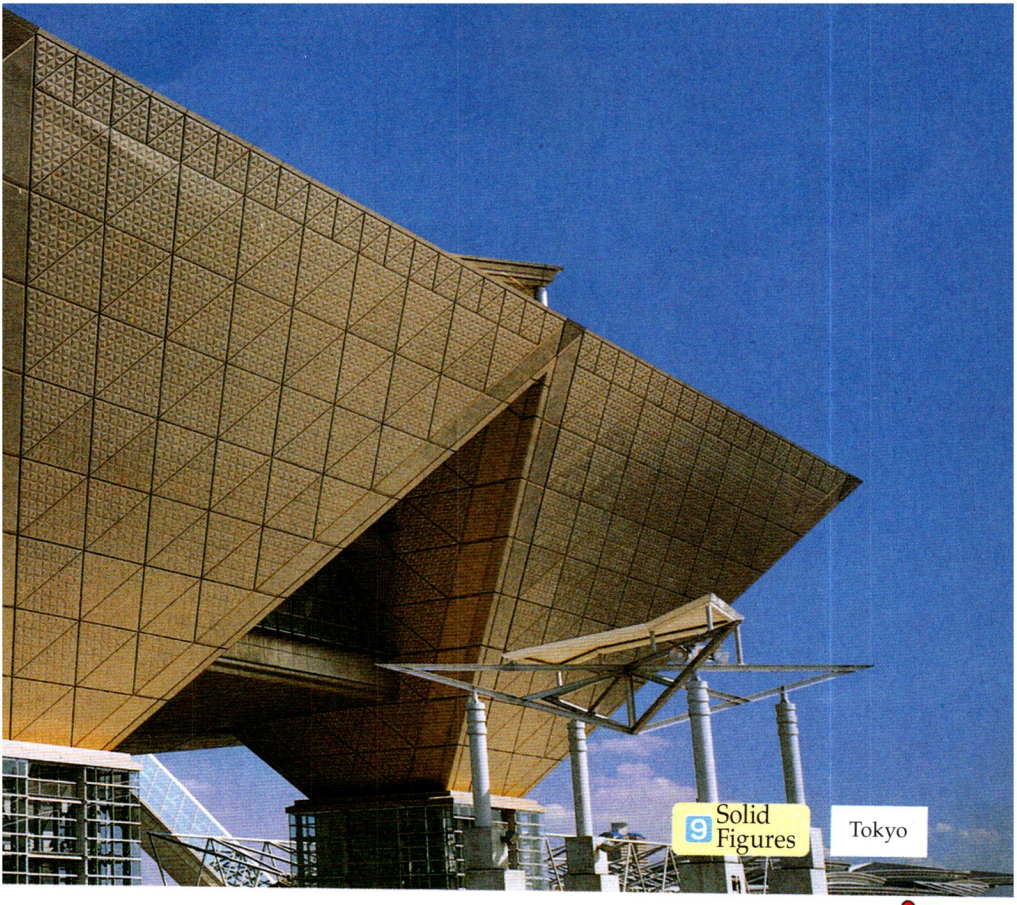
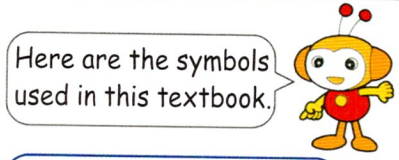
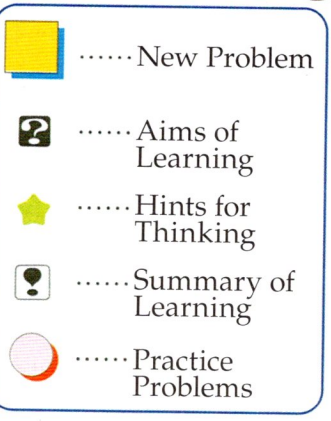
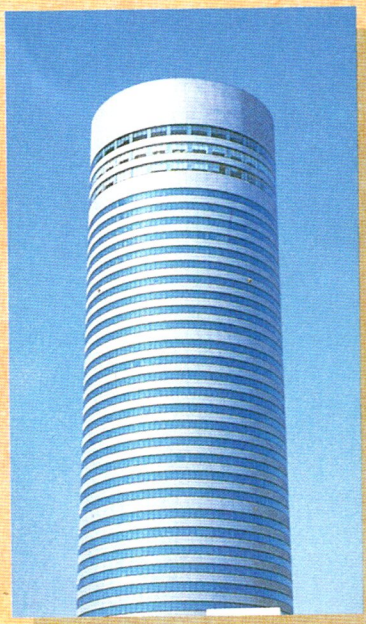


6 Grade 6A

1 Multiplication and Division of Fractions
2 Multiplication of Fractions
3 Division of Fractions
4 Symmetric Shapes
5 Ratio and Its Value
6 Enlarged Drawings and Reduced Drawings
7 Proportional Relationships
8 Inverse Proportions







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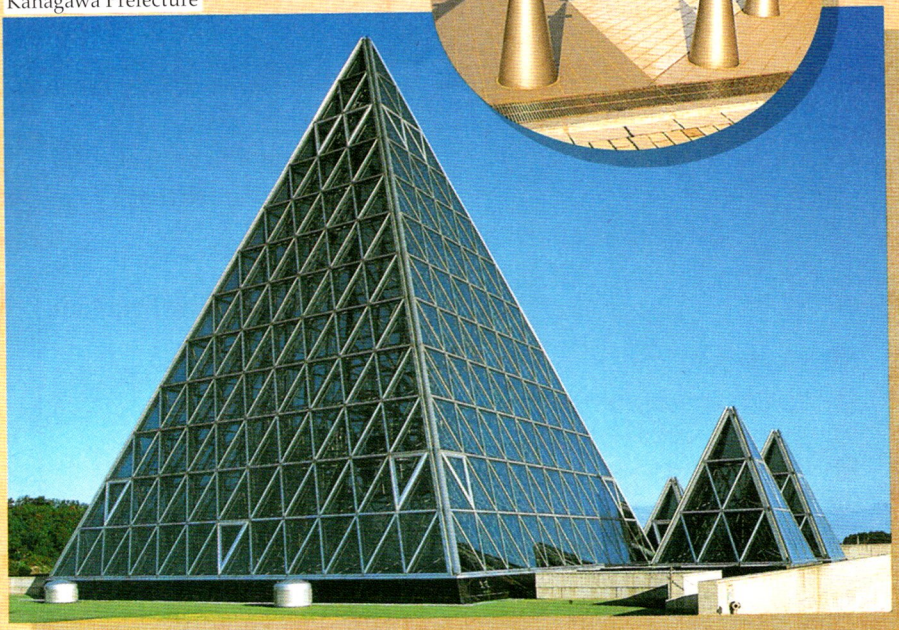


Nima Town,
Shimane Prefecture



Kanazawa City,
Ishikawa Prefecture

9 Solid Figures



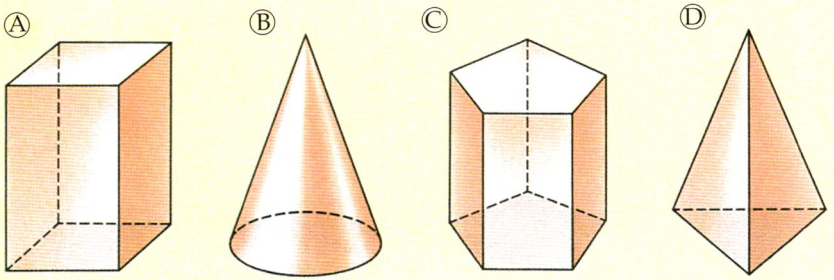
Yokohama City,
Kanagawa Prefecture

Tiling of Regular Polygons

9 Solid Figures

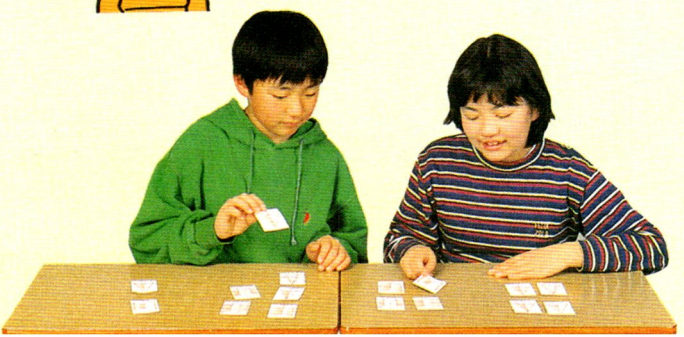




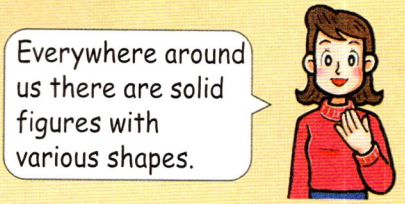




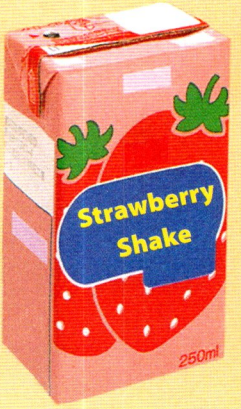
Using the solid figure cards on page 89, let's sort the figures above into two groups!











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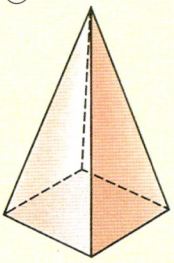

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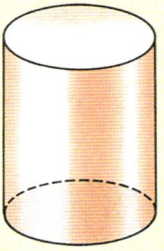

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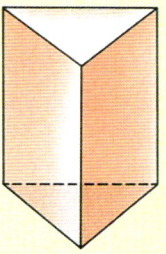

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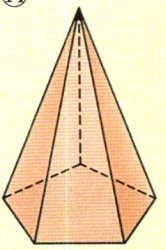


(1)

Ⓐ

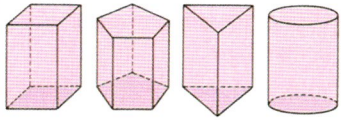
(2)

Ⓑ

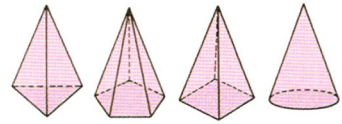
Let's investigate the characteristics of various solid figures!


Makoto

(1)



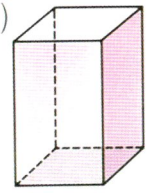
(2)



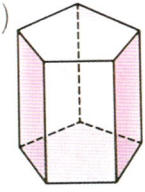
1 Prisms and Cylinders

1 Let's investigate the characteristics of the solids below!

(1)



(2)



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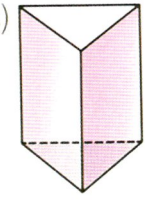
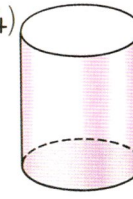


(4)



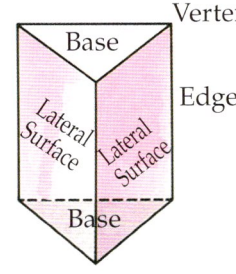
1 Which face is parallel to the bottom face?

2 Are the top and the bottom faces congruent?

3 Which solid figures are surrounded by planes only? What are the shapes of the planes around those figures?

Solids like (1), (2), and (3) above are called **prisms**. The two faces at the top and bottom of a prism are called **bases**. The quadrilateral faces around the bases are called **lateral surfaces**.

In the prisms above, all the lateral surfaces are rectangles.



The two bases of a prism are congruent polygons, and they are parallel.

4 Are the lateral surfaces of the prism above perpendicular to the bases?

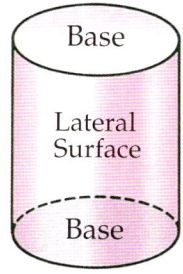
Prisms that have bases that are triangles, rectangles, and pentagons, are called triangular prisms, rectangular prisms, and pentagonal prisms, respectively.

5 Let's compare the number of lateral surfaces with the number of edges in the base of different prisms!

6 What kinds of prisms are boxes and cubes?

7 In the solid figure (4) on the previous page, what are the shapes of the base and the lateral surface?

Solid figures like (4) are called **cylinders**.



In a cylinder, the two bases are congruent circles and they are parallel.

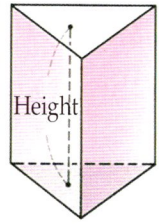
The lateral surface of a cylinder is a curved surface.

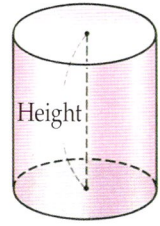
1 Let's find some objects around you that are shaped like prisms and cylinders!





The length of a perpendicular segment between a point on a base and a point on the other base is called the **height** of a prism or a cylinder.

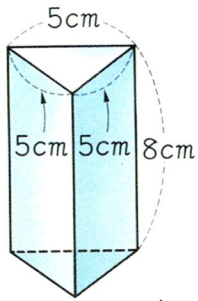




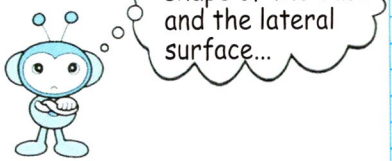
8 Let's figure out a way to measure the height of a prism and a cylinder!

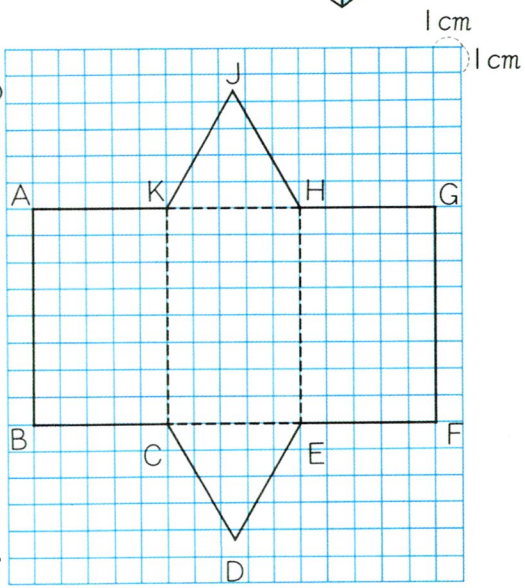
▶ The net of a prism

2 Let's make a triangular prism like the one on the right with construction paper!



? Let's draw a net of the triangular prism!

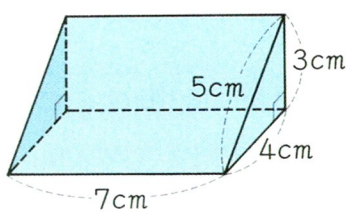




1 In the net on the right, where should you look to find the height?

2 Draw the net shown on the right and then construct a triangular prism.

1 See the prism on the right.



(1) What shape are the bases of this prism?

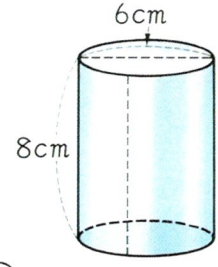
(2) What is the name of this prism?

(3) What is the height of this prism?

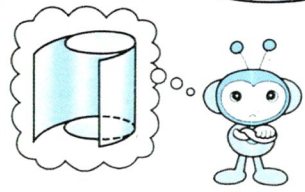
(4) Please draw a net of this prism.

▶ The net of a cylinder

3 Let's make a cylinder like the one on the right with construction paper!



? Let's think about how to draw a net of the cylinder!

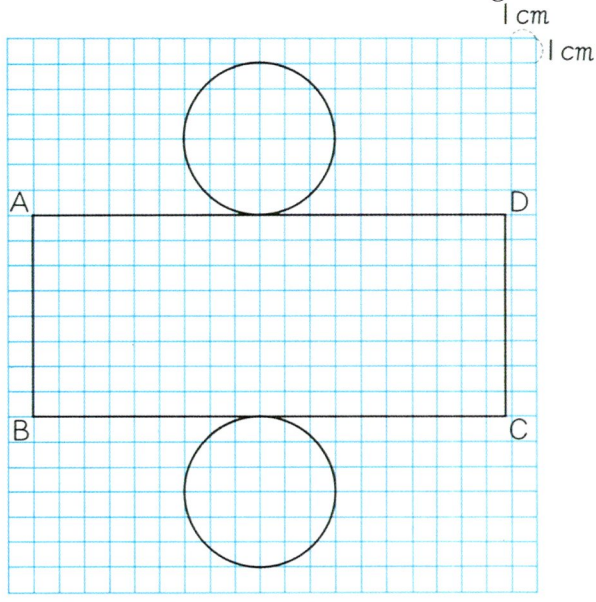


1 What shape will you get when you open up the lateral surface of the cylinder?

In a net of a cylinder, the lateral surface is a rectangle.

2 In the net on the right, which part of the base is the length of edge AD equal to?

3 How many cm is edge AD?

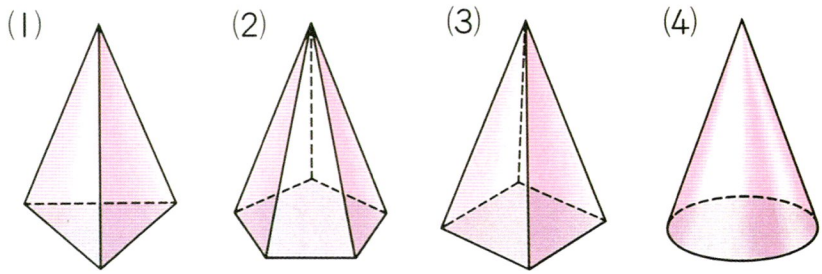


4 Draw the net like the one above and then construct a cylinder!

1 A cylinder has a height of 10cm and a base circumference of 15.7cm. Please draw a net of this cylinder.

2 Pyramids and Cones

1 Let's investigate the characteristics of the solids below!



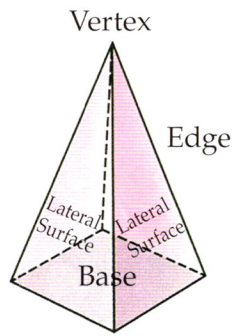
1 What shapes are the bottom faces?

2 Which solids are surrounded by planes only?
Also, what shapes are the faces surrounding the solid figures?

Figures like (1), (2), and (3) above are called **pyramids**.

In a pyramid, the polygon at the bottom is called the base. The triangles around the figure are called lateral surfaces.

All lateral surfaces of a pyramid meet at the vertex of the pyramid.



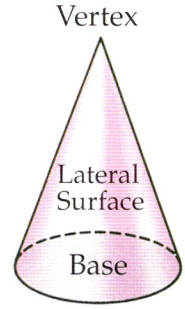
In the pyramids above, the bases are regular polygons. The lateral surfaces are congruent isosceles triangles.

Pyramids that have bases that are triangles, rectangles, and pentagons, are called triangular pyramids, rectangular pyramids, and pentagonal pyramids, respectively.

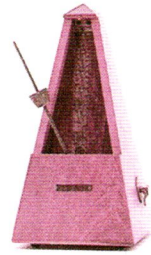
3 Let's investigate the number of lateral surfaces and the number of edges on the bases of various pyramids!

4 What shapes are the base and the lateral surface of the solid figure (4) on the previous page?

Solid figures like (4) are called **cones**.
The shape of the base of a cone is a circle.
The lateral surface of a cone is a curved surface.

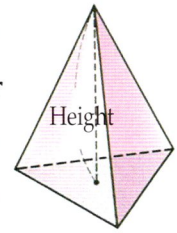


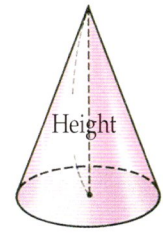
1 Let's find some objects around you that are in the shape of a pyramid or a cone!



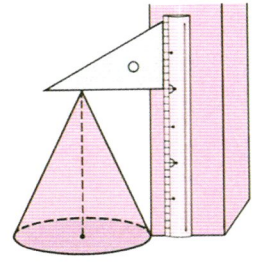


In a pyramid or a cone, the length of the perpendicular segment from the vertex to the base is called its height.



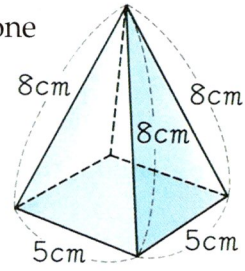


5 Let's figure out how to measure the height of a pyramid or a cone!



▶ The net of a pyramid

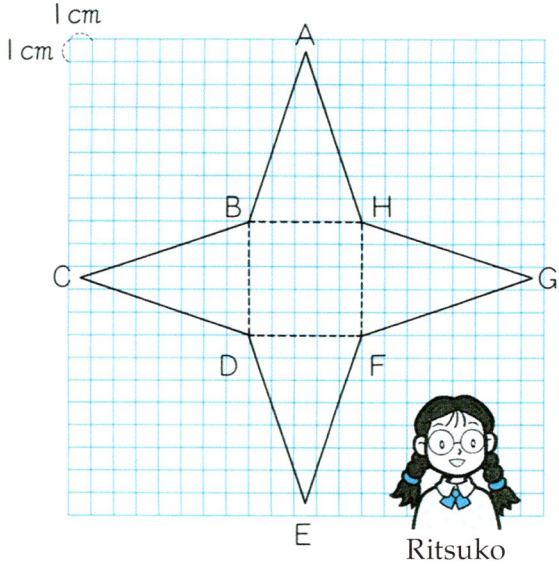
2 Let's make a square pyramid like the one on the right with construction paper!



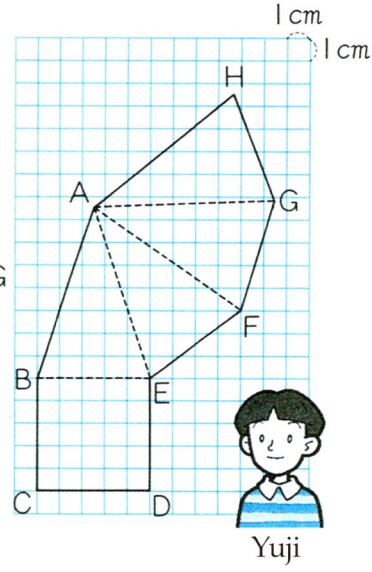
? Let's draw a net of a square pyramid!

Ritsuko and Yuji drew nets like the ones shown below.

1 Let's explain the way they drew their nets!



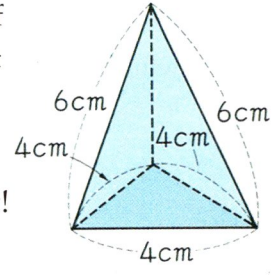
Ritsuko



Yuji

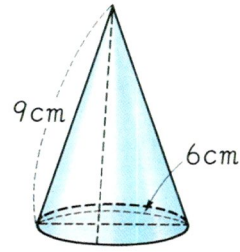
2 Please draw a net like either one of the ones above, and then construct a square pyramid.

1 Let's draw a net of a triangular pyramid like the one on the right!



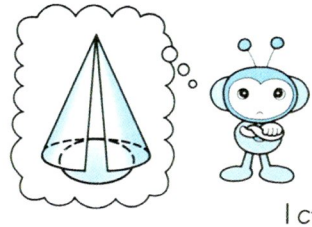
The net of a cone

3 Let's make a cone like the one on the right with construction paper!

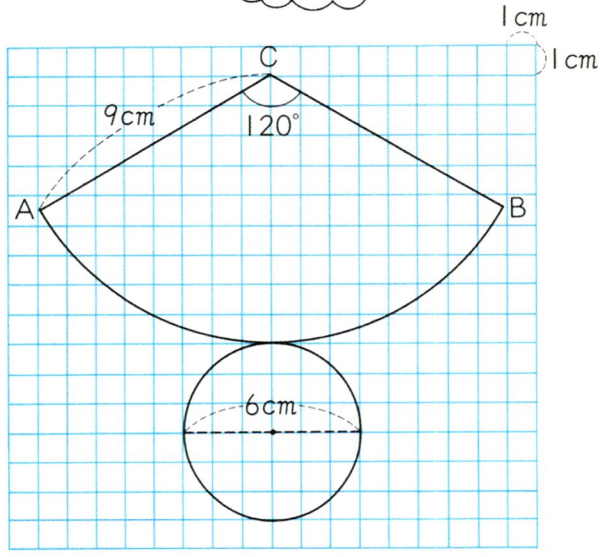


? Let's think about how to draw a net of a cone!

1 What shape will you get when you open up the lateral surface of a cone?



In a net of a cone, the shape of the lateral surface is a sector.

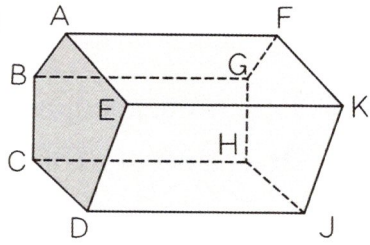


2 Let's draw a net like the one on the right and make a cone!

1 The net of a cone has a lateral surface that is a sector. The radius is 8cm and the central angle is 90° . Please draw a net of this cone.

Practice

① See the prism below:



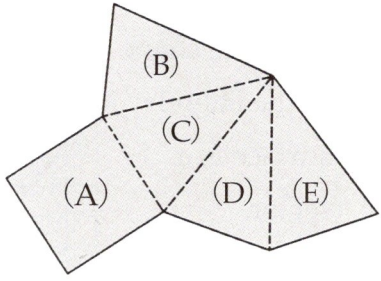
(1) What is the shape of the base of this prism?

(2) What is the name of this prism?

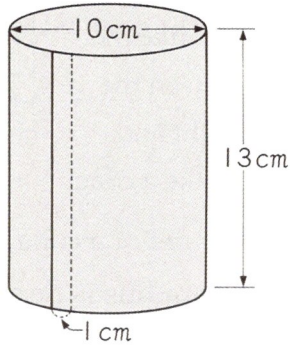
(3) Which face is parallel to face ABCDE?

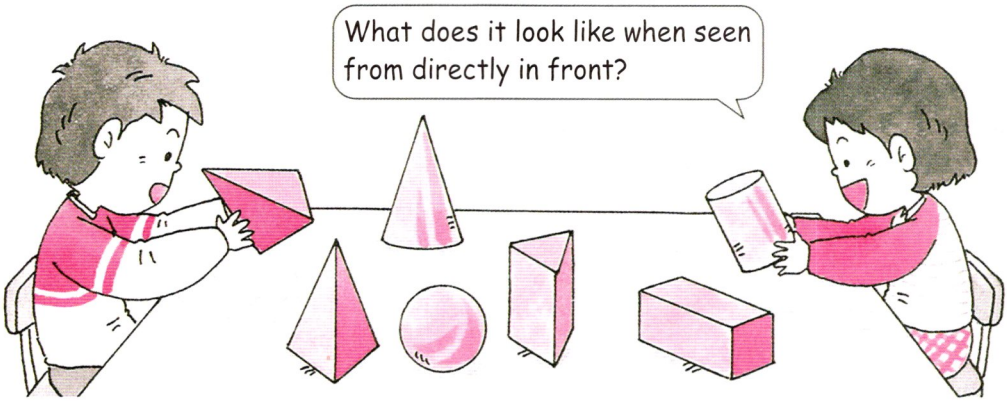
(4) Which edges are perpendicular to the base?
Are all of these edges the same length?

② The figure on the right is a net of what solid figure? Which face is the base and which faces are the lateral surfaces of this figure?



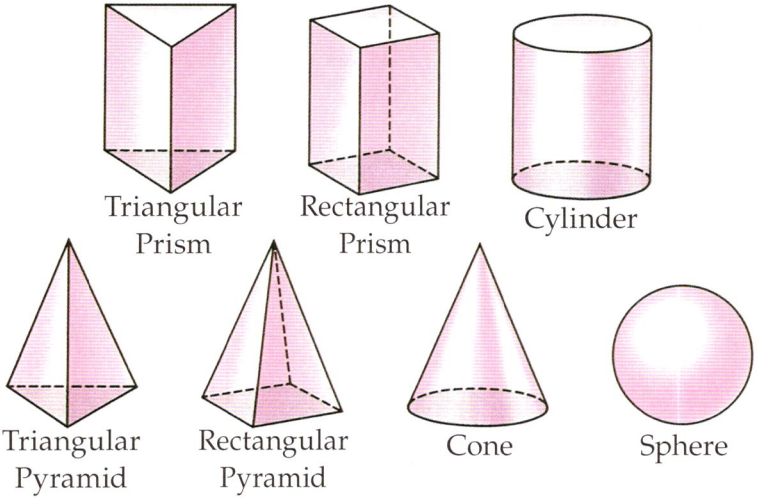
③ In order to make the lateral surface of the cylinder on the right out of construction paper, what size rectangle should you cut out of the paper?
Please make the glue area 1cm wide.





3 Looking at Shapes From Directly in Front and Directly Above

1 Let's investigate the shapes of various solid figures looking at from directly in front and directly above.

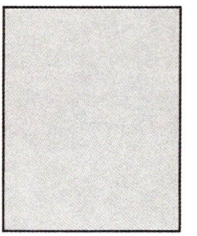


1 If you look at the solid figures above from directly in front, what shapes will you see?

2 If you look at these figures from directly above, what shapes will you see?

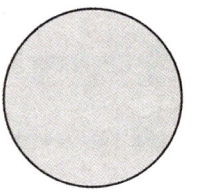
3 Let's collect solid figures that look like shape (1) on the right when seen from directly in front!
Can you tell whether that solid figure is a prism or a cylinder just by looking at.

(1)

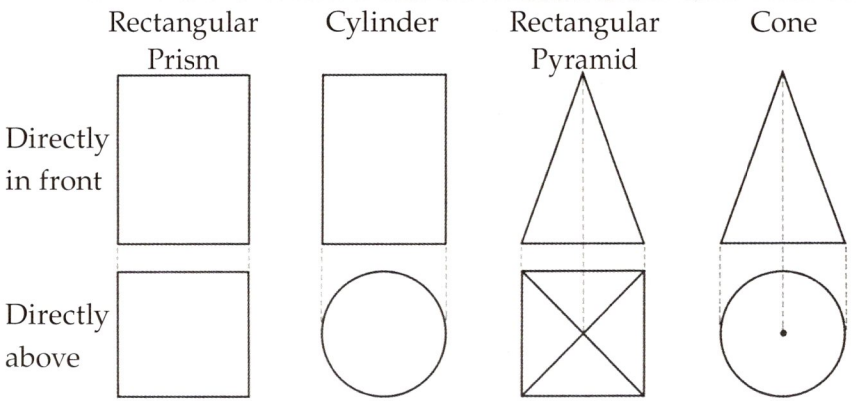


4 Let's collect solid figures that look like shape (2) on the right when seen from directly above!
Can you tell if the figure is a cylinder, cone, or sphere just by looking at.

(2)



2 Let's represent a solid on a plane by combining the shape seen from directly in front and the shape seen from directly above!

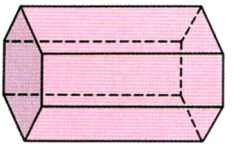
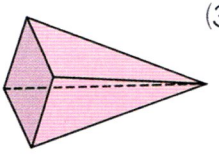
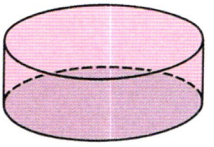


You can tell what a certain solid figure is by combining the shape seen from directly in front and the shape seen from directly above.

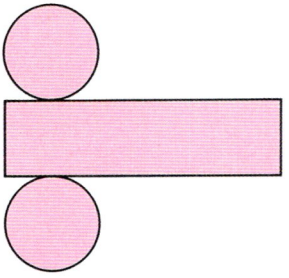
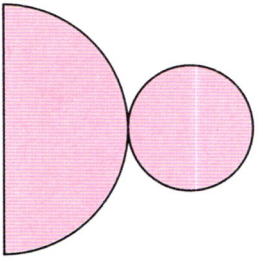
1 In the figures above, where are the shapes of the bases of the solid figures shown?

Check

1 What are the solid figures below called?

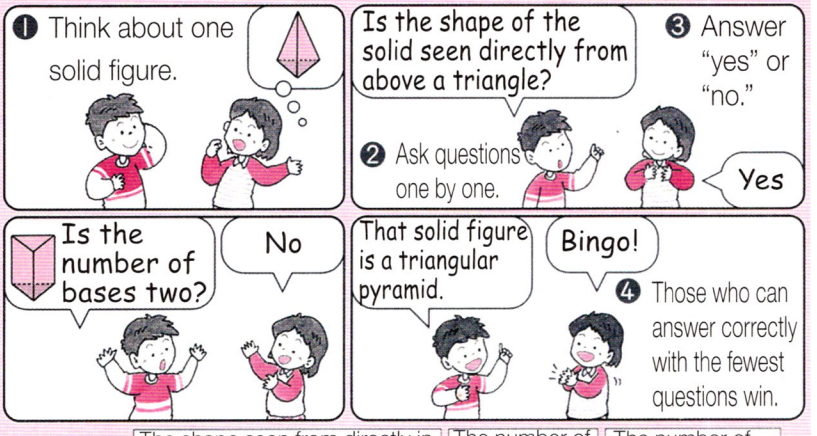
(1)  (2)  (3) 

2 The figures below are the nets of which solid figures?

(1)  (2) 

Challenge

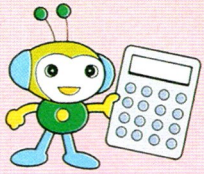
Let's Play "Name the Solid Figure" Game!

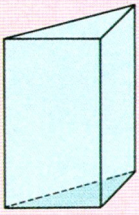


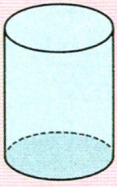
Question Cards

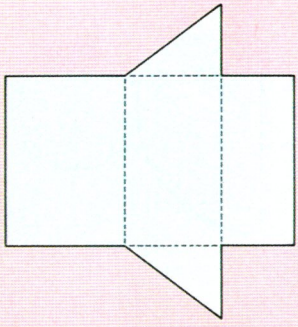
The shape seen from directly in front is ( )	The number of edges is ( )	The number of bases is ( )
The shape seen from directly above is ( )	The number of vertices is ( )	The number of lateral surfaces is ( )

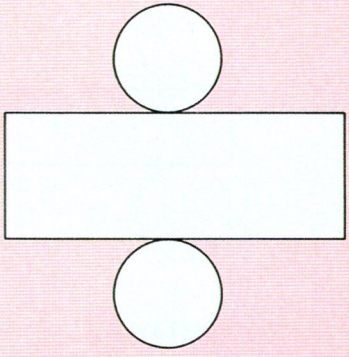
10 Surface Area and Volume of Solid Figures

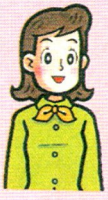












In the last chapter, we learned the characteristics of solid figures and how to draw their nets. What else do you want to learn?

I want to make a pretty box by pasting colored paper on the faces. What should I do to find the total area of faces?

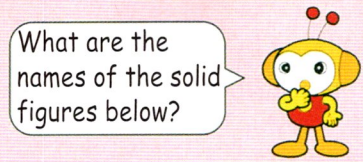


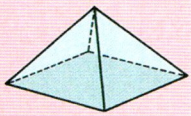
Kazuya

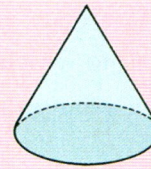
In 5th grade we learned how to find the volume of cubes and rectangular prisms. I wonder how we can find the volume of various solids.

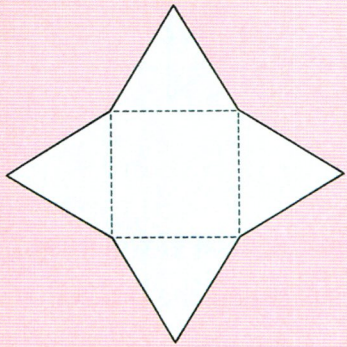


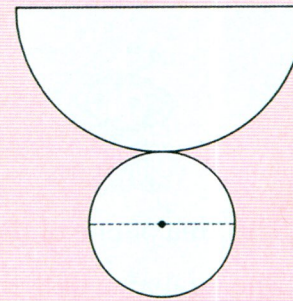
Mami









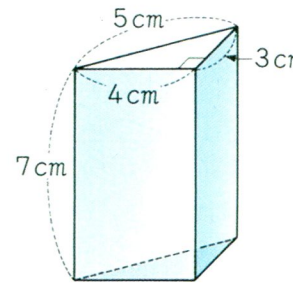


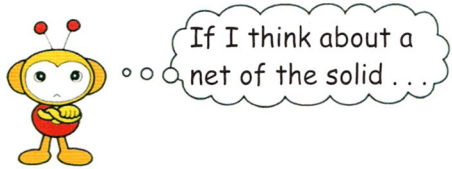
Let's think about how to find the total area of the surface and the volume of solid figures!

1 Surface Area of Solid Figures

▶ The surface area of prisms

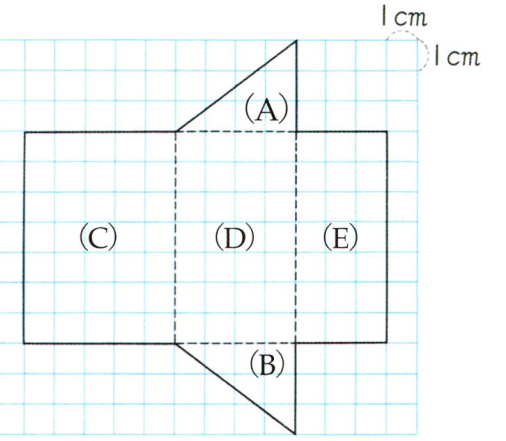
1 Let's think about how to find the surface area of the triangular prism on the right!

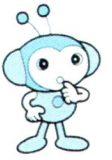




1 Let's think about it by drawing a net of the solid!

2 Faces (A), (B), (C), (D), and (E) correspond to which faces of the triangular prism?





The area of one of the bases is called the **area of the base**.

The total area of the lateral surfaces is called the **lateral surface area**.

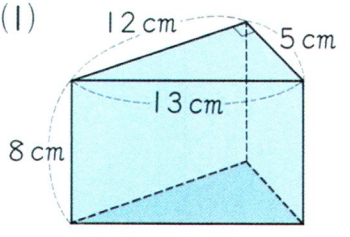
The total area of the faces is called the **surface area**.

3 How many cm^2 is the area of the base in the triangular prism above?
How many cm^2 is the lateral surface area?

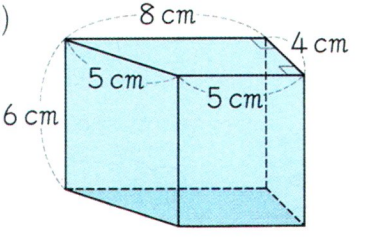
4 How many cm^2 is the surface area of the triangular prism above?

1 Let's find the surface areas of the prisms below!

(1)

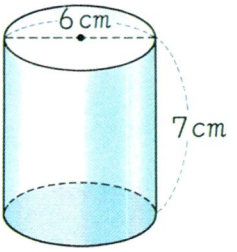


(2)



The surface area of cylinders

2 Let's think about how to find the surface area of the cylinder on the right!

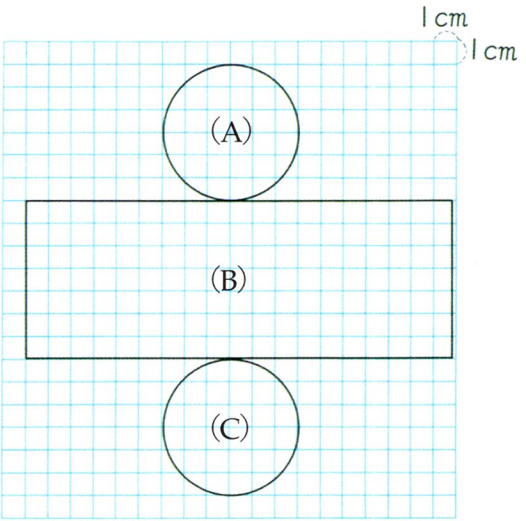


1 Let's think about it by drawing a net!

2 To which faces of the cylinder do faces (A), (B), and (C) correspond?

3 How many cm^2 is the area of the base of the cylinder? How many cm^2 is the lateral surface area?

4 How many cm^2 is the surface area of the cylinder above?

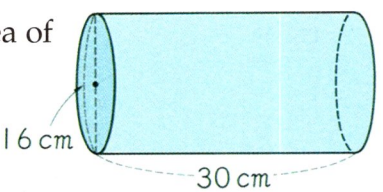


The lateral surface area and the surface area of prisms and cylinders can be found by using the following formulas:

Lateral surface area of a prism or a cylinders
= **Length around the base** × **Height**

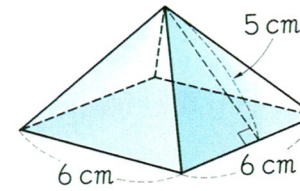
Surface area of a prism or a cylinders
= **Area of the base** × 2 + **Lateral surface area**

1 Please find the surface area of the cylinder on the right.



The surface area of pyramids

3 Let's think about how to find the surface area of the square pyramid on the right!

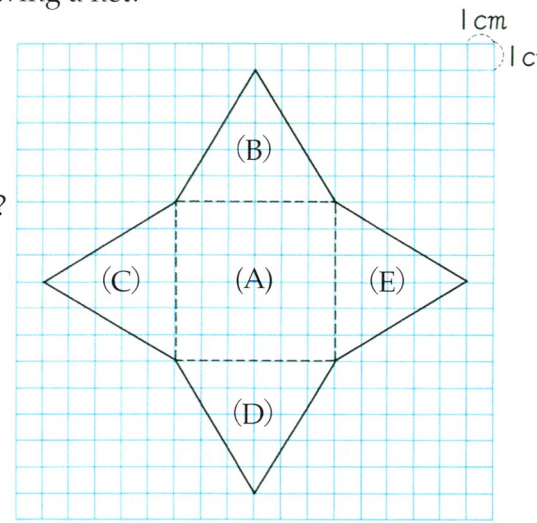


1 Let's think by drawing a net!

2 Faces (A), (B), (C), (D), and (E) correspond to which faces of the rectangular prism?

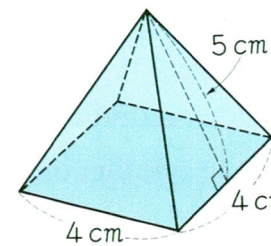
3 How many cm^2 is the area of the triangular face (B)?

4 How many cm^2 is the area of the base?
How many cm^2 is the lateral surface area?



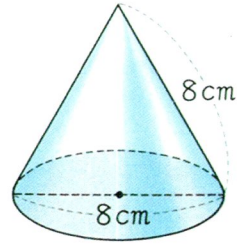
5 How many cm^2 is the surface area of the square pyramid above?

1 Please find the surface area of the square pyramid on the right.

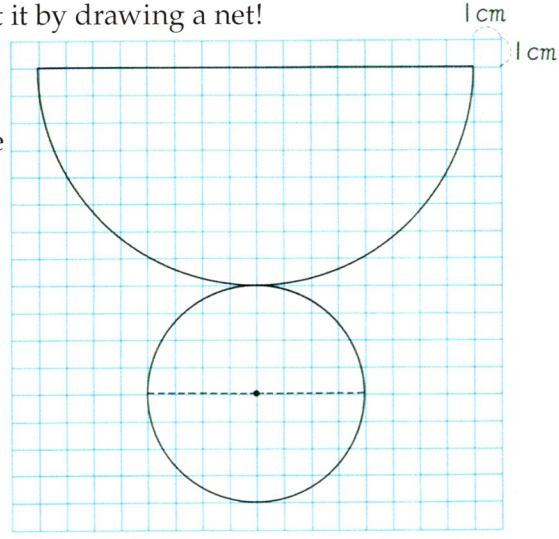


The surface area of cones

4 Let's think about how to find the surface area of the cone on the right!



1 Let's think about it by drawing a net!



2 What is the area of the base of the cone in cm^2? How many cm^2 is the lateral surface area?

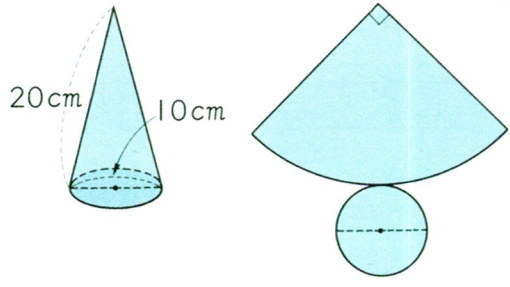
3 How many cm^2 is the surface area of the cone above?

The surface area of pyramids and cones can be found by using the following formula:

Surface area of a pyramid or a cone

= Area of the base + Lateral surface area

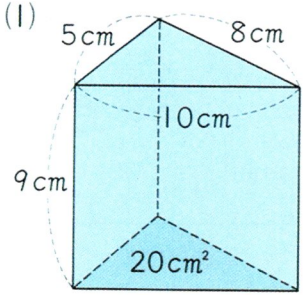
1 Please find the surface area of the cone on the right.



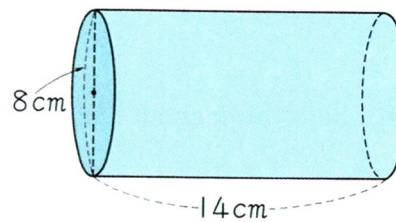
Practice 1

① Please find the surface areas of the prism and the cylinder below.

(1)

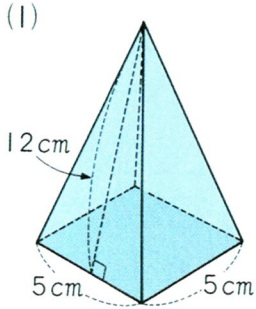


(2)

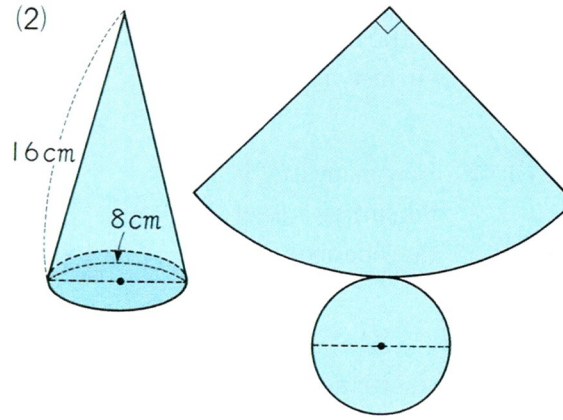


② Please find the surface areas of the pyramid and the cone below.

(1)



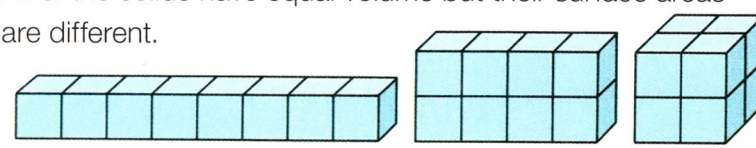
(2)



Story of Math

Surface Area of Figures that Have Equal Volume

We can make various solids using eight cubes of $1cm^3$ each. All of the solids have equal volume but their surface areas are different.

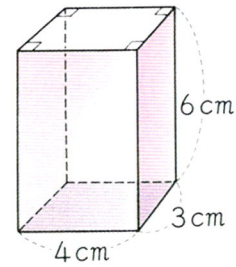


( ) cm^2 ( ) cm^2 ( ) cm^2

2 Volume of Solid Figures

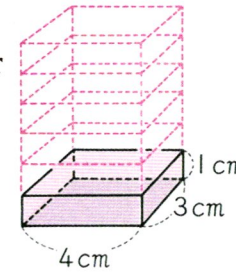
The volume of prisms

1 Let's think about how to find the volume of the rectangular prism on the right!



1 Please find the volume using the formula.

You can think of the rectangular prism on the right as a rectangular prism made by piling up six smaller rectangular prisms with heights of 1 cm each.



2 How many cm^2 is the area of the base in a smaller rectangular prism? How many cm^3 is its volume?

3 For the smaller rectangular prism, please compare the number that represents the area of the base with the number that represents the volume.

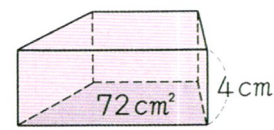
For the rectangular prism with a height of 1 cm, the number that represents the area of the base and the number that represents the volume are equal.

4 How many times as much is the volume of a 6 cm high rectangular prism as the volume of a 1 cm high rectangular prism with the same base area?

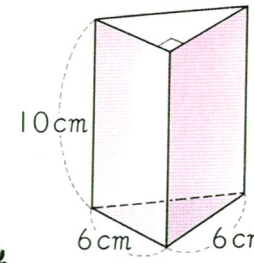
$$\underbrace{3 \times 4}_{\text{Area of the base}} \times 6 = \square \quad (cm^3)$$

The volume of a rectangular prism can be found by:
Area of the base × Height

1 Please find the volume of the prism on the right.



2 Let's think about how to find the volume of the triangular prism on the right!




Ritsuko

Since it is half of the rectangular prism ...

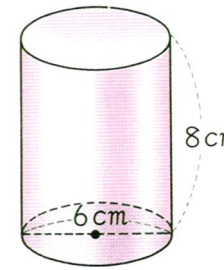

Minoru

By comparing the base area and the volume of the triangular prism that has a height of 1cm ...

The volume of prisms can be found by using the following formula:
Volume of prism = Area of the base × Height

▶ The volume of cylinders

3 Let's think about how to find the volume of the cylinder on the right!




Mami

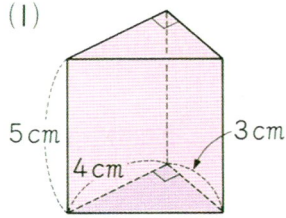
By comparing the area of the base and the volume of the cylinder with a height of 1cm...

The volume of prisms and cylinders can be found by using the following formula:

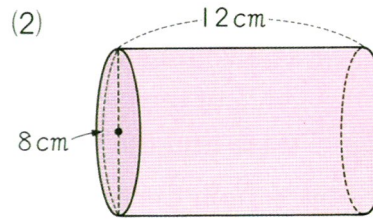
Volume of a prism or a cylinder = Area of the base × Height

1 Please find the volumes of the prism and the cylinder below.

(1)

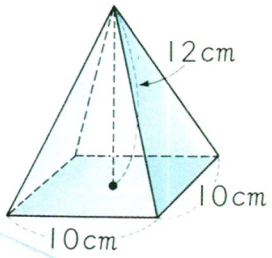


(2)

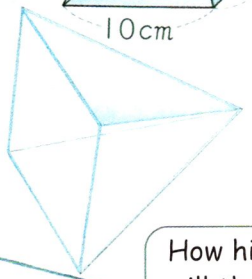


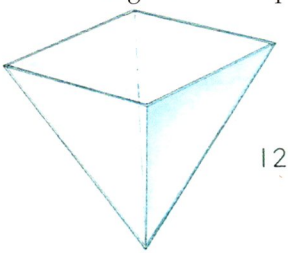
The volume of pyramids

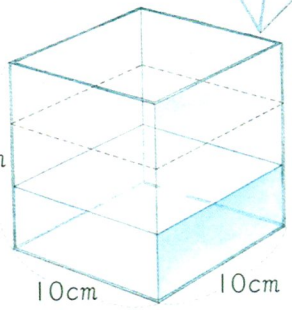
4 Let's think about how to find the volume of the square pyramid on the right!

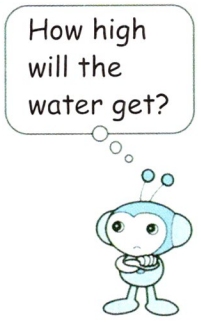


1 Put some water in the square pyramid-shaped container below, and then transfer the water to the square prism-shaped container which has the same base area and height and compare the volume.







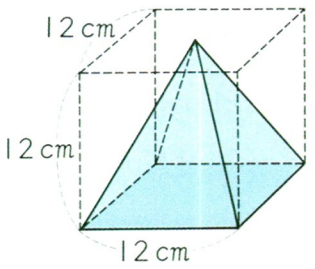


The volume of a pyramid is $\frac{1}{3}$ that of a prism that has the same base area and height.

2 What is the volume in cm^3 of a square pyramid?
The volume of a pyramid can be found by using the following formula:

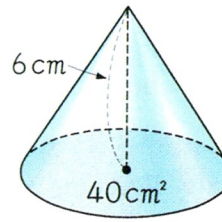
Volume of a pyramid = Area of the base × Height × $\frac{1}{3}$

1 Please find the volume of the square pyramid on the right.

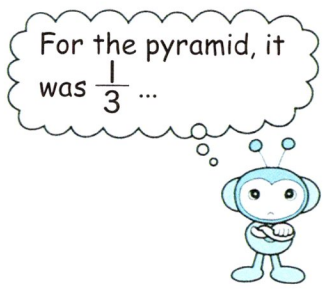


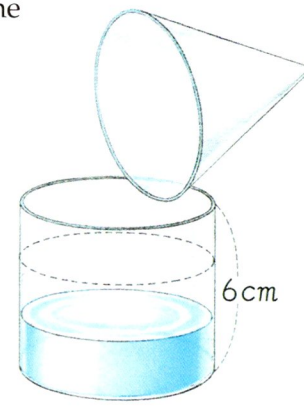
The volume of cones

5 Let's think about how to find the volume of the cone on the right!



1 As we did with the square pyramid, let's compare the volume of the cone with that of a cylinder that has the same base area and height.





The volume of a cone is also $\frac{1}{3}$ of the volume of a cylinder that has the same base area and height.

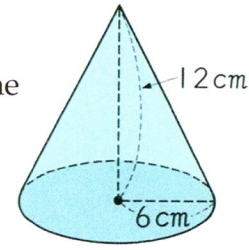
2 What is the volume of the cone above in cm^3?

The volume of pyramids and cones can be found by the following formula:

Volume of a pyramid or a cone

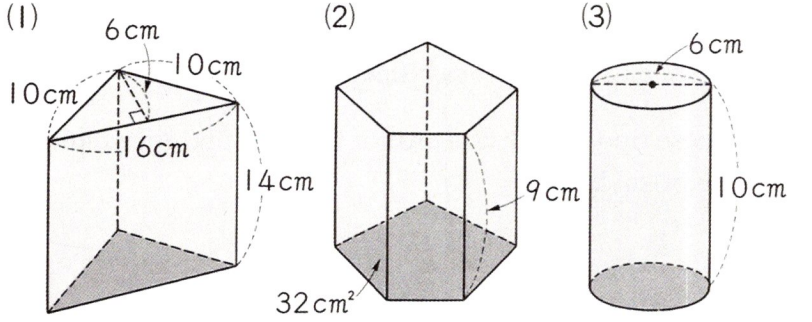
$$= \textbf{Area of the base} \times \textbf{Height} \times \frac{1}{3}$$

1 Please find the volume of the cone on the right.

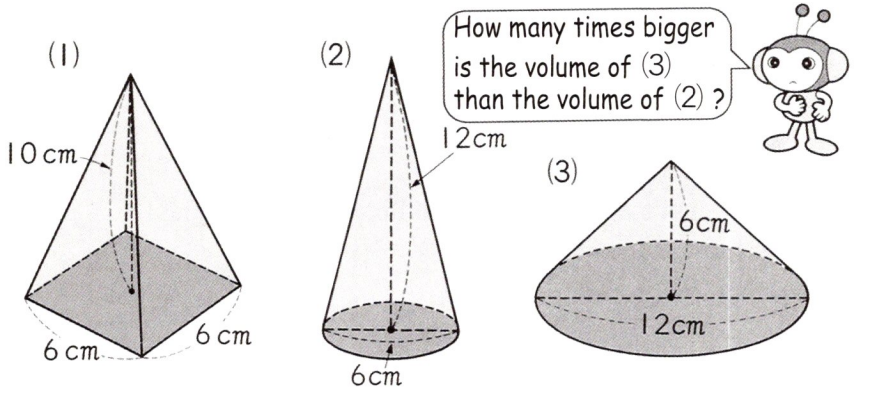


Practice 2

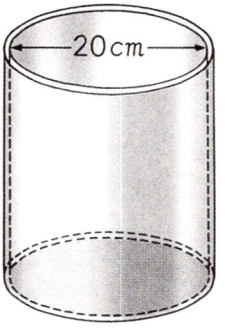
① Please find the volume of the prisms and the cylinder below:



② Please find the volume of the pyramid and the cones below:



③ The inside measure of the diameter of a cylinder-shaped container is 20cm.
If 4.7 1ℓ of water is put into this container, how many cm deep is the water?



Check

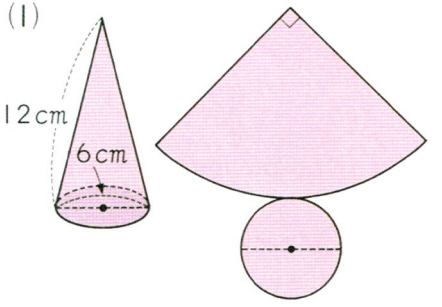
① Fill in the ☐ with the appropriate words.

(1) Surface area of a prism = ☐ × 2 + ☐

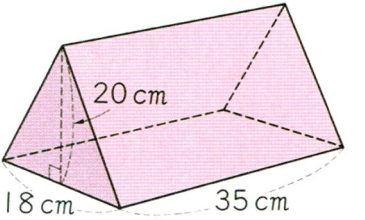
(2) Volume of a prism or a cylinder = ☐ × ☐

② Please find the surface area of the cone and the volume of the prism below:

(1)



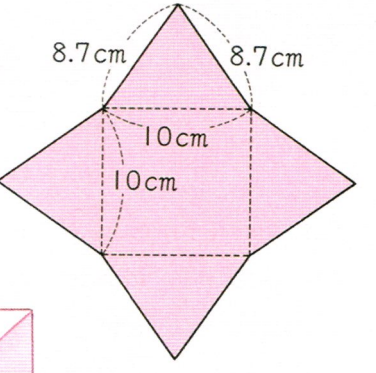
(2)

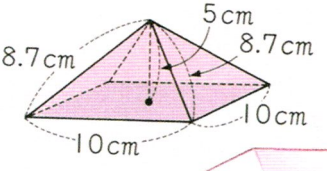


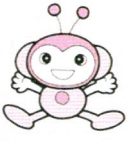
Challenge

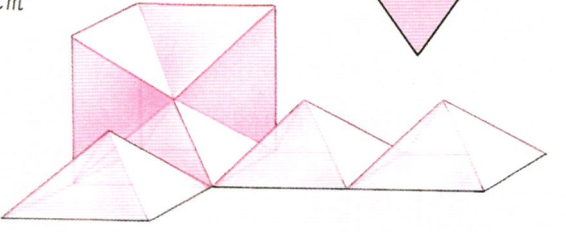
Let's Make a Cube with Six Square Pyramids!

Let's make six square pyramids like the one below and make a cube with edges of 10cm by combining these pyramids.





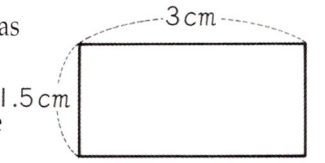




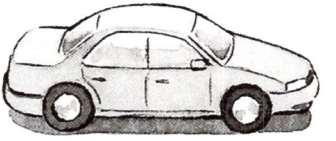
Review・4

① We made a drink by mixing 20g of syrup with 100g of water.
Please write the ratio of the weight of the syrup to the weight of the entire drink.
Please find the value of the ratio.

② Please draw an enlargement that is 3 times as big as the rectangle on the right.
Also, please find the value of the ratio of the length to the width of the enlargement, and compare that with the value of the ratio of the length to the width of the original rectangle.

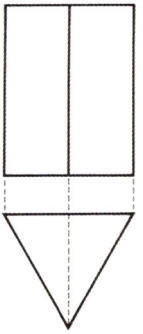


③ A car goes $9\frac{1}{3}$km on 1ℓ of gasoline.
How many km can this car go on $2\frac{1}{2}$ℓ of gasoline?

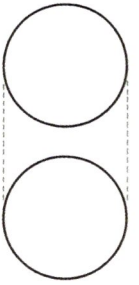


④ The drawings below are the drawings of solid figures seen from directly in front and directly above. What are these solid figures?

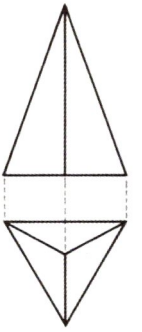
(1)



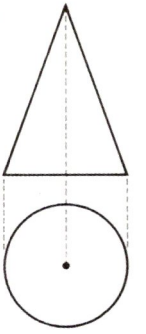
(2)



(3)

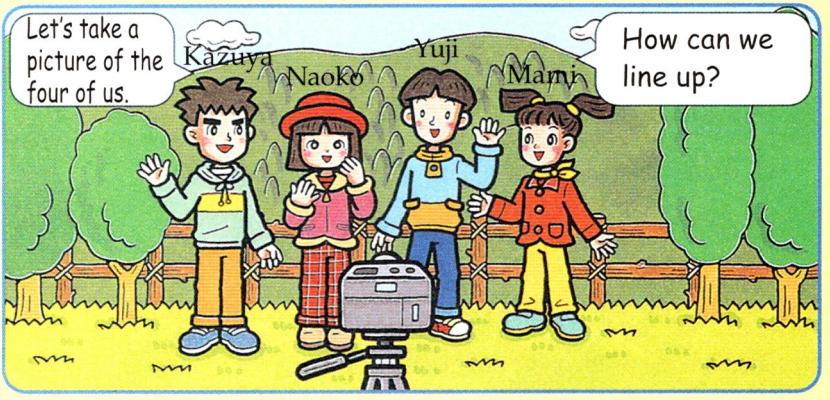


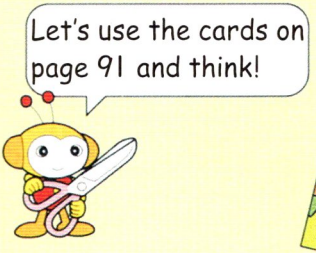
(4)



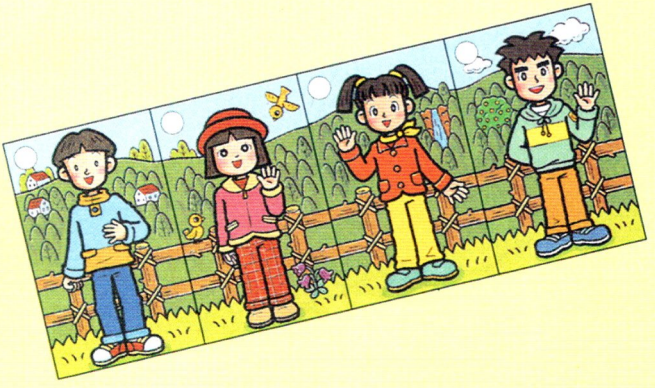
11 Number of Cases

Four people went hiking together.





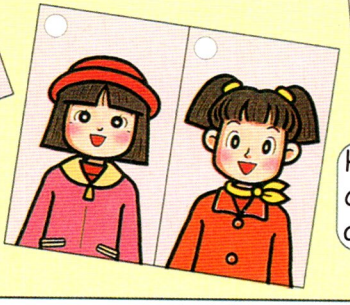


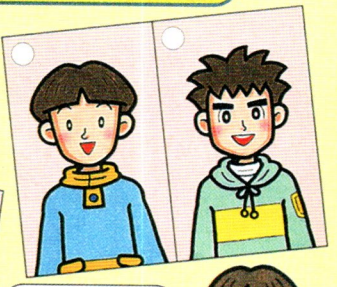


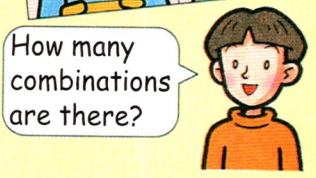












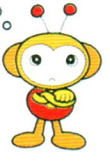
Let's think about how to find a way to order or combine!

▶ Ordering

1 We are going take a picture of four people in a line.

In how many different ways can they line up?

How many ways are there if there are 3 people ... ?



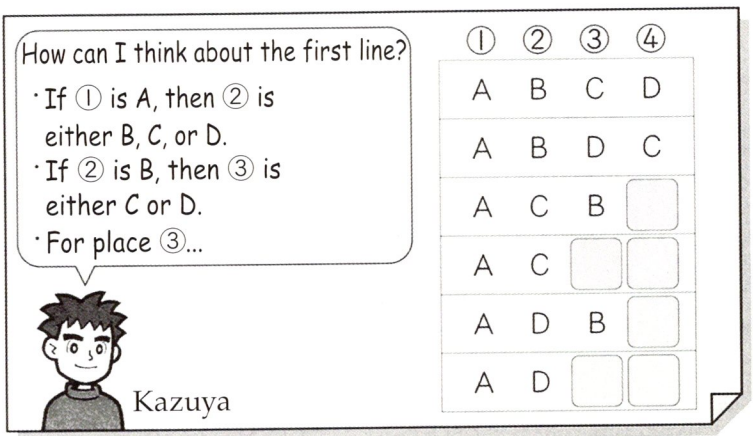
? Let's think about how to find all the ways to order without missing or repeating anything!

1 Let's think about it by representing the four people with A, B, C, and D!

Kazuya and Naoko investigated a way of deciding sequences by putting A in position ①.



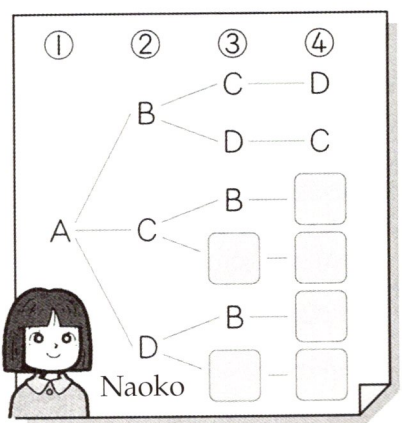
2 Please explain Kazuya's and Naoko's methods.



3 Let's compare Naoko's way of thinking with Kazuya's!

4 If ① is A, how many ways of ordering the rest are there?

5 In the cases where ① is B, C, and D, let's think about how many ways there are to order the rest by drawing a chart!



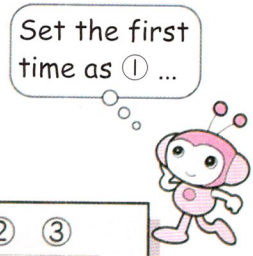
6 How many ways are there to arrange four people in a line?

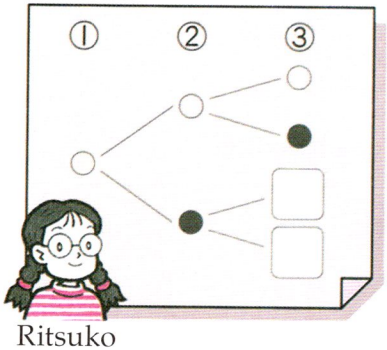
1 Suppose you have four cards, 1, 2, 3, and 4. If you are going to make three-digit numbers using these cards, how many different whole numbers can you make?

2 If you toss a medal three times, how many sequences of heads and tails are there?

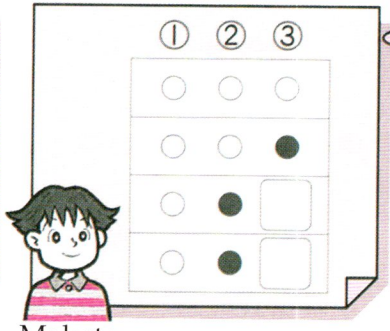


Ritsuko and Makoto thought about it by using charts and tables to represent heads with ○ and tails with ●.





Ritsuko



Makoto

1 Using a chart or a table, let's think about the case when you get tails first!

2 How many sequences of heads and tails are there?

▶ Ways of combining

3 Four people will play badminton.
Everyone will play each of the other players once. How many different matches are there?


Kazuya ··· A


Naoko ··· B


Yuji ··· C

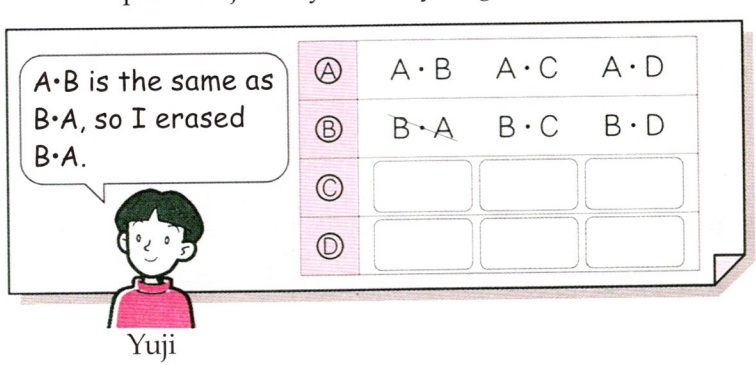

Mami ··· D

? Let's think about a way to find all the combinations without missing and repeating!

1 Please name the opponents of A.
Also, name the opponents of B, C, and D.

Yuji and Mami made the table below to find the answer.

2 Let's explain Yuji's way of analyzing!



Yuji

3 Let's explain the way Mami thought!

4 Please compare the ways that Mami and Yuji investigated this problem.

5 How many combinations of four people are there?

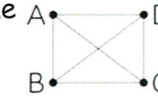


Mami

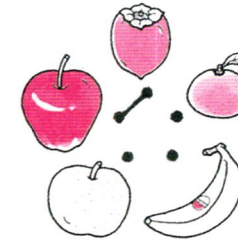
	A	B	C	D
A		○	○	○
B			○	○
C				○
D				



You can also use a chart like the one on the right. Each line represents one match.



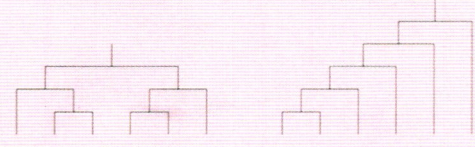
1 There are an orange, an apple, a persimmon, a pear, and a banana. How many ways are there to pick two of them?



Story of Math

Number of Games in a Tournament

The number of games in a tournament is the number of teams minus 1.



The number of games equals the number of teams who lost.



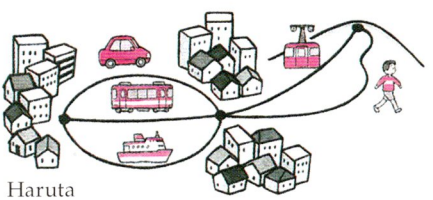
Check

1 Five people, A, B, C, D, and E, are going to climb a mountain. A is supposed to be in front all the time. The rest are to change their order in many different ways.
How many ways are there to line up?

2 There are four coins: 10 yen, 50 yen, 100 yen and 500 yen.
What amount will you get if you take two coins at a time?

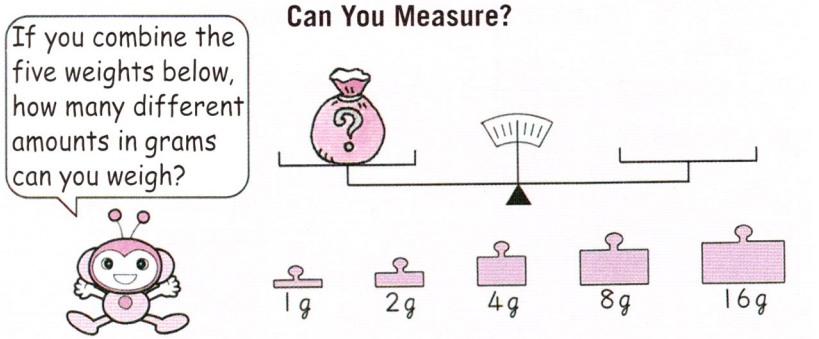


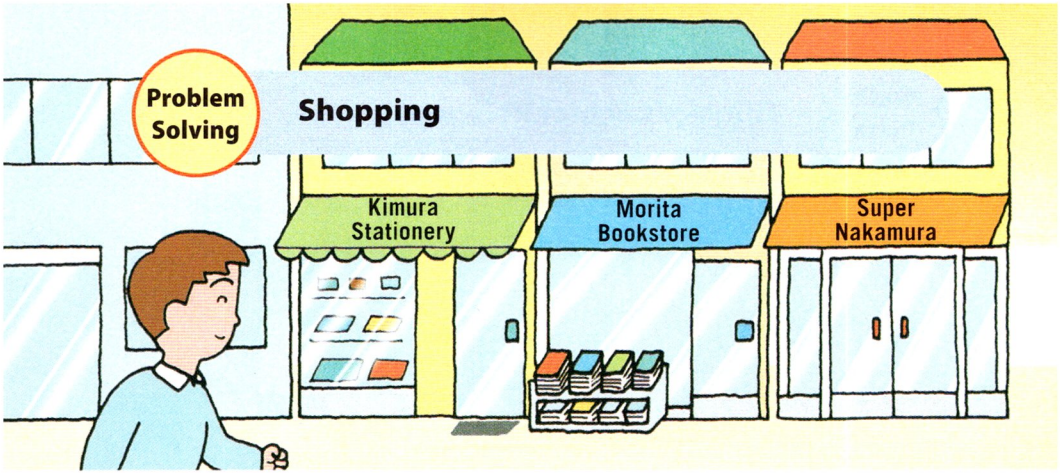
3 Look at the map on the right. How many different routes to go from Haruta City to the mountaintop via Natsuyama City are there?



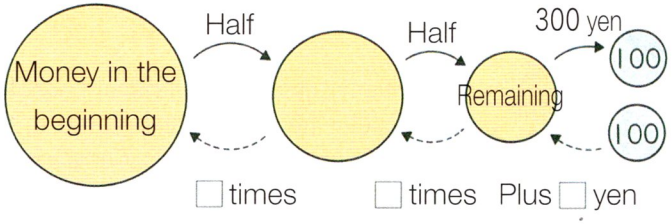
Challenge

How Many Different Weights Can You Measure?





1 Koji went to a stationery shop and bought a pen case with half the money he had. Next, he went to a bookstore and bought a magazine with half the money he had left. Finally, he bought a snack and paid 300 yen. He had 200 yen left.
How much money did he have at first?

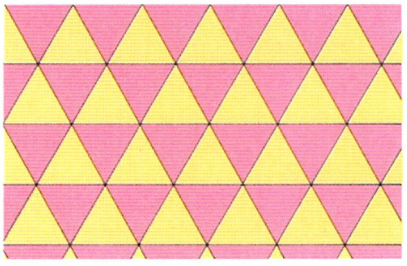


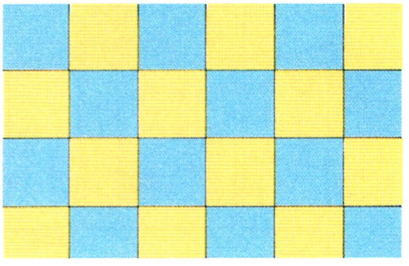
① Tomoko went to a department store for shopping. First, she bought a book that cost 600 yen. Next, she bought some paint with $\frac{1}{4}$ of the money she had left. Finally, she bought a cake that cost 400 yen. Now she has 800 yen left.
How much money did she have in the beginning?

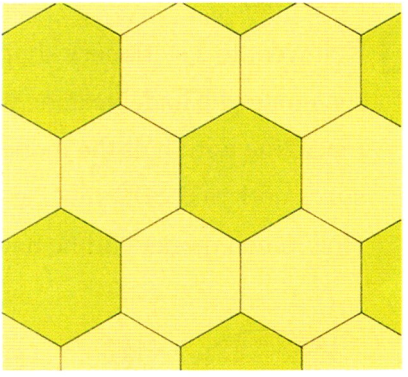


Math Plaza

Tiling of Regular Polygons

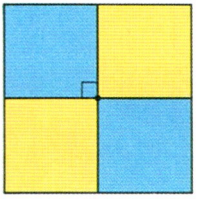


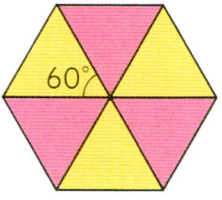


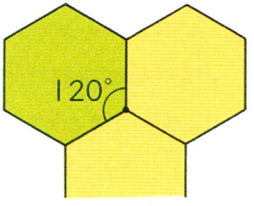




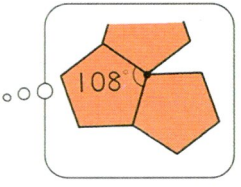






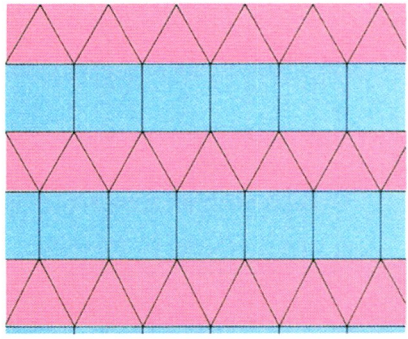


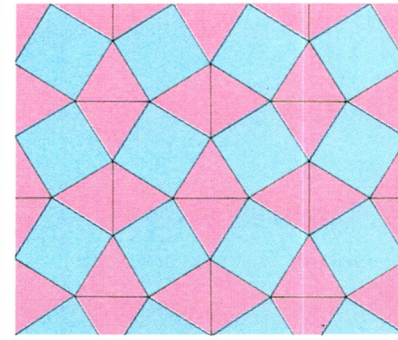


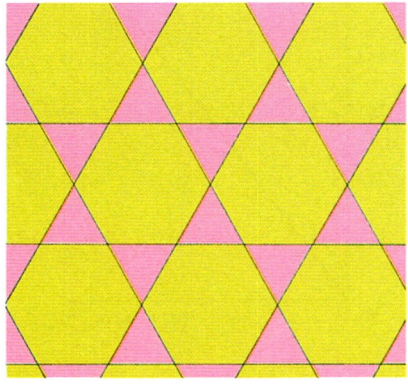


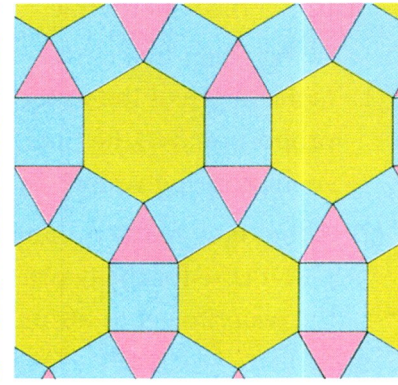


Cut out the congruent regular polygons on page 90 and use them to make tiling patterns.



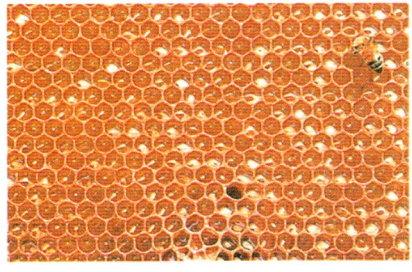






Let's find objects around you that show tiling patterns with regular polygons!







Review・5

① Please calculate the following.

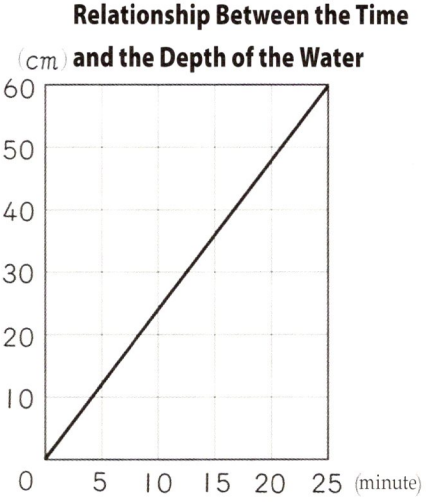
(1) $\frac{4}{5} \times \frac{3}{7}$ (2) $\frac{2}{11} \times \frac{7}{8}$ (3) $\frac{1}{9} \times \frac{3}{5}$ (4) $2\frac{1}{7} \times 1\frac{2}{5}$

(5) $\frac{2}{7} \div \frac{3}{4}$ (6) $\frac{5}{14} \div \frac{2}{7}$ (7) $1\frac{7}{8} \div \frac{3}{7}$ (8) $1\frac{5}{6} \div 2\frac{4}{9}$

② The graph on the right shows the relationship between the time and the depth of the water while pouring water in a tank.

(1) Is the depth of the water proportional to the time spent pouring water?

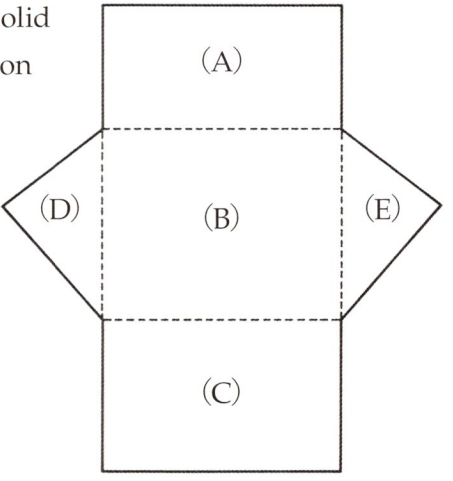
(2) About how many minutes will it take for the depth of the water to get to 50cm?



③ We are going to make a solid figure by folding the net on the right.

(1) What kind of figure will it be?

(2) Which face will be parallel to face (D)?

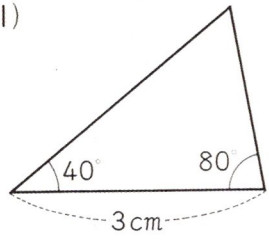


④ Please simplify the ratios. Also find the values of the ratios.

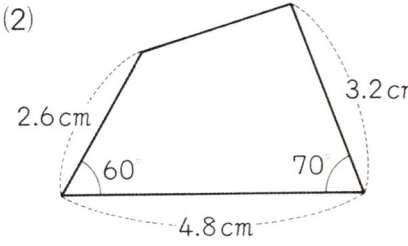
(1) 8 : 12 (2) 16 : 56 (3) 4.5 : 2.5 (4) $\frac{3}{5} : \frac{2}{3}$

⑤ Please draw enlargements that are twice as big and reductions that are $\frac{1}{2}$ as big.

(1)

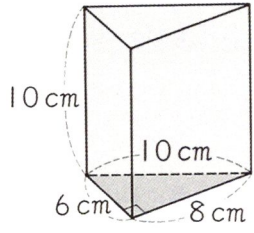


(2)

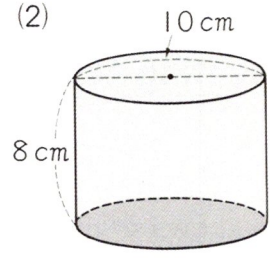


⑥ Please find the surface areas and volumes of the solids below.

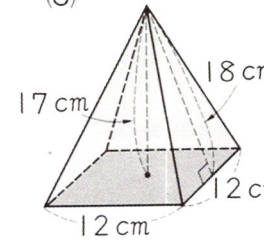
(1)



(2)



(3)



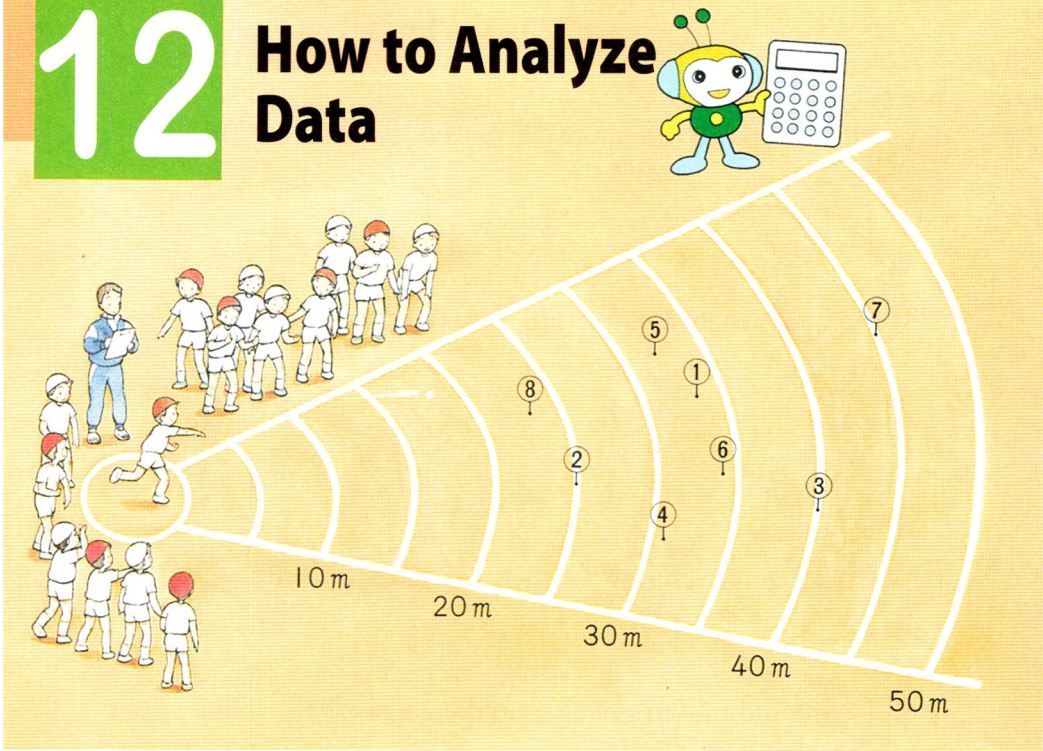
⑦ The number of 6th graders in Yumiko's school is shown in the table on the right. On average, how many students are in one class?



Number of Sixth Graders

Class	Number of students
1	33
2	30
3	29
4	32

12 How to Analyze Data



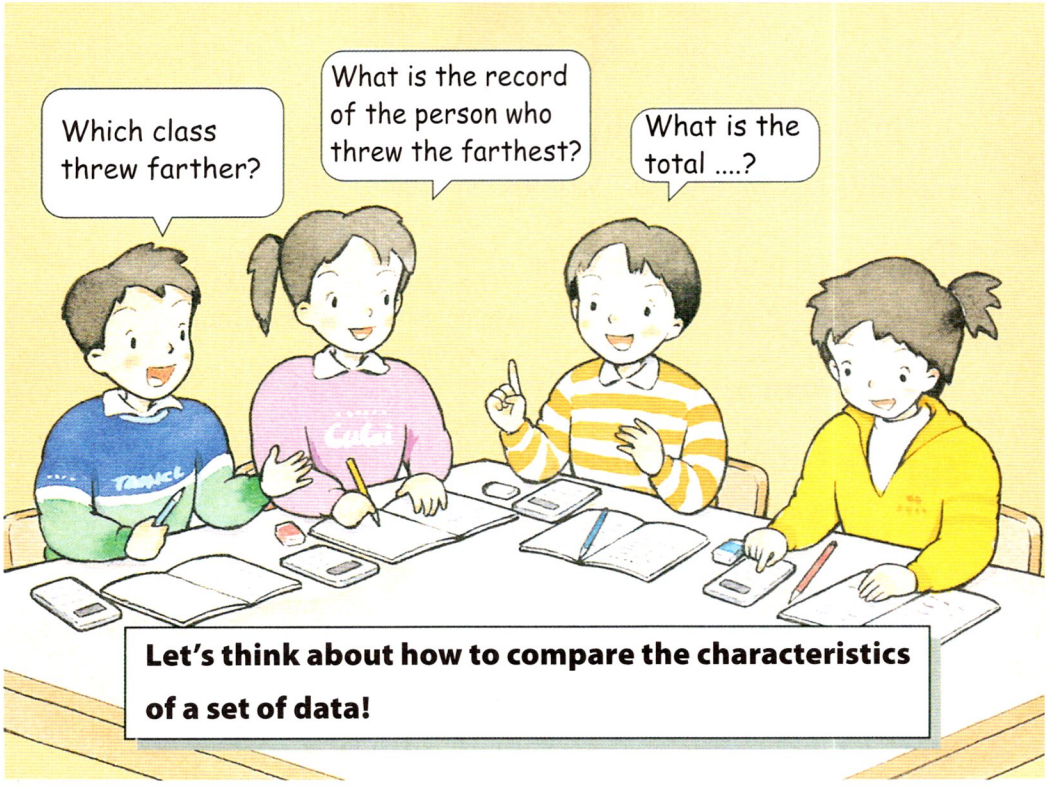
1 Tables Representing Average and Variation

Average

1 The table below shows softball throws for boys in two 6th grade classes.
Let's think about how we can compare the data of class A and class B!

Softball Throws of 6th Grade Boys in Class A (unit: meters)

①	33	②	25	③	40	④	31	⑤	32	⑥	34	⑦	45	⑧	23
⑨	27	⑩	36	⑪	34	⑫	33	⑬	32	⑭	33	⑮	37		



	Longest throw (m)	Shortest throw (m)	Total (m)
Class A			
Class B			

1 From what you have found above, which class of boys do you think can be said to have thrown farther?

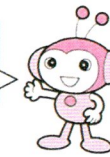
Softball Throws of 6th Grade Boys in Class B (unit: meters)

① 34	② 26	③ 42	④ 36	⑤ 21	⑥ 26	⑦ 35	⑧ 37
⑨ 26	⑩ 37	⑪ 24	⑫ 31	⑬ 35	⑭ 47	⑮ 36	⑯ 27

In cases like this, you can find the averages of class A and class B, and compare the results.

2 Let's find the averages of class A and class B!

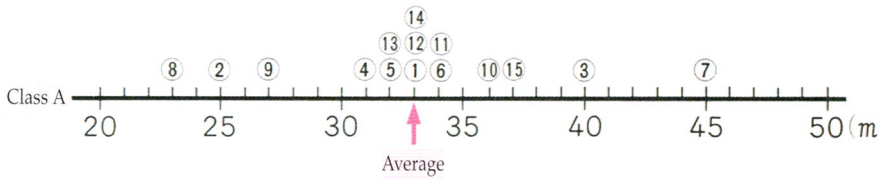
The tables below are the same as the tables on the previous pages.

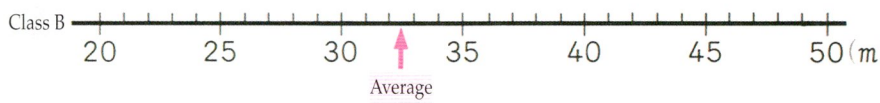


Variation

2 Let's find how the data for class A and class B are spread out!

1 Let's represent the data for class B on the number line!





2 In which class is the data spread out over a wider range?

3 Which class has more people around the average?

Softball Throws of 6th Grade Boys in Class A (unit: meters)

①	33	②	25	③	40	④	31	⑤	32	⑥	34	⑦	45	⑧	23
⑨	27	⑩	36	⑪	34	⑫	33	⑬	32	⑭	33	⑮	37		

3 2 For class A and class B, let's organize the data in tables, so we can make the variation within each class clear!

1 The tables on the right have distances divided into 5 meter intervals. Put the number of people in the tables.

More than or equal to 30*m* includes 30*m* and more than 30*m*.

Less than 35*m* means less than 35*m*, but does not include 35*m*.

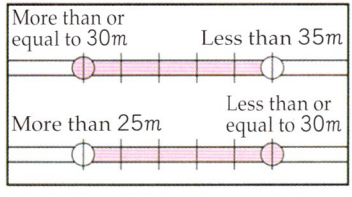
Less than or equal to 30*m* means 30*m* or less than 30*m*.

Softball Throws in Class A

Distance (*m*)	Number of people
More than or equal to 20 ~ Less than 25	1
25~30	2
30~35	
35~40	
40~45	
45~50	
Total	

Softball Throws in Class B

Distance (*m*)	Number of people
More than or equal to 20 ~ Less than 25	
25~30	
30~35	
35~40	
40~45	
45~50	
Total	



Softball Throws of 6th Grade Boys in Class B (unit: meters)

① 34	② 26	③ 42	④ 36	⑤ 21	⑥ 26	⑦ 35	⑧ 37
⑨ 26	⑩ 37	⑪ 24	⑫ 31	⑬ 35	⑭ 47	⑮ 36	⑯ 27

Softball Throws in Class A

Distance (m) (More than or equal to / Less than)	Number of people
20 ~ 25	1
25 ~ 30	2
30 ~ 35	8
35 ~ 40	2
40 ~ 45	1
45 ~ 50	1
Total	15

Softball Throws in Class B

Distance (m) (More than or equal to / Less than)	Number of people
20 ~ 25	2
25 ~ 30	4
30 ~ 35	2
35 ~ 40	6
40 ~ 45	1
45 ~ 50	1
Total	16

2 How many people in class A threw more than or equal to 30m and less than 35m? How many did the same in class B?

3 Approximately what percent of the people threw more than or equal to 40m in class A and class B, respectively?

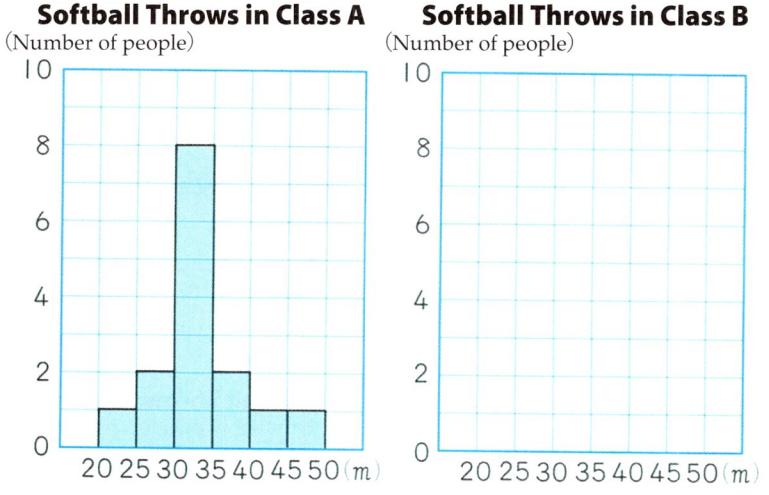
4 Kazuya, in class A, threw 33m. If we are to rank the data starting with the farthest throw, between what ranks will Kazuya's record be?

If you look at the way the records are spread out, you can see characteristics of the data that cannot be seen from the average alone.

1 Is 10 included in whole numbers "less than 10"?
Is 10 included in whole numbers "less than or equal to 10"?

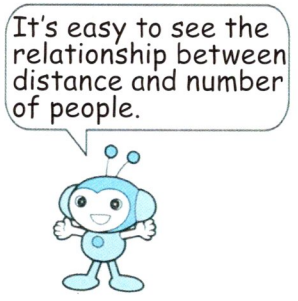
2 Histograms

1 The data table from class A on the previous page was put into the graph below.
Let's analyze how the records are spread out in class A and class B by looking at their graphs!



Graphs like this one are called **histograms**.

1. Let's show the records of class B in a histogram!
2. In class A and class B, which interval has the most people, respectively?
3. Let's find how the data varies for class A and class B in the graphs above!



Histograms are useful for finding how data varies.

Practice

① Please write numbers in the following range.

(1) Whole numbers more than or equal to 3 and less than or equal to 7.

(2) Whole numbers more than or equal to 5 and less than 9.

② The table below represents the heights of the children in Kenta's group. Compare the heights of the girls and the boys. Which can be said to be taller?

Heights of Children in the Group (Unit: cm)

Boys (4 boys)	Girls (5 girls)
148.0	147.4
152.9	158.0
135.3	147.5
154.2	144.7
	138.4

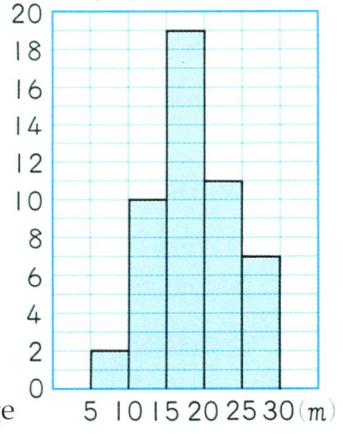


③ The histogram on the right shows the data for girls' softball throws in the 6th grade at Yoko's school.

(1) Which interval includes the most people?

(2) How many people threw more than or equal to 10m and less than 25m? Approximately what percentage of the 6th grade girls is the number of people in this range?

Softball Throws of 6th Grade Girls



3 Cumulative Total

1 We collected data on who borrowed books from the library. Let's compare the number of people in class A and class B who borrowed books from the library!



People Who Borrowed Books from the Library

	Class A	Class B
Mon	Hiroshi, Michiko	Tsutomu
Tue	Hiroshi	Susumu, Yoshie
Wed	Akiko, Kiyoshi	Susumu, Yoshie
Thu	Isamu, Kazuko	
Fri	Kazuko	Takeshi
Sat	Kazuko	Takeshi, Susumu

1 How many people in class A borrowed books?

$2 + 1 + 2 + 2 + 1 + 1 =$ ☐ (Number of people)

Sometimes when the same person borrows books several times we count that person as different each time and compute the total number, as in this case. The total number of people counted is called the "cumulative total."

2 What is the cumulative total of people in class B who borrowed books from the library?

3 On average, how many people in class A and class B borrowed books in a day?

4 Whole and Part

1 Takuya collected the data on the number of students at his school, and the population of the city and the prefecture where he lives. He also found out how many boys there were in each case.

Number of Elementary School Students (Number of people)

	Number of students	Number of boys
Higashi elementary school	808	415
City	26178	13423
Prefecture	420303	215833

Let's find the relative values of the number of boys to the total population in Higashi Elementary School, in the city and in the prefecture, and compare these values!

1 What percentage of the whole population are boys in each case?

2 Can you say that the relative values are almost the same even though the populations are different?

Depending on what you are researching, you can estimate the relative value for a large group by using the relative value of a smaller group within the large group.

Story of Math

Audience Rating of TV Programs

To estimate how many people in the whole population watched a particular program, we collect data on how many people in a smaller group watched the program.

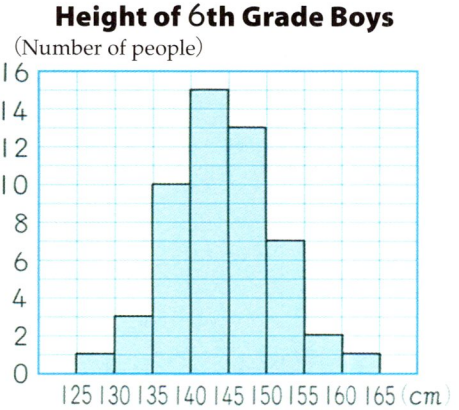


Check

1 From the numbers below, please find numbers that are greater than or equal to 0.1 and less than 0.7.

Ⓐ 0.2 Ⓑ 0.84 Ⓒ 0.1 Ⓓ 0.7 Ⓔ 0.23 Ⓕ 0.09

2 The histogram on the right represents the heights of 6th grade boys in Toshio's school.



(1) Which interval has the most boys?

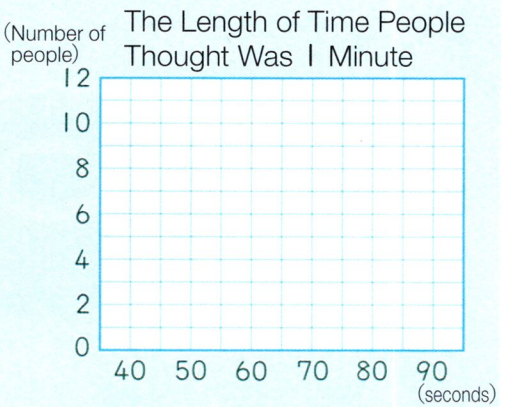
(2) How many boys have a height of greater than or equal to 140cm and less than 150cm?

Challenge

Let's Draw a Histogram!

Close your eyes. When you think a minute has passed, raise your hand, and record the time. Record your classmates' results with a histogram.

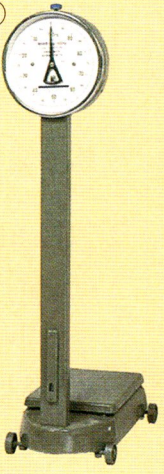


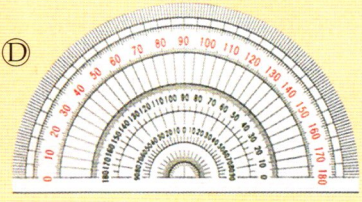


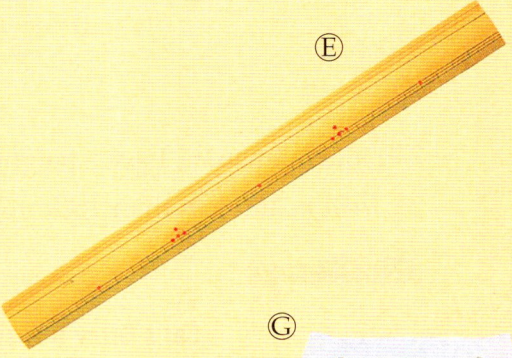
13 Units of Measurement

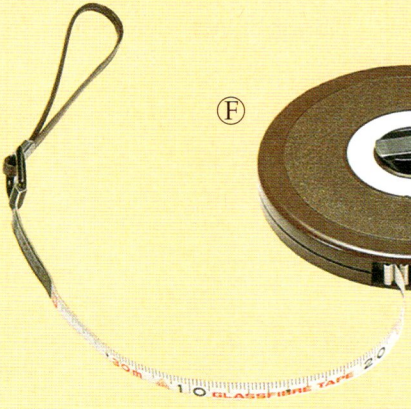


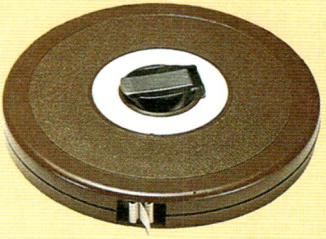


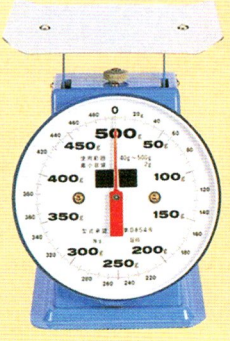






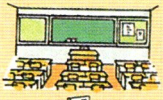
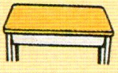






We have learned about various units to measure quantities.
Which unit will be used to describe the amount of quantities (1) through (8)?
Let's write the appropriate units in ☐!
What kinds of instruments are used to measure them? Choose the appropriate instruments from page 54.



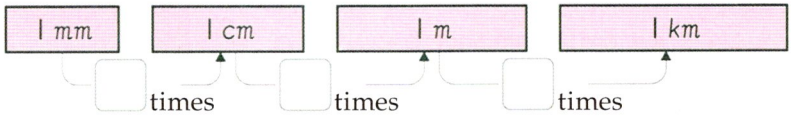
			Unit	Instrument
	(1)	Weight of a mathematics textbook	140	☐ ☐
	(2)	Length of a hallway	40	☐ ☐
	(3)	Area of a classroom	50	☐
	(4)	Volume of juice	1	☐ ☐
	(5)	Angle of an opened fan	90	☐ ☐
	(6)	Width of a desk	60	☐ ☐
	(7)	Elapsed time to run 50m	8	☐ ☐
	(8)	Body weight of a person	40	☐ ☐

Let's explore the system of units for different quantities!

Length and area

1 Let's explore the system of units for measuring length and area!

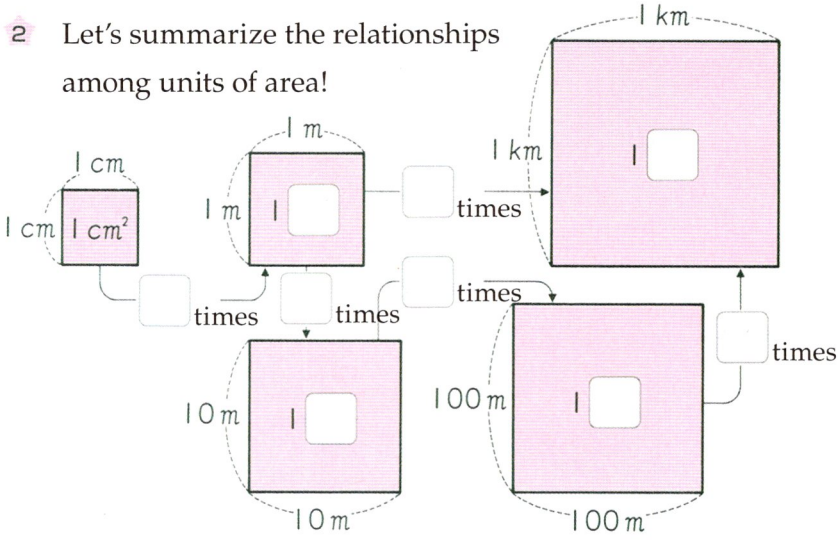
Let's say the units of length starting with the smallest one!
Also, let's summarize the relationship among these units!



1 Let's say the units of area starting from the smallest one!

Units of area are based on units of length.

2 Let's summarize the relationships among units of area!



Length of a side	1 cm	1 m	10 m	100 m	1 km
Area of square	1 cm^2	1 m^2	1 a	1 ha	1 km^2

Volume

2 Let's explore the system of units for measuring volume!

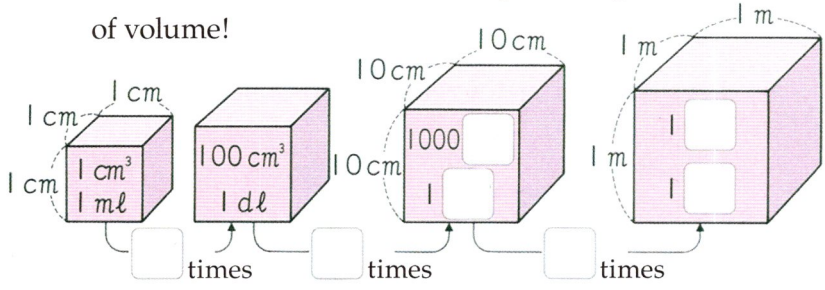
1 What units of volume are there?

1000 times 1 ℓ is called 1 **kiloliter** and it is written as 1 kℓ.

$$1\ k\ell = 1000\ \ell$$

Units of volume are also based on units of length.

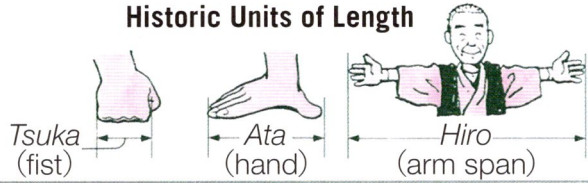
2 Let's summarize the relationships among the units of volume!



Length of a side	1 cm	—	10 cm	1 m
Volume of cube	1 cm^3 1 mℓ	100 cm^3 1 dℓ	1000 cm^3 1 ℓ	1 m^3 1 kℓ

Story of Math

Historic Units of Length



▶ Weight and volume

3 Let's explore the system of units for measuring weight!

1 What units of weight are there?

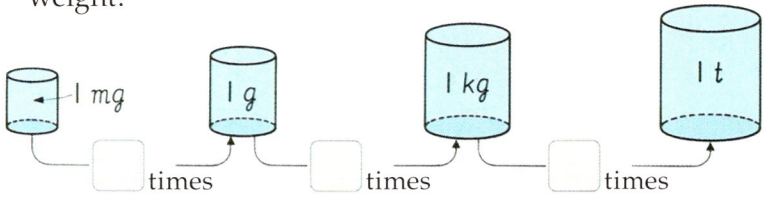
1000 times as much as 1kg is called 1 **ton** and is written as 1t.

$\frac{1}{1000}$ of 1g is called 1 **milligram** and is written as 1mg.



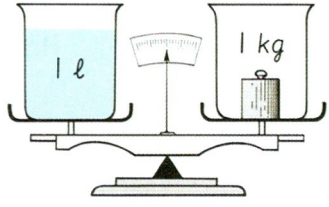
$1\,t = 1000\,kg$ $\qquad$ $1\,mg = \frac{1}{1000}\,g$

2 Let's summarize the relationships amoung units of weight!



4 What relationships are there among the units of weight and the units of volume?

The weight for $1000\,cm^3$ (1ℓ) of water is 1kg.



1 Please measure the weight of 1ℓ of water and check that it is actually 1kg.

The table below summarizes the relationships between the weight of water and the units of volume.

Unit of volume	$1\,cm^3$ $1\,m\ell$	$100\,cm^3$ $1\,d\ell$	$1000\,cm^3$ $1\,\ell$	$1\,m^3$ $1\,k\ell$
Weight of water corresponding to the volume above	$1\,g$	$100\,g$	$1\,kg$	$1\,t$

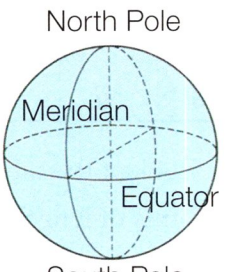
Story of Math

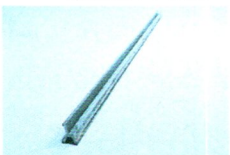
Meter and Kilogram

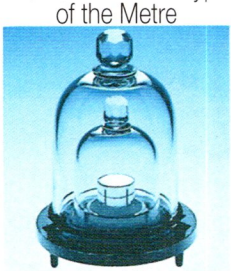
At the end of the eighteenth century in France, the meter and the kilogram were decided as common units for length and weight that could be used in any country. As the basis for the unit of length, $1m$ was decided to be $\frac{1}{10000000}$ of the length of the meridian from the North Pole to the equator. Thus, the international prototype of the metre was created.

Currently, $1m$ is the distance that light travels in a vacuum in $\frac{1}{299792458}$ of a second.

Also, as the basis for the unit of weight, it was decided that $1kg$ would be the weight of $1000cm^3$ (1ℓ) of water at 4 °C. Thus, the international prototype of the kilogram was created.




International Prototype of the Metre


International Prototype of the Kilogram

▶ The metric system

5 The system of units that we use to measure length, volume, weight, etc. is called the metric system. Let's explore the structure of units in the metric system!

	Kilo (k)	Hecto (h)	Deca (da)		Deci (d)	Centi (c)	Milli (m)
	1000 times	100 times	10 times	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Length	km	—	—	m	—	cm	mm
Volume	kℓ	—	—	ℓ	dℓ	—	mℓ
Weight	kg	—	—	g	—	—	mg

1 Look at the table above, and explain the system of the units of length, volume, and weight, respectively.

In the metric system, you can convert one unit to another by adding or deleting 0, or by moving the decimal point.

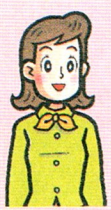
Story of Math

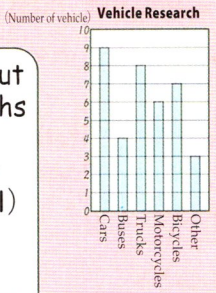
Metric Prefixes

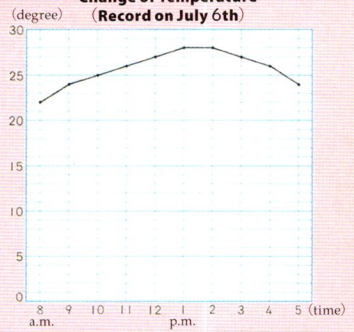
Mega	1000000 times	Micro	$\frac{1}{1000000}$
Giga	1000000000 times	Nano	$\frac{1}{1000000000}$

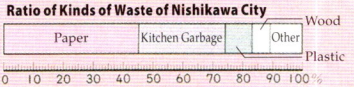
14 Various Graphs

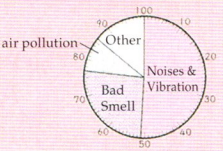
We have learned about various kinds of graphs so far.
What graph will best represent the data (1) through (4) below?
Let's choose appropriate ones from the graphs on the right and write their names!

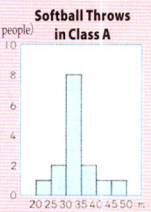












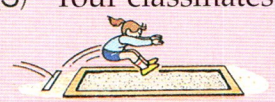
(1) The relative amount of apples produced by individual prefectures compared to the national total.



(2) The change in your weight, measured each month.



(3) Your classmates' records for the running long jump.



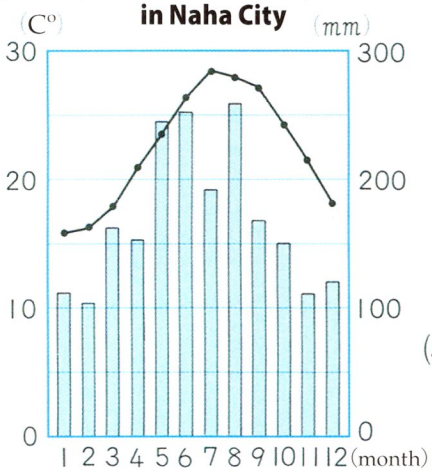
(4) The favorite type of food of the students in your class.



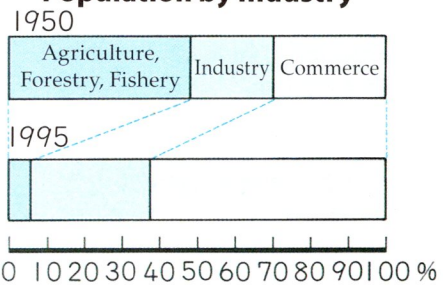
Let's explore the characteristics of different graphs!

1 The following graphs are modifications of the graphs we learned about previously.
Let's investigate their characteristics!

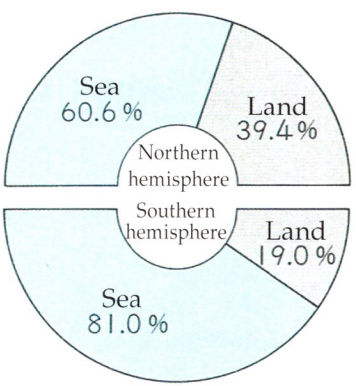
(1) **Temperature and Precipitation in Naha City**



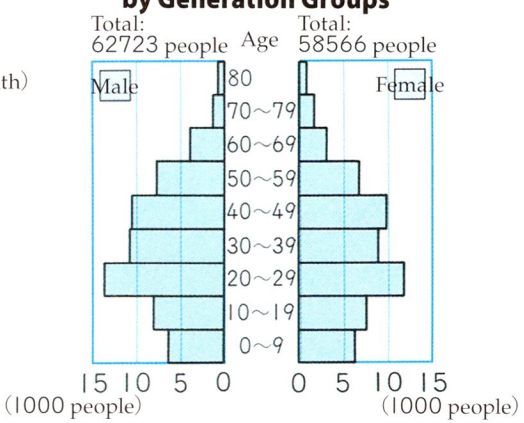
(2) **Relative Values of the Population by Industry**

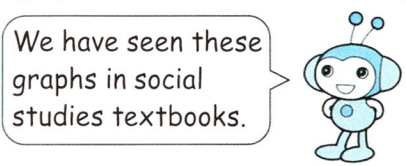


(3) **Relative Amount of Land and Sea on the Earth**



(4) **The Population of Urayasu City by Generation Groups**



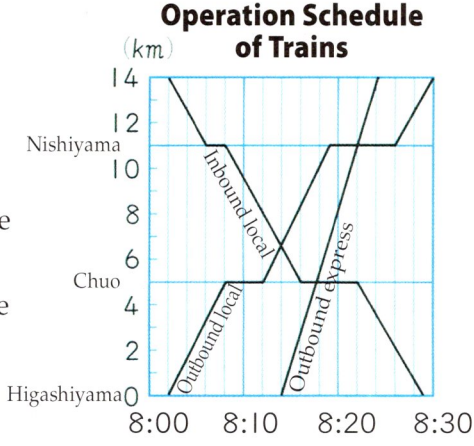


1 Please share what you notice about these graphs.

2

The graph below shows the operation schedule of trains. The vertical axis represents the location of the stations, and the horizontal axis represents the time. Let's investigate the characteristics of this graph!

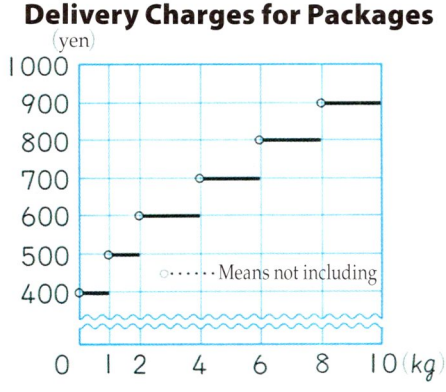
1. What time does the outbound local train arrive at Chuo Station?
2. What time and where does the outbound express train pass the local train?
3. What else do you notice?

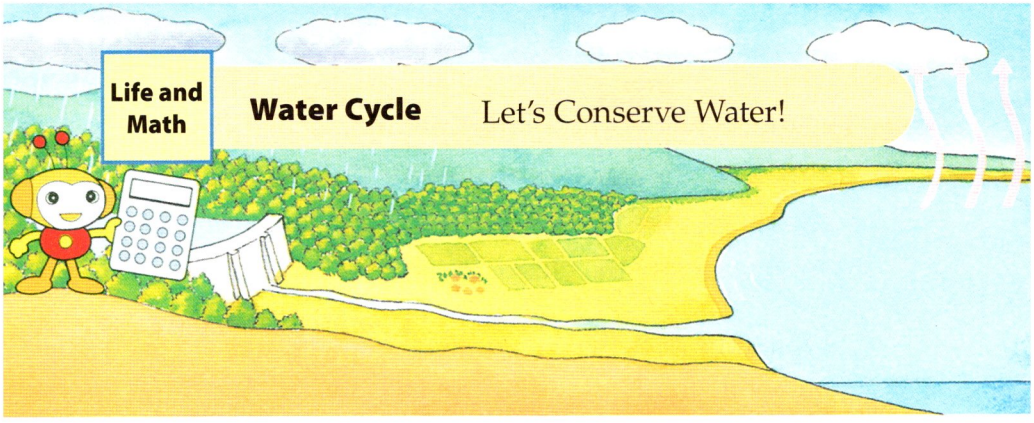


3

The graph below shows the delivery charges of a certain delivery company.
Let's explore the characteristics of this graph!

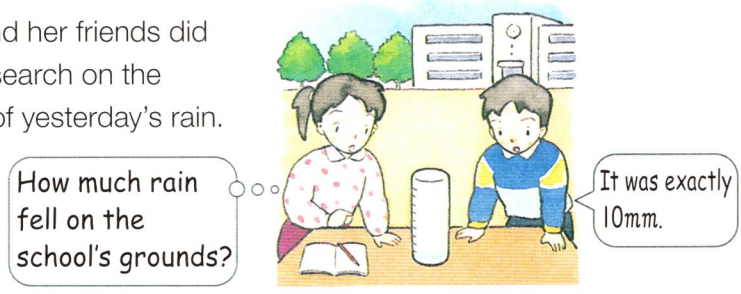
1. What are the charges for the packages with weights of 1.2kg and 6kg, respectively?
2. What else do you notice?



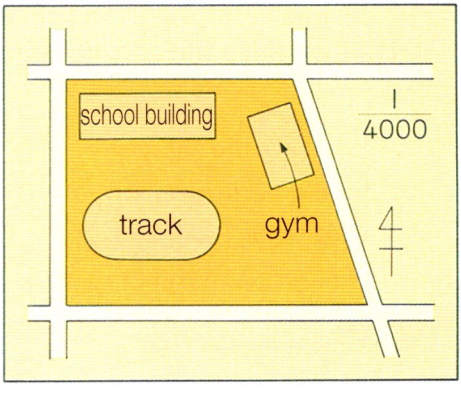


Raindrops become underground water or go into rivers and oceans. Some of them evaporate, become clouds, and become raindrops again.

Emiko and her friends did some research on the amount of yesterday's rain.



1 The figure on the right is a reduced drawing of the school's grounds.

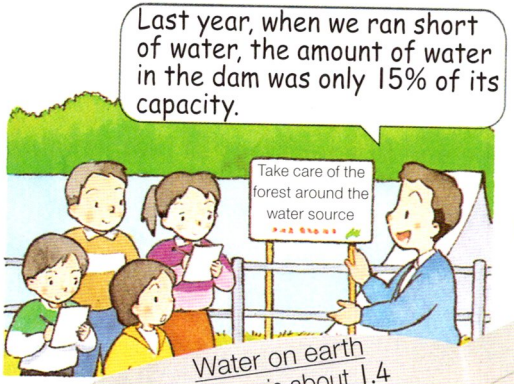


(1) How many m^2 are the school's grounds?

(2) How many m^3 of rain fell on the school's grounds?

(3) If you measure the amount of rainfall with an 8- liter bucket, how many bucketfuls of water is it?

On Sunday, Emiko's group went to Minamiyama dam.



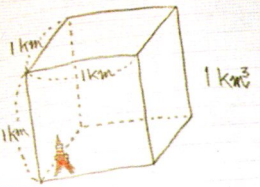
Minamiyama Dam
Amount of water at its full capacity:
28 *oku* m^3 (2.8 billion)
Amount of water on February 5:
85%

Water on earth

On earth, there is about 1.4 billion km^3 of water. This amount hasn't changed since ancient days.
About 97.5% of the water on earth is sea water and only 2.5 % is fresh water. The amount of water available to us is only 0.3% of the total amount of water on earth.

Unit: km^3

1km^3 is the volume of a cube with 1km sides.



2 How many m^3 of water was in the Minamiyama dam on February 5?

3 At Emiko's school, people used 13920 m^3 of tap water last year.
On average, how many m^3 of tap water did they use in one month?

① Let's examine a variety of information about water!
Also, from the information you examined on your own, as well as the information above, please create and solve different problems.

Summary of Mathematics

You are approaching the end of your study of mathematics. Since you started school, you have learned many things. Let's summarize what you have learned in mathematics in your six years of study!



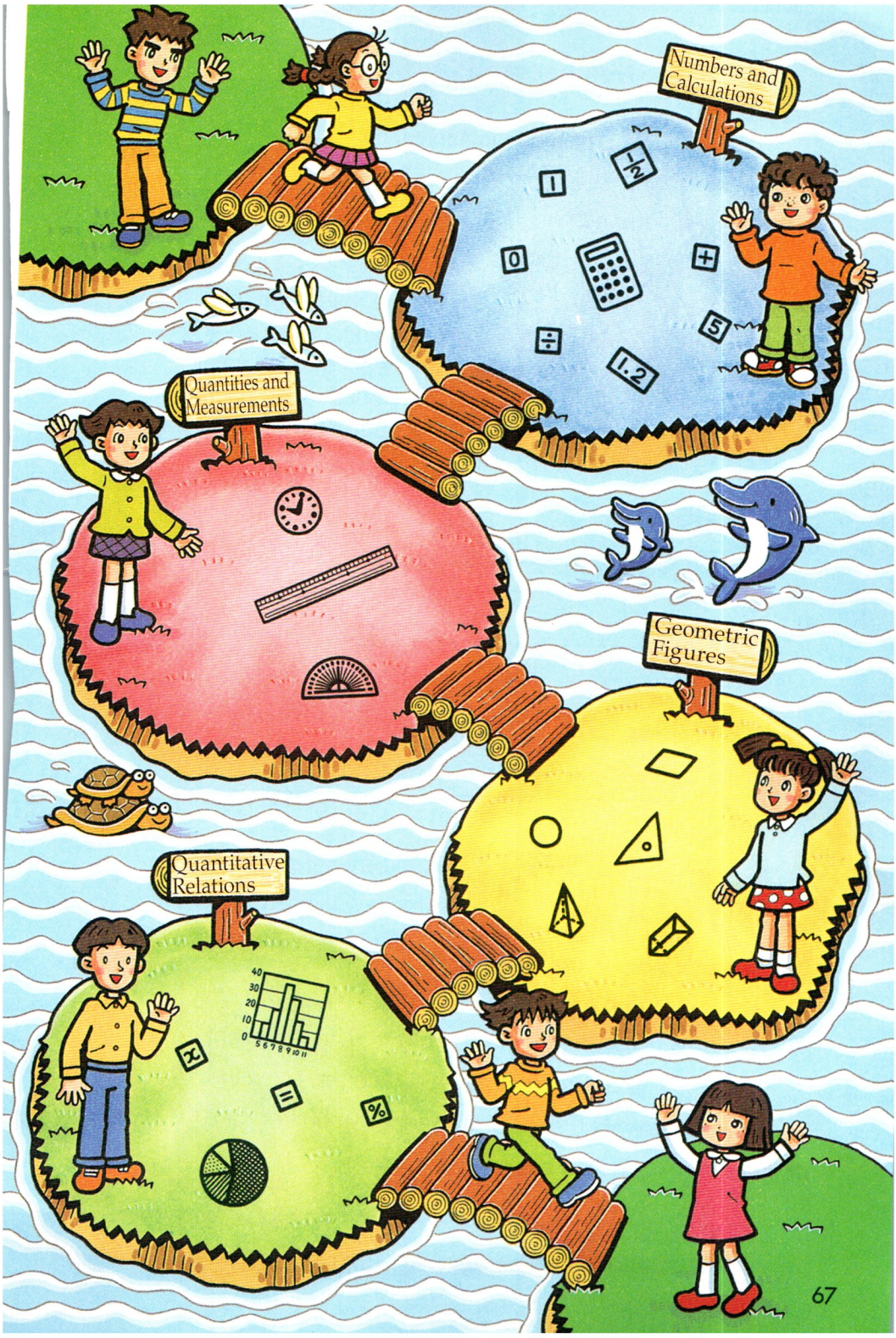






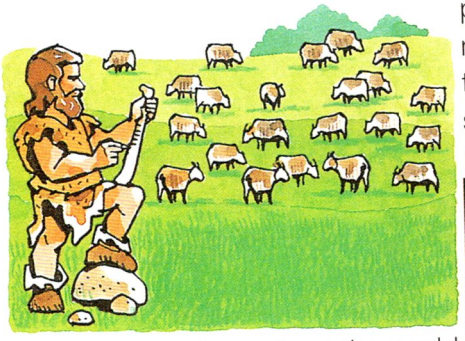




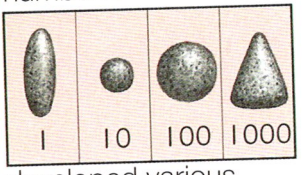

Numbers and Calculations
Quantities and Measurements
Geometric Figures
Quantitative Relations

Numerals in Ancient and Modern Times

What kinds of symbols did ancient people use to represent numbers?

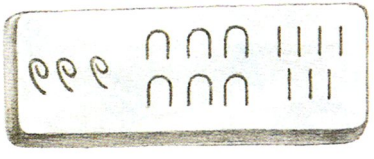


In the beginning, people made cutting marks on bones and trees, or used small stones to represent numbers.



Later, people throughout the world developed various symbols.

Egypt	𓏺	𓏻	𓏼	𓏽	𓏾	𓏿	𓐀	𓐁	𓐂	𓍢	𓆼	𓆽
Rome	I	II	III	IV	V	VI	VII	VIII	IX	X	C	M
China	一	二	三	四	五	六	七	八	九	十	百	千
	1	2	3	4	5	6	7	8	9	10	100	1000



367

In ancient Egypt, a new symbol was used when a number became big.

Now, we use 10 numerals, from 0 to 9, to represent a number, and a number's size is determined by the place where each symbol is used. Any number can now be expressed easily.

① Please read the following numbers.

(1) 370028469 (2) 600790508520000

② Please write each number in numerals.

(1) Seven *oku* four hundred three *man* nine thousand

(2) Fifty *cho* two thousand six hundred eighty *oku* seventy *man*

③ Please fill in the ☐ with the appropriate numbers.

(1) $2076 = 1000 \times \square + 100 \times \square + 10 \times \square + 1 \times \square$

(2) $5.184 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

④ Please convert fractions to decimals, and decimals to fractions.

(1) $\frac{3}{5}$ (2) $1\frac{7}{16}$ (3) $\frac{9}{8}$ (4) 0.4 (5) 2.6 (6) 1.003

⑤ Please write the least common multiple of each set of numbers in ( ). Also, write the greatest common factor of each set of numbers in ( ).

(1) (8, 12) (2) (14, 21) (3) (18, 24)

⑥ Please simplify the following fractions.

(1) $\frac{6}{9}$ (2) $\frac{9}{12}$ (3) $\frac{7}{21}$ (4) $\frac{16}{24}$ (5) $\frac{20}{32}$ (6) $\frac{14}{35}$ (7) $\frac{25}{45}$

⑦ Please rewrite the fractions in ( ) using a common denominator. Then write them in order from the largest to the smallest.

(1) $\left(\frac{5}{8}, \frac{3}{5}\right)$ (2) $\left(\frac{23}{24}, \frac{5}{6}\right)$ (3) $\left(\frac{1}{3}, \frac{7}{10}, \frac{8}{15}\right)$

⑧ Please calculate the following.

(1) $\begin{array}{r} 2983 \\ +\ 4079 \\ \hline \end{array}$ (2) $\begin{array}{r} 36.4 \\ +\ 45.29 \\ \hline \end{array}$ (3) $\begin{array}{r} 58000 \\ -\ 27843 \\ \hline \end{array}$ (4) $\begin{array}{r} 91 \\ -\ 20.08 \\ \hline \end{array}$

(5) $\begin{array}{r} 326 \\ \times\ 418 \\ \hline \end{array}$ (6) $\begin{array}{r} 1407 \\ \times\ 506 \\ \hline \end{array}$ (7) $\begin{array}{r} 7.04 \\ \times\ 5.2 \\ \hline \end{array}$ (8) $\begin{array}{r} 24.5 \\ \times\ 0.34 \\ \hline \end{array}$

(9) $56\overline{)3248}$ (10) $73\overline{)4964}$ (11) $208\overline{)6864}$

(12) $125\overline{)287.5}$ (13) $0.43\overline{)0.301}$ (14) $3.54\overline{)424.8}$

⑨ Please calculate the following.

(1) $\frac{3}{8}+\frac{5}{6}$ (2) $1\frac{3}{4}+\frac{3}{7}$ (3) $\frac{2}{5}-\frac{2}{7}$ (4) $5\frac{1}{3}-2\frac{2}{5}$

(5) $\frac{1}{4}\times\frac{2}{7}$ (6) $2\frac{2}{9}\times 3\frac{3}{10}$ (7) $\frac{2}{5}\div\frac{1}{4}$ (8) $3\frac{5}{7}\div 5\frac{7}{9}$

Story of Math

Representing Numbers with Computers

The numbers we use are represented by 10 numerals, from 0 to 9. Computers use only two numerals, 0 and 1. The table below shows how computers represent numbers from 0 to 9.



0	1	10	11	100	101	110	111	1000	1001
0	1	2	3	4	5	6	7	8	9

⑩ A number was supposed to be divided by 21, but by mistake it was divided by 27. This calculation led to a quotient of 508 and a remainder of 18.
What is the correct answer to the original problem?

⑪ Using the relationships $5=10\div2$, $25=100\div4$, and $125=1000\div8$, think about ways to calculate the following more efficiently.

(1) 180×5 (2) 36×25 (3) 32×125

⑫ $\frac{3}{8}$ of the book has been read, but there are still 200 pages left.
How many pages are in this book?

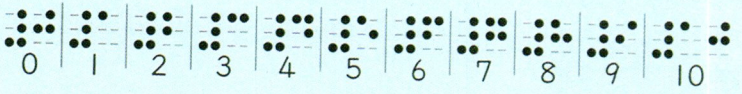


Story of Math

Numbers in Braille

In Braille, symbols are represented as arrangements of raised dots so that they can be understood by touch.
The following illustration shows numbers written in Braille.





If you multiply a whole number or a fraction by 10 or $\frac{1}{10}$, the decimal point will move by one place to the right or left, respectively.

⑬ Please write down the numbers that result from multiplying the following numbers by 10, 100, $\frac{1}{10}$, or $\frac{1}{100}$.

(1) 12.34 (2) 0.098 (3) 6,000,000

Whole numbers and decimals can be expressed as fractions.

⑭ Please fill in the ☐ with the appropriate numbers.

(1) $1 = \frac{1}{\square} = \frac{\square}{2} = \frac{7}{\square}$ (2) $3 = \frac{\square}{1} = \frac{6}{\square} = \frac{\square}{10}$

(3) $0.3 = \frac{3}{\square}$ (4) $2.8 = 2\frac{\square}{5}$ (5) $0.53 = \frac{53}{\square}$

Story of Math

Interesting Multiplication



What is 63×67?

It is 4221.



$63 \times 67 = \underset{(6+1)\times 6}{42}\ \underset{3\times 7}{21}$

Like the calculation above, if the digits in the tens place are the same and the sum of the digits in the ones place is 10, the answer can be calculated quite easily.
Write the product of the tens digit and 1 more than the tens digit on the left. Then write the product of the ones digits on the right.
Calculate 52×58, and 96×94.

There are the following kinds of fractions:

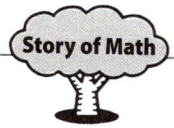
A. Fractions that can be expressed as whole numbers

B. Fractions that can be expressed as decimal numbers

C. Fractions that cannot be expressed as either whole numbers or decimal numbers

⑮ To which group above does each fraction on the right belong, A, B, or C?

$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{6}{2}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$



Simplifying Calculations

About 200 years ago, a boy named Gauss in Germany simplified the calculation of 1+2+ ... +10 in the following way:

$$1+2+3+4+5+6+7+8+9+10 = 11\times 5 = 55$$

(11, 11, 11, 11, 11)

Calculate 1+2+ ... +100 in the same way.

$$1+2+3+\cdots\cdots+99+100 = \square \times \square = \square$$

When a whole number cannot be divided evenly by another whole number, a fraction can be used to express the quotient without using a remainder.

⑯ Please write the quotients in fractions.

(1) $1 \div 3$ (2) $2 \div 3$ (3) $4 \div 3$ (4) $5 \div 3$

The reciprocal of a can be obtained by $1 \div a$.

⑰ Please find the reciprocal of each number.

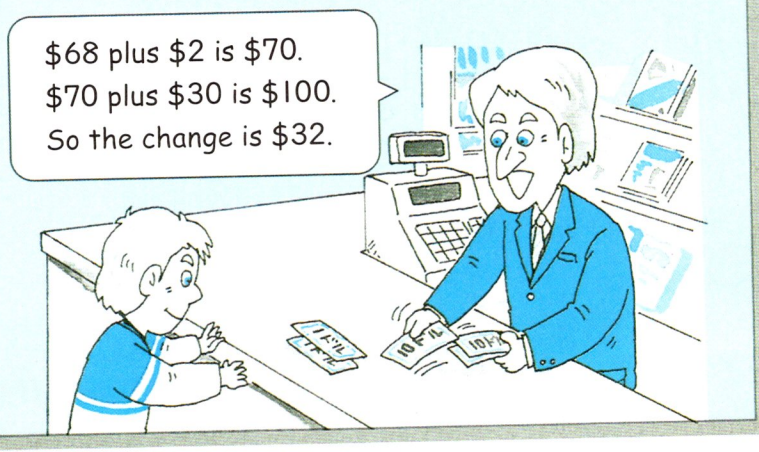
(1) 3 (2) $\frac{2}{5}$ (3) $1\frac{4}{7}$ (4) 0.9 (5) 2.4

Story of Math

Calculating Change

When we calculate change during shopping, we use subtraction. But in foreign countries, addition is sometimes used.

When you pay with a $100 bill to buy a $68 item, the change may be calculated as following.



Division is the same as multiplication by the reciprocal of the divisor.

⑱ Please rewrite the math sentences so that only multiplication is used, then calculate.

(1) $\frac{7}{12} \div \frac{14}{15} \times \frac{3}{10}$

(2) $\frac{4}{9} \div 6 \times \frac{15}{16} \div \frac{5}{18}$

Calculations of whole numbers, decimal numbers, and fractions can be done after the whole numbers and decimal numbers are converted to fractions.

⑲ Please convert the whole numbers and decimal numbers to fractions and calculate.

(1) $4.5 - 1\frac{2}{3} \times 2$

(2) $3\frac{1}{4} - 0.7 \div \frac{2}{3}$

(3) $2\frac{1}{3} \div 28 \times 0.8$

(4) $5 - \frac{4}{5} \times \frac{3}{8} + 7.2$

Story of Math

Fractions with Numerators of 1

In ancient Egypt, a fraction with a numerator of 2 was expressed as the sum of two fractions with numerators of 1, as shown below.

$$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}$$

Let's think about this in other fractions.

$\frac{2}{7} = \frac{1}{4} + \frac{1}{\square}$

$\frac{2}{9} = \frac{1}{6} + \frac{1}{\square}$

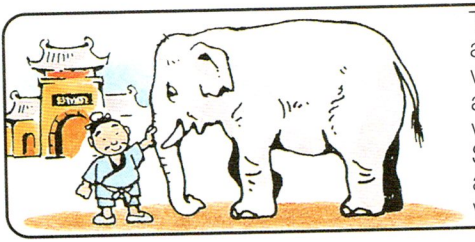
$\frac{2}{11} = \frac{1}{6} + \frac{1}{\square}$

$\frac{2}{13} = \frac{1}{\square} + \frac{1}{52} + \frac{1}{104}$

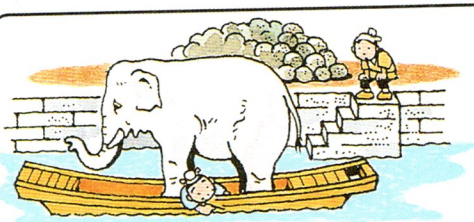
$\frac{2}{15} = \frac{1}{10} + \frac{1}{\square}$

$\frac{2}{17} = \frac{1}{\square} + \frac{1}{51} + \frac{1}{68}$

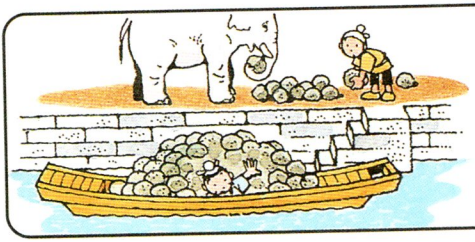
How to Measure the Weight of an Elephant



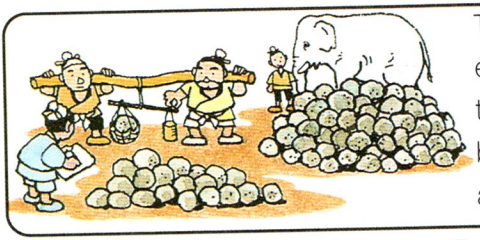
This is a story from long ago, in ancient China, when people did not have a scale to measure the weight of heavy things. Some people tried to find a way to measure the weight of an elephant.



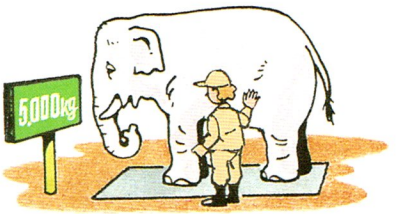
First, they put the elephant on a boat, and marked the water level on the side of the boat.



Next, they let the elephant off the boat and put some rocks on the boat until the water came up to the mark they recorded before.



Then they weighed each rock. The weight of the elephant was found by adding the weights of all of the rocks.



Today, we weigh heavy things using a metal scale buried in the ground.

Quantities include length, area, volume, weight, time, angles, etc.

① Please fill in the ☐ with the appropriate units.

(1) The area of a square with $1cm$ side is 1 ☐.

(2) The volume of a cube with $1m$ edge is 1 ☐.

In the metric system, units for length, area, volume, and weight are 10 times, 100 times, ..., or $\frac{1}{10}$, $\frac{1}{100}$, ... of each other.

② Please fill in the ☐ with the appropriate numbers.

(1) $1\,kg =$ ☐ g

(2) $1\,m =$ ☐ cm

(3) $1\,m^2 =$ ☐ cm^2

(4) $1\,m^3 =$ ☐ cm^3

Speed is represented as the distance traveled per unit of time.

③ Please think about the following questions.

(1) Write the formula to find speed if you know distance and time.

(2) What is the speed per second of an airplane that goes $900km$ in one hour?

(3) How many km does a car traveling $40km$ per hour go in 1 hour and 15 minutes?

The circumference of a circle is 3.14 times as long as the diameter.

④ Please answer the following questions.

(1) Write a formula to find the circumference if you know the length of the diameter.

(2) How long in cm is the circumference of a circle with a diameter of $10cm$?

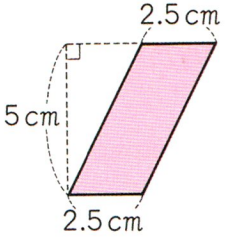
(3) How long in m is the radius of a circle with a circumference of $21.98m$?

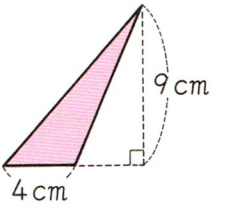
When finding the area of plane figures or the volume of solid figures, it is convenient to use formulas.

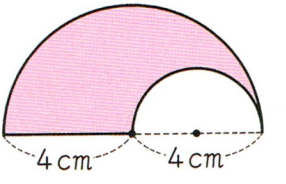
⑤ Please answer the following questions.

(1) Please write the formulas for finding the areas of rectangles, parallelograms, triangles, trapezoids, and circles.

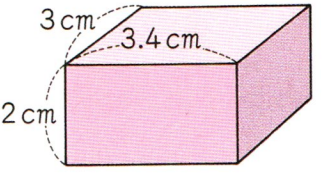
(2) Please find the area of the figures below.







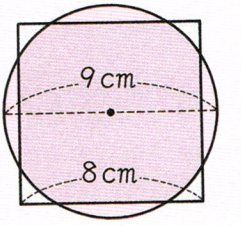
(3) Please find the volume of the rectangular prism on the right in cm^3.



Story of Math

Circle and Square

In ancient Egypt, people found the area of a circle by finding the area of a square with a side that was $\frac{8}{9}$ of the diameter of the circle. Compare the area of a circle with a diameter of $9cm$ and the area of a square with a side of $8cm$.

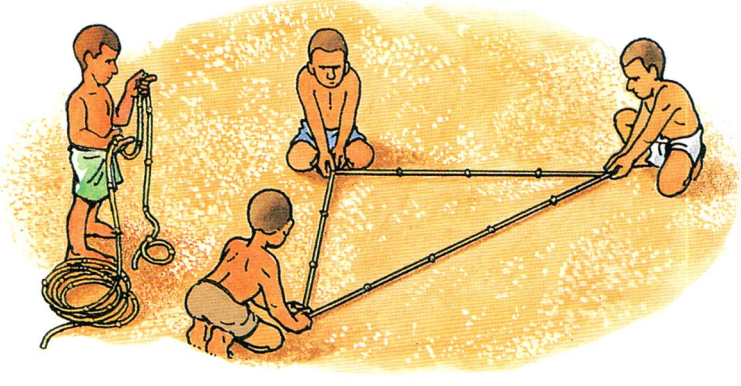


How to Make a Right Angle



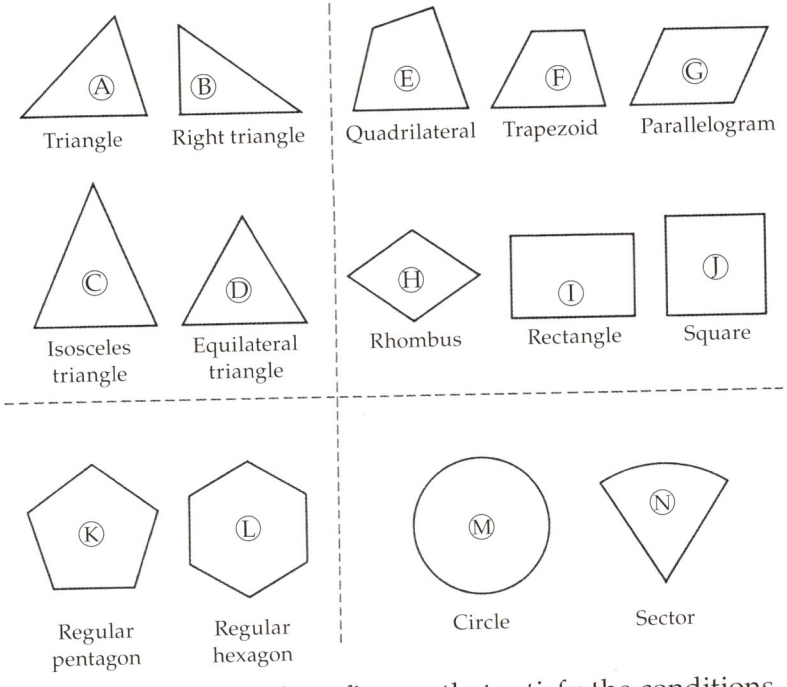
In ancient Egypt, people built pyramids with bases that were squares.

How could they make a right angle even though they did not have a tool like a set-square?



At that time, there were people whose job was to measure things by using ropes and making right angles. They made 12 evenly spaced knots on the rope and then used the rope to make a right triangle whose sides were 3 knots, 4 knots, and 5 knots long.

We have learned about plane figures like the ones shown below.



① Please choose the plane figures that satisfy the conditions below.

(1) Triangle(s) with 2 sides of equal length.

(2) Quadrilateral(s) with 2 pairs of parallel sides.

(3) Quadrilateral(s) with 2 diagonals of equal length.

(4) Quadrilateral(s) that have 2 diagonals that are perpendicular to each other.

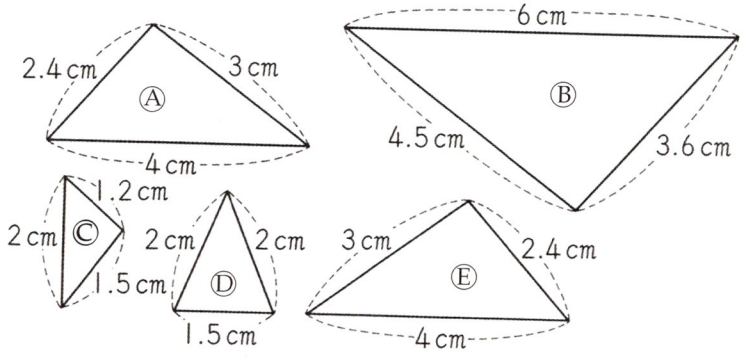
(5) Figures in which all sides have the same length and all angles have the same measure.

(6) Figures with line symmetry.

(7) Figures with point symmetry.

The shape and size of congruent figures are the same.

② Please see the figures below and answer the questions.

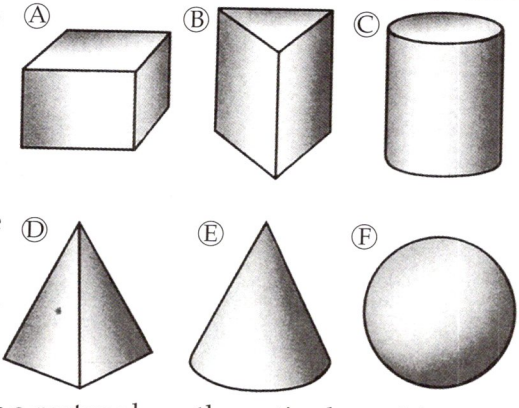


(1) Which triangle is congruent to A?

(2) Which triangles are enlarged drawings of A? Which triangles are reduced drawings of A?

We have learned about solid figures like the ones shown below.

③ Please look at the figures on the right and answer the questions.



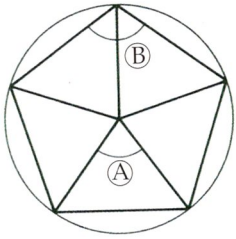
(1) Which ones are prisms?

(2) Which ones are pyramids?

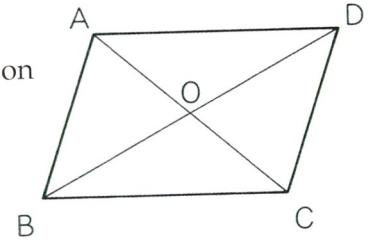
(3) Which solid has a rectangle as the entire lateral face when its net is drawn?

(4) Which one looks circular when seen from both directly in front and directly above?

④ How big are the measures of angles Ⓐ and Ⓑ of the regular pentagon on the right?



⑤ Please name all the congruent triangles in the parallelogram on the right.



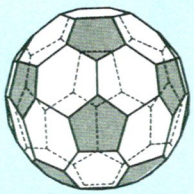
Story of Math

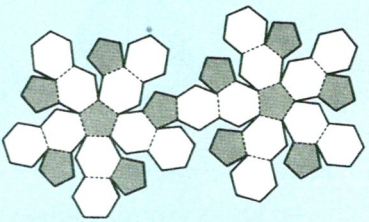
Soccer Ball



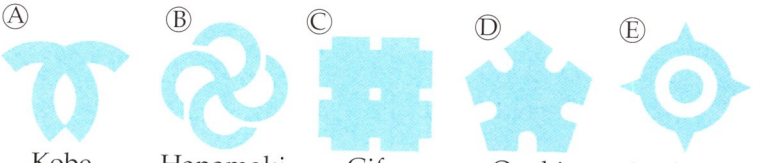
Of what kinds of figure is a soccer ball composed?
It is composed of 12 regular pentagons and 20 regular hexagons.





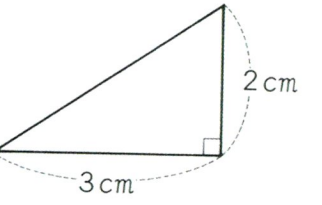


⑥ The figures below are symbols that represent various cities. Which ones have line symmetry? Which ones have point symmetry?



Kobe City　Hanamaki City　Gifu City　Ogaki City　Nichinan City

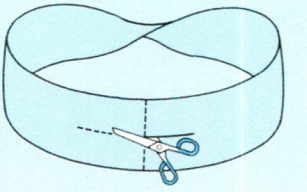
⑦ Please make an enlargement that is twice the size and a reduction that is $\frac{4}{5}$ the size of the right triangle on the right.

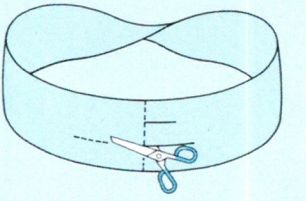


Story of Math

Moebius Strip

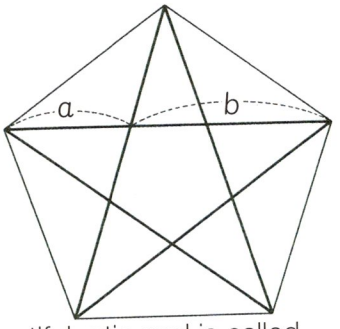
Take a strip of paper, twist the strip once and glue its ends together. Next, cut the strip down the middle lengthwise. How many rings are there? Next, make another ring with one twist and cut it lengthwise at one third of the width of the tape.
How many rings are there now?



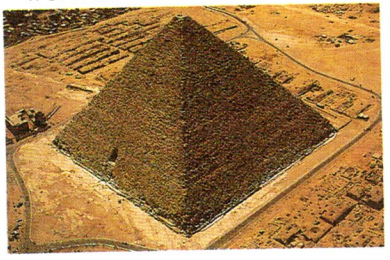


The Golden Ratio

If you draw diagonal lines in a regular pentagon like the figure on the right, you get a star shape. Find the ratio of the lengths a and b. The ratio of a to b is about 1 to 1.6.



This ratio is thought to be a beautiful ratio and is called the golden ratio. This ratio has been used in many ways from the ancient times.



Half of the length of 1 side of the base to the height of the triangular lateral surface of the pyramids in Egypt.



The height to the length of the base of the Parthenon in Greece.



The lengths of the short side to the long side of phone cards.

The inverse of addition is subtraction. The inverse of multiplication is division.

① Please find the number represented by x.

(1) $x + 2.9 = 5.5$ (2) $\frac{5}{6} \times x = \frac{7}{9}$

Even if you multiply or divide a and b in the ratio $a : b$ by the same number, the ratio stays the same.

② In the following ratios, which ones are equivalent to 4 : 6?

Ⓐ 2 : 3 Ⓑ 6 : 9 Ⓒ 8 : 10 Ⓓ 12 : 18

Percent is a way to represent ratio by using 100 as the base.

③ Please write the following ratios using percent.

(1) 0.5 (2) 1.03 (3) $\frac{3}{8}$ (4) $1\frac{1}{4}$ (5) 4-*bu* 5-*rin* (4分5厘) (6) 10-*wari* (10割)

When y is proportional to x, the equation, $y =$ fixed number $\times x$, holds true.

④ Please answer the following questions.

(1) Which of the following is proportional to the length of one side of a square?

Ⓐ The area of the square

Ⓑ The perimeter of the square

(2) When the length of one side of a square becomes 3 times longer, how many times as long as the original perimeter will the perimeter be?

When y is inversely proportional to x, the equation, y = fixed number $\div$ x, holds true.

⑤ Please answer the following questions.

(1) When you travel a fixed distance, is the time needed inversely proportional to the running speed?

(2) If you make the length of a rectangle 3 times as long without changing the area, the width becomes what fraction of the original width?

⑥ What kinds of graphs are used for the following purposes?

(1) To see how things change

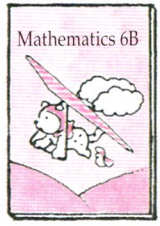
(2) To compare the sizes of things

(3) To see the spread of the data

(4) To see the ratio to the whole

Story of Math

Shapes of Pictures and Books







Let's find the ratio of the width to the length of various books! In most books, the ratio is about 1 to 1.4.

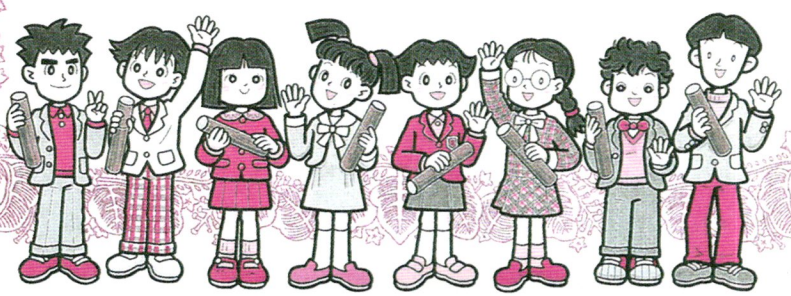
To:

Congratulations!

You have now completed all of your studies of mathematics in elementary school.

You have studied various things in the last 6 years. In the first grade, you only knew numbers up to about 100. But now you know decimals, fractions, proportions, and inverse proportions. You have also studied the properties of various shapes. You have become able to do many things because you carefully studied each topic one by one.

Next school year, you will be a junior high school student. In junior high school, you will study more advanced mathematics. It may seem difficult, but if you continue to reason based on what you have studied, like you did with mathematics in elementary school, you will be able to understand and do more things, and the world around you will expand further.



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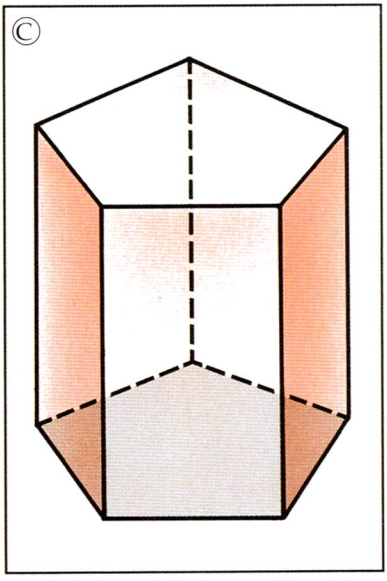

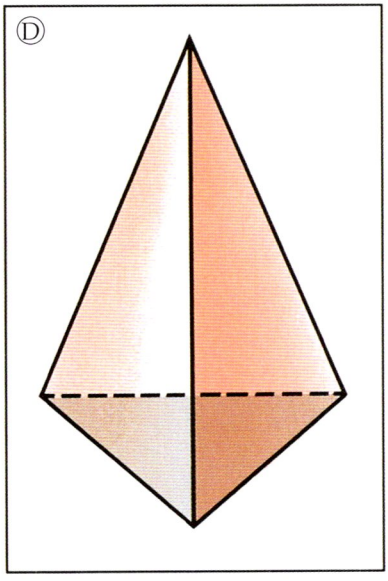

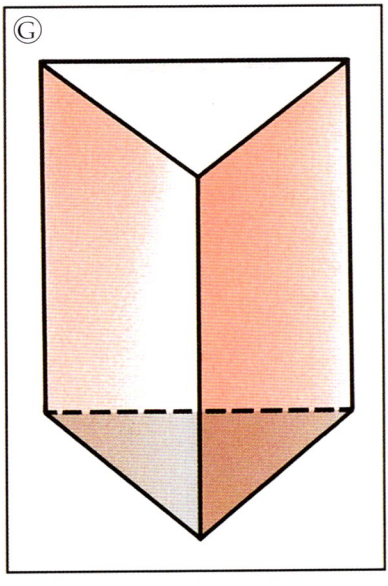

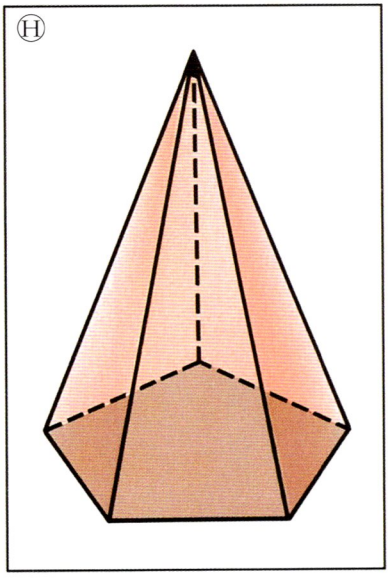

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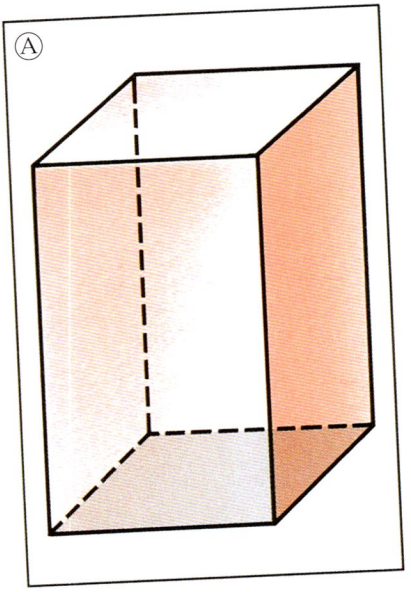

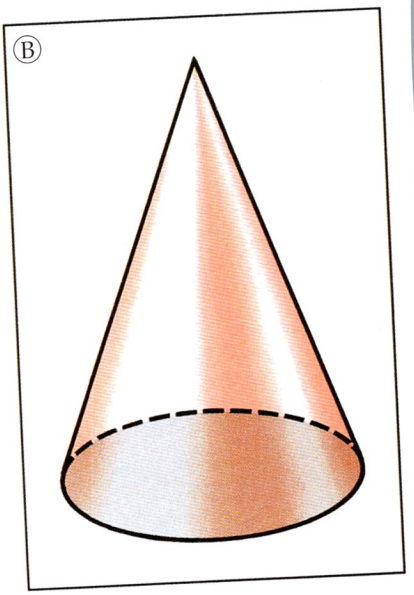

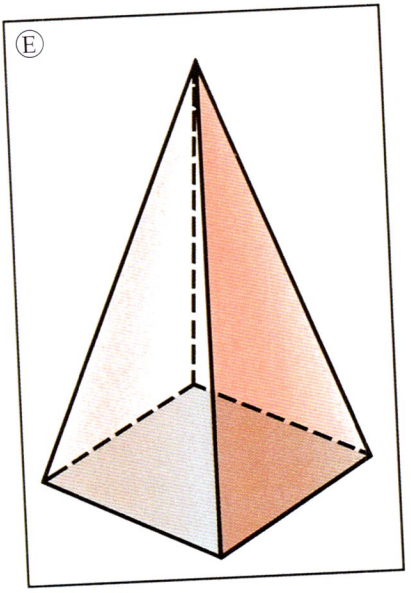

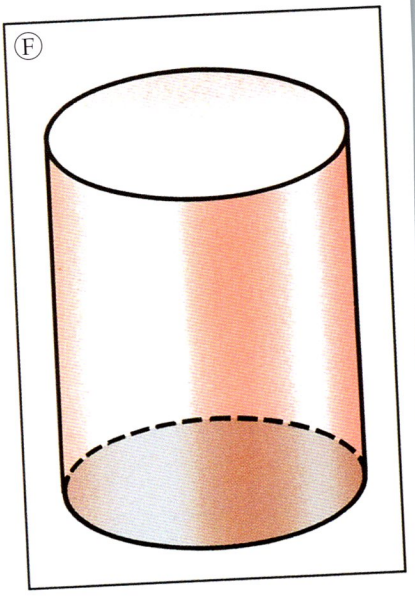

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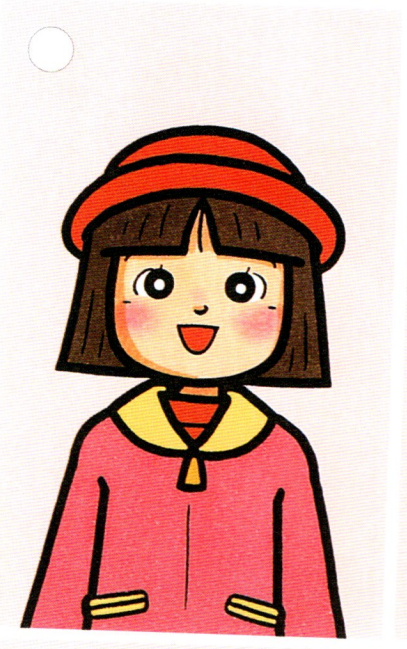
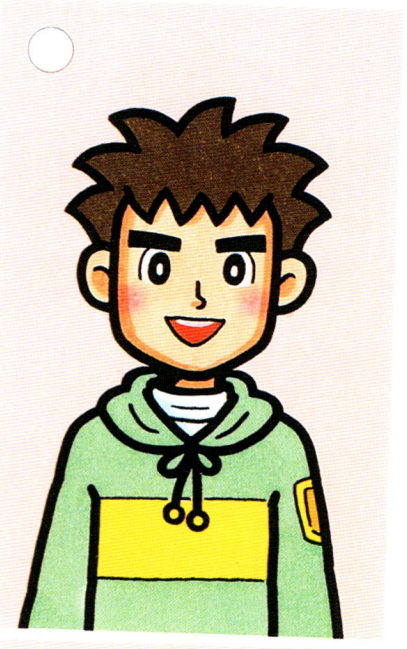
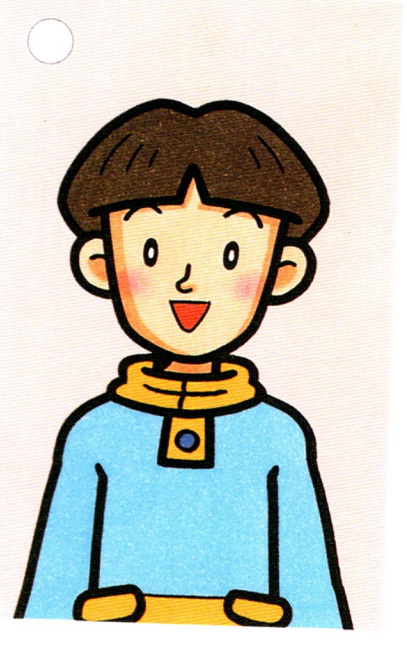
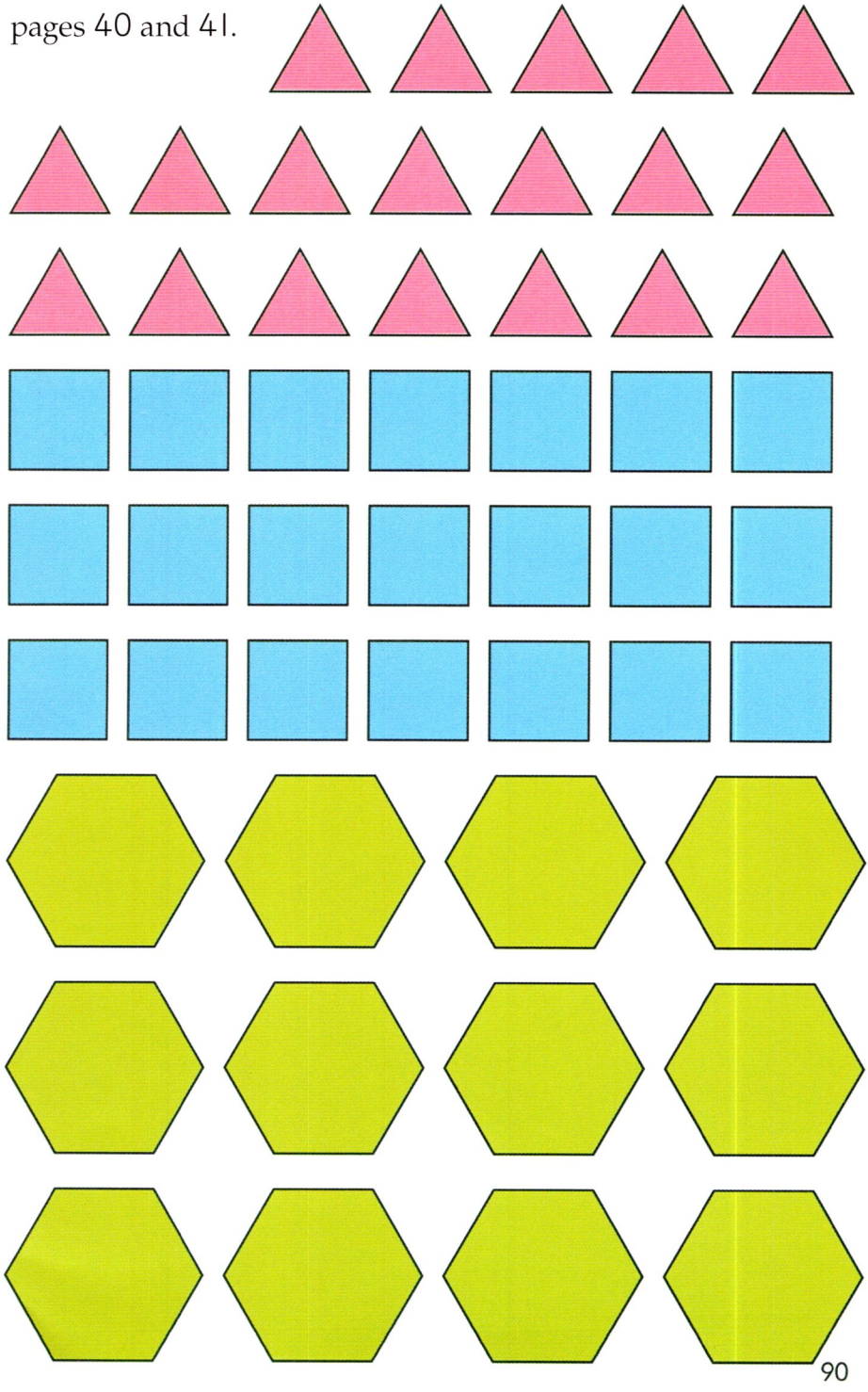






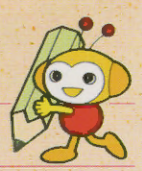


This is used on pages 40 and 41.



Memories of Mathematics

You have learned many things in mathematics classes.
Write down what you thought was good or what impressed you about learning mathematics.



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